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Entanglement of Memories and Reality in Models of Livings

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Abstract: This paper discusses quantum-inspired models of Livings from the viewpoint of the second law of thermodynamics. It concentrates on Hadamard's instability of the corresponding parabolic PDE for motions against the time arrow. The instability is removed by adding imaginary components to state variables. The evolution from present to past in a virtual (mental) space is interpreted as memories of the Livings under consideration. Quantum-like entanglement of memories and real motions are found and discussed.

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1. Introduction

This paper is based upon models of Livings introduced in our earlier publications ([1-6]) and motivated by an attempt to interpret special properties of parabolic PDE associated with these models. We will start with a brief description of mathematical models of Livings.

*a. Dynamical model of Livings***.** In this paper, the underlying dynamical model that captures behavior of Livings is based upon extension of the First Principles of classical physics to include phenomenological behavior of living systems, i.e. to develop a new mathematical formalism within the framework of classical dynamics that would allow one to capture the specific properties of natural or artificial living systems such as formation of the collective mind based upon abstract images of the selves and non-selves, exploitation of this collective mind for communications and predictions of future expected character-

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istics of evolution, as well as for making decisions and implementing the corresponding corrections if the expected scenario is different from the originally planned one. The approach is based upon our previous publications (see References) that postulate that even a primitive living species possesses additional non-Newtonian properties which are not included in the laws of Newtonian or statistical mechanics. These properties follow from a privileged ability of living systems to possess a self-image (a concept introduced in psychology) and to interact with it. The proposed mathematical formalism is quantum-inspired: it is based upon coupling the classical dynamical system representing the motor dynamics with the corresponding Liouville equation describing the evolution of initial uncertainties in terms of the probability density and representing the mental dynamics. (Compare with the Madelung equation that couples the Hamilton-Jacobi and Liouville equations via the quantum potential.)The coupling is implemented by the information-based supervising forces that can be associated with the self-awareness. These forces fundamentally change the pattern of the probability evolution, and therefore, leading to a major departure of the behavior of living systems from the patterns of both Newtonian and statistical mechanics. Further extension, analysis, interpretation, and application of this approach to complexity in Livings and emergent intelligence have been addressed in the papers referenced above.

In the next introductory sub-sections we will briefly review these models without going into mathematical details. Instead we will illustrate their performance by the Figure 1.

The model is represented by a system of nonlinear ODE and a nonlinear parabolic PDE coupled in a master-slave fashion. The coupling is implemented by a feedback that includes the first gradient of the probability density, and that converts the first order PDE (the Liouville equation) to the second order PDE (the Fokker-Planck equation). Its solution, in addition to positive diffusion, can display negative diffusion as well, and that is the major departure from the classical Fokker-Planck equation. The nonlinearity is generated by a feedback from the PDE to the ODE. As a result of the nonlinearity, the solutions to PDE can have attractors (static, periodic, or chaotic) in *probability* space. The multi-attractor limit sets allow one to introduce an extension of neural nets that can converge to a prescribed type of a stochastic process in the same way in which a regular neural net converges to a prescribed deterministic attractor. The solution to ODE represents another major departure from classical ODE: due to violation of Lipchitz conditions at states where the probability density has a sharp value, the solution loses its uniqueness and becomes random. However, this randomness is controlled by the PDE in such a way that each random sample occurs with the corresponding probability, (see Fig. 1).

The model represents a fundamental departure from both Newtonian and statistical mechanics. In particular, negative diffusion cannot occur in isolated systems without help of the Maxwell sorting demon that is strictly forbidden in statistical mechanics. The only conclusion to be made is that the model is non-Newtonian, although it is fully consistent with the theory of differential equations and stochastic processes. Strictly speaking, it is a matter of definition weather the model represents an isolated or an open system since the

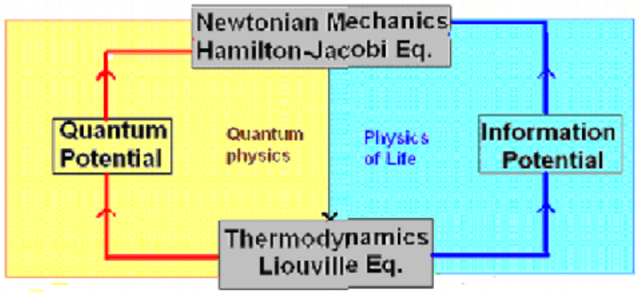

Figure 1 . Classical Physics, Quantum physics, and Physics of Life

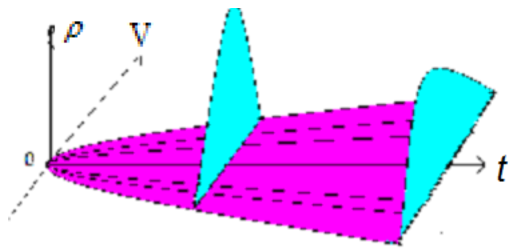

Figure 2 Stochastic process and probability density

additional energy applied via the information potential is generated by the system "itself" out of components of the probability density. In terms of a *topology* of its dynamical structure, the proposed model links to quantum mechanics: if the information potential is replaced by the quantum potential, the model turns into the Madelung equations that are equivalent to the Schrödinger equation. The system of ODE describes a mechanical motion of the system driven by *information* forces. Due to specific properties of these forces, this motion acquires characteristics similar to those of quantum mechanics. These properties are discussed below. The most important property is *Superposition*. In quantum mechanics, any observable quantity corresponds to an eigenstate of a Hermitian linear operator. The linear combination of two or more eigenstates results in quantum superposition of two or more values of the quantity. If the quantity is measured, the state will be randomly collapsed onto one of the values in the superposition (with a probability proportional to the square of the amplitude of that eigenstate in the linear combination). Let us compare the behavior of the model of Livings from that viewpoint, Fig. 3.

As follows from Fig. 3, all the particular solutions intersect at the same point $v = 0$ at $t = 0$,and that leads to non-uniqueness of the solution due to violation of the Lipcshitz condition. Therefore, the same initial condition$v = 0$ at $t = 0$yields infinite number of different solutions forming a family; each solution of this family appears with a certain probability guided by the corresponding Fokker-Planck equation. For instance, in case of the solution plotted in Fig. 3, the "winner" solution is $v \equiv 0$since it passes through the maxima of the probability density. However, with lower probabilities, other solutions of the same family can appear as well. Obviously, this is a non-classical effect. Qualitatively, this property is similar to those of quantum mechanics: the system keeps all the solutions simultaneously and displays each of them "by a chance", while that chance is controlled

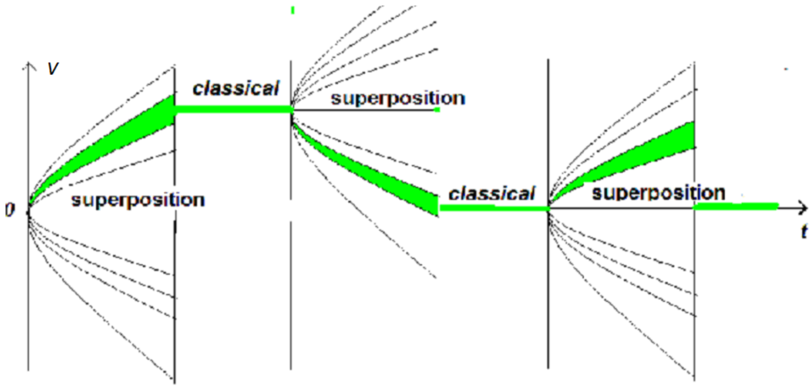

Figure 3 Switching between superposition and classical states

by the evolution of probability density. It should be emphasized that the choice of displaying a certain solution is made by the Livings model only once, and in particular, at the instant of time when the feedback is removed and the dynamical system becomes a Newtonian's one. Therefore, the removal of the feedback can be associated with a quantum measurement. Modified versions of such quantum properties as uncertainty and entanglement are also described in the referenced papers.

The model illuminates the "border line" between living and non-living systems. The model introduces a biological particle that, in addition to Newtonian properties, possesses the ability to process information. The probability density can be associated with the *self-image* of the biological particle as a member of the class to which this particle belongs, while its ability to convert the density into the information force - with the *self-awareness* (both these concepts are adopted from psychology). Continuing this line of associations, the equation of motion can be identified with a motor dynamics, while the evolution of density –with a mental dynamics. Actually the mental dynamics plays the role of the Maxwell sorting demon: it rearranges the probability distribution by creating the information potential and converting it into a force that is applied to the particle. One should notice that mental dynamics describes evolution of the whole class of state variables (differed from each other only by initial conditions), and that can be associated with the ability to generalize that is a privilege of living systems. Continuing our biologically inspired interpretation, it should be recalled that the second law of thermodynamics states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (that is the state with the highest entropy) follows from a simple combinatorial analysis. However, this statement is correct only if there is no Maxwell' sorting demon, i.e., nobody inside the system is rearranging the probability distributions. But this is precisely what the Liouville feedback is doing: it takes the probability density $\rho$from the mental dynamics, creates functions of this density, converts them into a force and applies this force to the equation of motor dynamics. As already mentioned above, because of that property of the model, the evolution of the probability density becomes nonlinear, and the entropy may decrease "against the second law of thermodynamics", Fig.4. Obviously the last statement should not be taken literary; indeed, the proposed model captures only those aspects of the living systems that are associated with their behavior, and in particular, with their motor-mental dynamics, since other properties are beyond the dynamical formalism. Therefore, such physiological processes that are needed for the metabolism are not included into the model. That is why this model is in a formal disagreement with the second law of thermodynamics while the living systems are not. In order to further illustrate the connection between the life/non-life discrimination and the second law of thermodynamics, consider a small physical particle in a state of random migration due to thermal energy, and compare its diffusion i.e. physical random walk, with a biological random walk performed by a bacterium. The fundamental difference between these two types of motions (that may be indistinguishable in physical space) can be detected in probability space: the probability density evolution of the physical particle is always linear and it has only one attractor: a stationary stochastic process where the motion is trapped. On the contrary, a *typical* probability density evolution of a biological particle is nonlinear: it can have many different attractors, but eventually each attractor can be departed from without any "help" from outside.

That is how H. Berg, [7], describes the random walk of an E. coli bacterium:" If a cell can diffuse this well by working at the limit imposed by rotational Brownian movement, why does it bother to tumble? The answer is that the tumble provides the cell with a mechanism for *biasing its random walk*. When it swims in a spatial gradient of a chemical attractant or repellent and it happens to run in a favorable direction, the probability of tumbling is reduced. As a result, favorable runs are extended, and the cell diffuses with drift". Berg argues that the cell analyzes its sensory cue and generates the bias *internally*, by changing the way in which it rotates its flagella. This description demonstrates that actually a bacterium interacts with the medium, i.e., it is not isolated, and that reconciles its behavior with the second law of thermodynamics. However, since these interactions are beyond the dynamical world, they are incorporated into the proposed model via the self-supervised forces that result from the interactions of a biological particle with "itself," and that formally "violates" the second law of thermodynamics. Thus, this model offers a unified description of the progressive evolution of living systems.

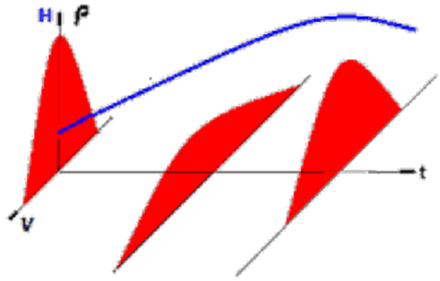

Figure 4. Deviation from thermodynamics

2. Analytical Formulation

The proposed model that describes *mechanical behavior* of a Living can be presented in the following compressed invariant form

$$\dot{v} = -\zeta\alpha \bullet \nabla_v \ln \rho, \tag{1}$$

$$\dot{\rho} = \zeta \nabla_V^2 \rho \bullet \bullet \alpha, \tag{2}$$

where ν is velocity vector, ρ is probability density, ζ is universal constant, and α *(D,w)* is a tensor co-axial with the tensor of the variances D that may depend upon these variances. Eq. (1) represents the second Newton's law in which the physical forces are replaced by information forces via the gradient of the information potential$\Pi = \zeta \ln \rho$, while the constant ζ connects the information and inertial forces formally replacing the Planck constant in the Madelung equations of quantum mechanics. Eq. (2) represent the continuity of the probability density (the Liouville equation), and unlike the classical case, it is non-linear because of dependence of the tensor α upon the components of the variance D. This model is equipped by a set of parameters w that control the properties of the solutions discussed above. The only realistic way to reconstruct these parameters for an object to be discovered is to solve the inverse problem: given time series of sensor data describing dynamics of an unknown object, find the parameters of the underlying dynamical model of this object within the formalism of Eqs(1) and (2). As soon as such a model is reconstructed, one can predict future object behavior by running the model ahead of actual time as well as analyze a hypothetical (never observed) object behavior by appropriate changes of the model parameters. But the most important novelty of the proposed approach is the capability to detect Life that occurs if, at least, some of "non-Newtonian" parameters are present. The methodology of such an inverse problem is illustrated in Figure 5.

Our further analysis will be based upon the simplest version of the system (1), (2)

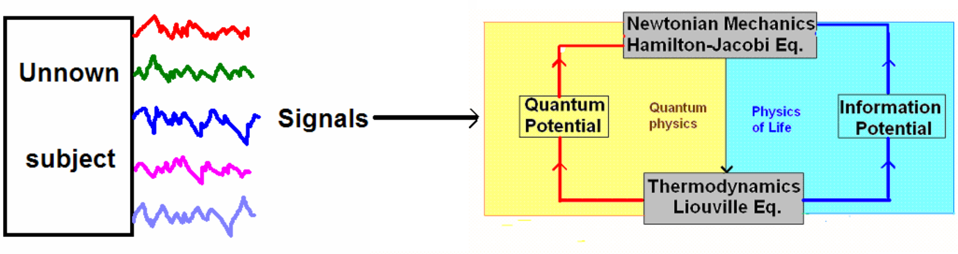

Figure 5. Data-driven model discovery.

$$\dot{v} = -\sigma^2 \frac{\partial}{\partial v} \ln \rho, \tag{3}$$

$$\frac{\partial \rho}{\partial t} = \sigma^2 \frac{\partial^2 \rho}{\partial V^2} \tag{4}$$

Here v stands for the velocity, and σ^2 is the constant diffusion coefficient.

Remark 1. Here and below we make distinction between the random *variable* $v(t)$ and its *values* V in probability space.

The solution of Eq. (3) subject to the sharp initial condition

$$\rho = \frac{1}{2\sigma\sqrt{\pi t}} \exp(-\frac{V^2}{4\sigma^2 t}) \tag{5}$$

describes diffusion of the probability density. Substituting this solution into Eq. (3) at $\boldsymbol{V=v}$ one arrives at the differential equation with respect to $v(t)$

$$\dot{v} = \frac{v}{2t} \tag{6}$$

and, therefore,

$$v = C\sqrt{t} \tag{7}$$

where C is an arbitrary constant. Since $v = 0$ at $t = 0$ for any value of C, the solution (7) is consistent with the sharp initial condition for the solution (5) of the corresponding Liouville equation (4).

The solution (7) describes the simplest irreversible motion: it is characterized by the "beginning of time" where all the trajectories intersect (that results from the violation of the Lipcsitz condition at $t = 0$, Figure 2), while the backward motion obtained by replacement of t with $(-t)$ leads to imaginary values of velocities. One can notice that the probability density (5) possesses the same properties. Further analysis of the solution (7) demonstrates that it is *unstable* since

$$\frac{d\dot{v}}{dv} = \frac{1}{2t} > 0 \tag{8}$$

And, therefore, an initial error always grows generating *randomness.* Initially, at $t = 0$, this growth is of infinite rate since the Lipchitz condition at this point is violated

$$\frac{d\dot{v}}{dv} \to \infty \text{ at } t \to 0 \tag{9}$$

3. Modification of the Model

One of the most distinct properties of the model (3), (4) (as well as its general version (1), (2)) is the irreversibility. This property can be linked to ill-posedness of parabolic PDE if they are considered backward in time, (see Appendix). The irreversibility has a clear interpretation in physics: it represents a violation of the second law of thermodynamics that states that in an isolated system entropy cannot decrease. However in *physics of Livings*, the interpretation could be different if one is interested in evolution of memories that is directed toward the past. This will be the main subject of our paper.

Let us turn to the solutions (5) and (7) and try to find the values of ρ and v at some negative time $(-t)$. One finds that the both values are imaginary

$$\rho = -i\frac{1}{2\sigma\sqrt{\pi t}} \exp(\frac{V^2}{4\sigma^2 t}) \tag{10}$$

$$v = iC\sqrt{t} \tag{11}$$

This suggests a hint that we can get rid of ill-posedness by enlarging the class of functions (in which $\rho$and $v$are sought) from real to complex one, i.e. by introducing complex variables

$$\rho = \rho_1 + i\rho_2 \tag{12}$$

$$v = v_1 + iv_2 \tag{13}$$

It is reasonable to redefine the normalization constraint for the probability density as

$$\int\limits_{-\infty}^{\infty} |\rho| d|V| = 1 \tag{14}$$

where

$$|\rho| = \sqrt{\rho_1^2 + \rho_2^2}, \text{ and } |v| = \sqrt{v_1^2 + v_2^2} \tag{15}$$

are the modules of the corresponding complex variables.

Let us introduce the following simplification by assuming that

$$\rho_1, v_1 = 0 \text{ at } t < 0 \tag{16}$$

$$\rho_2, v_2 = 0 \text{ at } t > 0 \tag{17}$$

Then we can reduce the original problem to a much simpler one rewriting Eq. (4) in the form

$$\frac{\partial \rho_1}{\partial t} = \sigma^2 \frac{\partial^2 \rho_1}{\partial V_1^2} \tag{18}$$

$$i\frac{\partial \rho_2}{\partial(-t)} = i\sigma^2 \frac{\partial^2 \rho_2}{\partial(iV_2)^2} \text{ i.e.}$$

$$\frac{\partial \rho_2}{\partial t} = \sigma^2 \frac{\partial^2 \rho_2}{\partial V_2^2} \tag{19}$$

Thus after reformulating the problem in the enlarged class of functions, one arrived at equivalent system of two PDE that a well-posed in the corresponding time regions.

Eq. (3) now can be rewritten in the following form

$$\dot{v}_1 = -\sigma^2 \frac{\partial}{\partial v_1} \ln \rho_1, \tag{20}$$

$$-i\dot{v}_2 = -\sigma^2 \frac{\partial}{\partial(iv_2)} \ln \rho_2, \text{ i.e.}$$

$$\dot{v}_2 = -\sigma^2 \frac{\partial}{\partial v_2} \ln \rho_2, \tag{21}$$

Now one can see that Eqs. (18) and (19) play the role of the Liouville equations for Eqs. (20) and (21), respectively.

In order to write a solution of the system (18), (19), (20), and (21), we have to reformulate the initial condition

$$\int_{-\infty}^{\infty} \sqrt{\rho_1^2+\rho_2^2} d\sqrt{V_1^2+V_2^2} = \delta(0) \text{ at } t=0 \tag{22}$$

whence

$$\int_{-\infty}^{\infty} \rho_1 dV_1 = \lambda_1\delta(0), \int_{-\infty}^{\infty} \rho_2 dV_2 = \lambda_2\delta(0) \text{ at } t=0 \tag{23}$$

where λ_1 and λ_2 are constants to be found from the normalization constraint.

Since the regions where the variables are defined have one point in common, that is $t=0$ (see Eqs. (16) and (17)), the initial condition (23) should satisfy the normalization constraint that couples these two system (see Eq. (14))

$$\int_{-\infty}^{\infty} \sqrt{\rho_1^2+\rho_2^2} d\sqrt{V_1^2+V_2^2} = 1 \tag{24}$$

i.e.

$$\int_{-\infty}^{\infty} \rho_1 dV_1 = \lambda_1 \text{ at } t>0 \tag{25}$$

$$\int_{-\infty}^{\infty} \rho_2 dV_2 = \lambda_2 \text{ at } t<0 \tag{26}$$

$$\int_{-\infty}^{\infty} \rho_1 dV_1 + \int_{-\infty}^{\infty} \rho_2 dV_2 = \lambda_1\delta(0)+\lambda_2\delta(0) = 1 \text{ at } t=0 \tag{27}$$

whence

$$\lambda_1+\lambda_2=1, \text{ i.e. } \lambda_1=\lambda, \lambda_2=1-\lambda \tag{28}$$

Thus one arrives at the following system of differential equations

$$\frac{\partial\rho_1}{\partial t} = \sigma^2\frac{\partial^2\rho_1}{\partial V_1^2}, t\geq 0, \tag{29}$$

$$\dot{v}_1 = -\sigma^2\frac{\partial}{\partial v_1}\ln\rho_1, \quad t\geq 0 \tag{30}$$

$$\frac{\partial\rho_2}{\partial t} = \sigma^2\frac{\partial^2\rho_2}{\partial V_2^2}, t\leq 0 \tag{31}$$

$$\dot{v}_2 = -\sigma^2\frac{\partial}{\partial v_2}\ln\rho_2, t\leq 0 \tag{32}$$

to be solved subject to initial conditions

$$\int_{-\infty}^{\infty} \rho_1 dV_1 = \lambda_1 \delta(0), \int_{-\infty}^{\infty} \rho_2 dV_2 = \lambda_2 \delta(0) \text{ at } t = 0 \tag{33}$$

and a global constraint that

$$\int_{-\infty}^{\infty} \rho_1 dV_1 = \lambda, \int_{-\infty}^{\infty} \rho_2 dV_2 = 1 - \lambda, 0 \leq \lambda \leq 1 \tag{34}$$

In order to deal with the constraint (34), let us integrate Eq. (29) over the whole space assuming that $\rho_1 \to 0$ and $|d\rho_1/dV_1| \to 0 at |V_1| \to \infty$ Then

$$\frac{\partial}{\partial t} \int_{-\infty}^{\infty} \rho_1 dV_1 = 0, \int_{-\infty}^{\infty} \rho_1 dV_1 = const, \tag{35}$$

Hence, the constraint (34) is satisfied for $t > 0$ if it is satisfied for $t = 0$. Obviously the same is true for the variable ρ_2 . But according to Eq. (27), the global constraint is satisfied for $t = 0$. Therefore it is satisfied for the entire time interval.

Now the solution can be written as following

$$\rho_1 = \frac{\lambda}{2\sigma\sqrt{\pi t}} \exp(-\frac{V_1^2}{4\sigma^2 t}), t \geq 0 \tag{36}$$

$$v_1 = C\sqrt{t}, t \geq 0 \tag{37}$$

$$\rho_2 = \frac{1-\lambda}{2\sigma\sqrt{\pi|t||}} \exp(-\frac{V_2^2}{4\sigma^2 |t|}), t \leq 0 \tag{38}$$

$$v_2 = C\sqrt{|t|}, t \geq 0 \tag{39}$$

4. Analysis and Interpretation of the Model

The purpose of the modification performed above was to interpret and to remove an abnormal behavior of the model of Livings when time is running backward: the solution becomes Hadamard unstable and that leads to ill-posedness of the Cauchy problem. In physics, similar abnormalities could be anticipated since they represent violation of the second law of thermodynamics. However in the model of Livings, a more constructive interpretation can be suggested if the backward-in-time "evolution" is associated with memories of the living subject. Fig.6.

It is reasonable to assume that the family of trajectories in the solution (37) describes the real actions in physical space that evolve from Present to Future, while the family of trajectories in the solution (39) describes virtual actions in the "imaginary" space that evolve from Present to Past, and that can be associated with memories. As follows from the solutions (36)-(39), the family of real trajectories (37) and memory trajectories

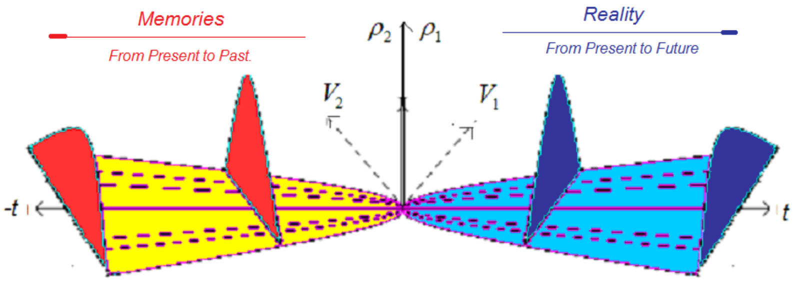

Figure 6. Entanglement of memories and reality.

(39) are mirror-symmetric with respect to the plane t = 0. However, realization of each of these trajectories is random: it is controlled by the probability densities (36) and (38), respectively. A mechanism of such random realization was described above (see the comments to Fig.3). As will be shown below, the occurrence of these trajectories, in general, is not necessarily simultaneous, and these trajectories are not necessarily identical. Indeed, as follows from Eqs. (36) and (38), simultaneous occurrence of real action and memory has the probability

$$\rho_{12} = \lambda(1-\lambda), \quad 0 \le \rho_{12} \le \frac{1}{4} \tag{40}$$

Occurrence of only real action has the probability

$$\rho_1 = \lambda \tag{41}$$

and occurrence of only memory (sleep dream) has the probability

$$\rho_2 = 1 - \lambda \tag{42}$$

Occurrence of action *or* memory has the probability one

$$\rho_1 + \rho_2 = 1. \tag{43}$$

Here the free parameter $\lambda$is a specific property of the Livings under consideration

$$0 \le \lambda \le 1 \tag{44}$$

Thus the real actions and memories are entangled via the probability of their occurrence: with the highest probability they can be identical, or similar, but with non-zero probability they can be different. Such a "weak" entanglement results from the global normalization constraint (43) followed from Eq. (34). And even for the simplest model under consideration, the concept of weak entanglement fits well into "relationships" between memories and reality as we know from human experience.

Turning to the general model of Livings described by Eqs. (1) and (2), we should recall that as shown in [5], its solution being considered for small times possesses the same ill-posedness, i.e. it is Hadamard unstable when small time is reversed, and therefore, all the results described above are applicable to the general case. However, if Eqs. (1) or (2) include *even* functions of time of state variables, then the mirror-symmetry could be broken leading to a significant difference between the memories and reality. This difference can be exploited for description of absurdity of sleep dreams in case (42).

Conclusion

Reconciliation of evolution of living systems with the second law of thermodynamics still attracts attention of mathematicians, physicists and biologists. In 1944, Schredinger wrote[8]“life is to create order in the disordered environment against the second law of thermodynamics”. This paper represents the next step in analysis of the connection between models of Livings and the second law of thermodynamics. It discusses quantum-inspired models of Livings from the viewpoint of the second law of thermodynamics. It concentrates on Hadamard’s instability of the corresponding parabolic PDE for motions against the time arrow. The instability is removed by adding imaginary components to state variables. The evolution from present to past in a virtual (mental) space is interpreted as memories of the Livings under consideration. Quantum-like entanglement of memories and real motions are found and discussed.

Appendix

Negative diffusion and Hadamard instability

Since a parabolic PDE with negative diffusion coefficients is of fundamental importance for the proposed model, we will take a closer look at its properties associated with the so called Hadamard’s instability, or the ill-posedness of the initial value problem. Without loss of generality, the analysis will be focused on the one-dimensional case.

Consider a parabolic PDE

$$\frac{\partial \rho}{\partial t} = -q^2 \frac{\partial^2 \rho}{\partial X^2} \tag{A1}$$

subject to the following initial conditions

$$\rho^{00} = \rho|_{t=0} = \{\frac{\frac{1}{\lambda_0^2}\sin\lambda_0 X \text{ if } |X| \leq X_0}{0 \text{ if } |X| > X_0}\} \tag{A2}$$

with the parameter λ_0 being made as large as necessary,i.e

$$\lambda_0 \to \infty \tag{A3}$$

The region of the initial disturbance can be arbitrarily shrunk, i.e.

$$|X_0| \to 0 \tag{A4}$$

The solution to Eq. (A1) can be sought in the form

$$\rho = \frac{1}{\lambda_0^2} e^{\lambda \Delta t} \sin \lambda_0 X. \tag{A5}$$

Substituting this solution into Eq. (A1), one obtains

$$\lambda = q^2 \lambda_0^2 \to \infty \text{ at } \lambda_0 \to \infty. \tag{A6}$$

Thus, the solution to Eq. (A1) subject to the initial conditions (A2) is

$$\rho = \frac{1}{\lambda_0^2} e^{q^2 \lambda_0^2 \Delta t} \sin \lambda_0 X \tag{A7}$$

This solution has very interesting properties: its modulus tends to infinity if

$$\lambda_0 \to \infty \tag{A8}$$

within an arbitrarily short period of time Δt_0 and within an infinitesimal length around the point $X = X_0$. In other words, vanishingly small changes in initial conditions lead to unboundedly large changes in the solution during infinitesimal period of time.

The result formulated above was obtained under specially selected initial conditions (A2), but it can be generalized to include any initial conditions. Indeed, let the initial conditions be defined as

$$\rho|_{t=0} = \rho * (X) \tag{A9}$$

and the corresponding solution to Eq. (A1) is

$$\rho = \rho * *(X, t) \tag{A10}$$

Then, by altering the initial conditions to

$$\rho|_{t=0} = \rho * (X) + \rho^{00}(X) \tag{A11}$$

where ρ^{00}(x) is defined by Eq. (A2), one observes the preceding argument by superposition that vanishingly small change in the initial condition (A9) leads to unboundedly large change in the solution (A10) that occurs during an infinitesimal period of time. Such an unattractive property of the solution (that represents so called Hadamard's instability) repelled scientists from using Eq. (A1) as a model for physical phenomena. However, the situation becomes different if the variable ρ in Eq. (A1) cannot be negative, i.e. when Eq. (A1) is complemented by the constraint

$$\rho \geq 0 \tag{A12}$$

This constraint is imposed, for instance, when ρ stands for the probability density, or for the absolute temperature. It is easily verifiable that the proof of the Hadamard's instability presented above fails if the constraint (A12) is imposed, since negative values of ρ is essential for that proof. Thus, if the models of negative diffusion have attractors separating positive and negative areas of the solutions, they are free of the Hadamard's instability.

References

[1] Zak, M. 2005, From Reversible Thermodynamics to Life. Chaos, Solitons&Fractals, 1019-1033,

[2] Zak, M., 2006 Expectation-based intelligent control, Chaos, Solitons&Fractals, 28, 5. 616-626,

[3] Zak, M., 2007, Complexity for Survival of Livings, Chaos, Solitons&Fractals, 32, 3, 1154-1167

[4] Zak, M., 2007, From quantum entanglement to mirror neuron, Chaos, Solitons&Fractals, 34, 344-359.

[5] Zak, M., 2007, Physics of Life from First Principles, EJTP **4**, No. 16(II)(2007) 11–96

[6] Zak, M., 2010, Introduction to quantum-inspired intelligence, PHYSICS ESSAYS **23**, 1.

[7] Berg,H., 1983, Random walks in biology, Princeton Univercity Press, New Jersey.

[8] Schrödinger, E**.** What is life, Cambridge University Press, 1944.

[9] Prigogine, I. 1961, Thermodynamics of irreversible processes, John Wily,

[10] Haken,H. 1988, Information and self-organization, Springer, N. Y,