The Lagrange-Gordeyev's Method and the Lorentz-Dirac Equation

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Abstract: By using Lagrange-Gordeyev's method in order to calculate the retarded and advanced electromagnetic fields, the Lorentz-Dirac equation is obtained. Since the electromagnetic radiation field is represented by an average between the retarded and the advanced fields, the mass renormalization is avoided. Landau-Lifshitz equation of motion for a spinless charged point particle is also derived.

Keywords: Lagrange-Gordeyev's Method; Lorentz-Dirac Equation; Electromagnetic Fields; Radiation; Landau-Lifshitz Equation
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1. Introduction

Based on a Laplace expansion for retarded functions, Gordeyev [1] developed a technique which describes the electromagnetic fields by using simultaneous characteristics of the motion of the charge. A non quantal relativistic statistical physics was supposed to be described by this method. However, due to the complicated expansion, Gordeyev [1], himself, abandoned a statistical applications. It is possible that he was not aware about the non-interacting theorem [2]. However, he obtained, in a non rigorous form, the Lorentz-Dirac [LD] equation [3] of motion for a point charged particle. However, by means of the Gordeyev's approach, the flux of energy through a sphere which contains a point charged particle may be calculated. Consequently, by considering the effect of the

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motion of the sphere in the integrals [4], the Abraham [A] equation [5] has been obtained by Ares de Parga et al. [6]. Nevertheless, the mass renormalization was not avoided. Even this approach is not relativistic, by using the Ansatz of Pauli, the deduction turns covariant and the LD equation is obtained based on the Gordeyev expansion [1].

By using a surface integral formalism developed by Penrose [8], Unruh [9] obtained the LD equation. The same result has been found by Wheeler and Feynman [10] by defining a perfect absorber universe as one in which all radiation is eventually absorbed due to interactions with matter. In both cases the mass renormalization is avoided. However, even if there are other elegant derivations which use the average technique in order to find the LD equation, it is important to mention the derivation done by Teiltelboim [11] and Plebañski [12]. This average technique is so powerful that López-Bonilla et al [13] used it in order to obtain the equation of motion in a General Relativity with an arbitrary curvature for a spinless point particle, that is, Hobbs [14] [H] equation of motion.

However, LD equation presents many physical difficulties as the runaway solutions and the preaccelerations. Therefore, with over the years many different proposals have appeared; the most important are described in the book of Parrot´s book [15]. Nowadays, the Landau-Lifshitz [LL] equation [16] of motion has been considered by many authors as the correct equation which describes the motion of a spinless point particle [17], [18]. However, many discussions about the validity of the LL proposal have been done in the last years. Among others, we can cite the interesting analysis done by Baylis and Huschilt [19] where the validity of the use of the LL equation has been questioned and a mixed LD and LL theory is proposed. Consequently, Ares de Parga [20] and later Medina [21] have argued in favor of changing the classical relativistic Larmor formula claiming compatibility with the LL equation. It is important to mention the review works done by Hammond [22] and Griffith [23], where both articles undertake some numerical studies to determine which of the two equation, the LD or the LL equation, is superior. Finally, the average method has been used by Quinn and Wald [24] to obtain a generalization of the LL equation in general relativity.

The main purpose of the article consists of obtaining the Abraham equation just by using the average of the retarded and the advanced fields obtained with the Lagrange-Gordeyev method [1] at first order in the velocity. By applying the Pauli Anzats [7], the result is generalized to special relativity and LD equation [3] is obtained. The LL equation is deduced through a review of the meaning of radiation term.

The paper is organized as follows: Sec. 2 is advocated to calculate the retarded and advanced electromagnetic fields in a sphere of radius R. In Sec. 3, the flux of energy through a sphere moving along the path of the charge is calculated. The Abraham equation of motion is deduced without mass renormalization. Consequently, the Lorentz-Dirac equation is derived. LL equation is deduced in Sec. 4. Some concluding remarks are exposed in Sec. 5.

2. The Lagrange Gordeyev method

By considering a function $u(y)$ with $y = x + \alpha\phi(y)$, Gordeyev [1] found that

$$u(y) = u(x) + \sum_{k=1}^{\infty} \frac{\alpha^k}{k!} \frac{\partial^{k-1}}{\partial x^{k-1}} \left(\phi^k(x) \frac{\partial u}{\partial x}(x) \right)_{\alpha=0}. \tag{1}$$

In the particular case of looking for a retarded function, that is, for $\alpha = -\frac{1}{c}$ being c the speed of light and $\phi = R(t')$ where R represents the distance between the charge at the retarded time t' and the considered point at t; that is: $t' = t - \frac{R(t')}{c}$. One has,

$$u(t') = u(t) + \sum_{k=1}^{\infty} \frac{(-1)^k}{c^k k!} \frac{d^{k-1}}{dt^{k-1}} \left(R^k(t) \frac{du}{dt}(t) \right) \tag{2}$$

The retarded electric field results to be:

$$\overrightarrow{E} = e \sum_{j=0}^{\infty} E_j, \tag{3}$$

where e represents the charge of the particle and

$$\overrightarrow{E}_j = (-1)^j \left[-C_j^{-1} \overrightarrow{R} + \sum_{i=0}^{j} \frac{(j-i)}{(j-i+1)!} C_i^{j-i} \overset{j-i}{\overrightarrow{v}} \right] \tag{4}$$

with

$$C_0^k = \Omega_k \text{ and } C_l^k = \sum_C \frac{1}{i!j!\ldots h!} \frac{\partial^s \Omega_{k+l}}{\partial v^s} \left(\frac{\overset{\bullet}{\overrightarrow{v}}}{2!} \right)^i \left(\frac{\overset{\bullet\bullet}{\overrightarrow{v}}}{3!} \right)^j \cdots \left(\frac{\overset{q}{\overrightarrow{v}}}{(q+1)!} \right)^h, \tag{5}$$

where the index C in the sum means all the possible combinations such that

$$\begin{aligned} i + j + \ldots + h &= s, \\ i + 2j + \ldots + qh &= l, \end{aligned} \tag{6}$$

and

$$\Omega_k = R^{k-2} \gamma^{k+2} \frac{\left[z + (1+z^2)^{1/2} \right]^k \left[k(1+z^2)^{1/2} - z \right]}{(1+z^2)^{3/2}}, \tag{7}$$

with

$$z = \gamma(\widehat{n}.\overrightarrow{v}),\ \widehat{n} = \frac{\overrightarrow{R}}{R} \text{ and } \gamma = (1-v^2)^{-1/2}. \tag{8}$$

The speed of the light has been taken as $c = 1$. For the retarded magnetic field, the result is:

$$\overrightarrow{B}^{ret} = e \sum_{j=0}^{\infty} \overrightarrow{B}_j^{ret}, \tag{9}$$

where

$$\overrightarrow{B}_j^{ret} = (-1)^j \sum_{i=0}^{j} \overrightarrow{B}_i^{jret}, \tag{10}$$

being $\overrightarrow{B}_i^{jret}$described by

$$\overrightarrow{B}_i^{jret} = C_i^{j-i-1} \frac{\overrightarrow{R} \times \overset{j-i}{\overrightarrow{v}}}{(j-i)!} + C_i^{j-i} \sum_{k=0}^{E[(j-i)/2]} \left[\frac{(j-i-2k) \overset{k}{\overrightarrow{v}} \times \overset{j-i-k}{\overrightarrow{v}}}{(k+1)!\,(j-i-k+1)!} \right], \tag{11}$$

where $E\left[(j-i)/2\right]$ represents the integral part of $(j-i)/2$.

After some calculations [6], we obtain:

$$\begin{aligned}
\overrightarrow{E}_0^{ret} &= \frac{1}{R^2}\widehat{n}, \\
\overrightarrow{E}_1^{ret} &= -\frac{\overset{\bullet}{v}}{2R}\left(\cos\theta \widehat{n} + \widehat{k}\right), \\
\overrightarrow{E}_2^{ret} &= -\frac{3\sin^2\theta}{2}\left(\frac{\overset{\bullet}{v}^2}{4} + \frac{v\overset{\bullet\bullet}{v}}{3}\right)\widehat{n} + \left(\frac{1}{3}\left(2 + 3v\cos\theta\right)\overset{\bullet\bullet}{v} + \left(2v + \frac{3}{4}\cos\theta\right)\overset{\bullet}{v}^2\right)\widehat{k}, \\
\overrightarrow{B}_0^{ret} &= \frac{v}{R^2}\widehat{k} \times \widehat{n}, \\
\overrightarrow{B}_1^{ret} &= -\frac{3v\overset{\bullet}{v}\cos\theta}{2R}\widehat{k} \times \widehat{n}, \\
\overrightarrow{B}_2^{ret} &= -\left(\frac{\overset{\bullet\bullet}{v}}{2} + \frac{15v\overset{\bullet}{v}^2}{8}\sin^2\theta\right)\widehat{k} \times \widehat{n}
\end{aligned} \tag{12}$$

By an inspection of Eq. (2) for the advanced electric field, it is easy to see that the difference with the retarded electric field corresponds to the $(-1)^j$. Therefore,

$$\overrightarrow{E}^{adv} = e\sum_{j=0}^{\infty} E_j^{adv}, \tag{13}$$

where

$$\overrightarrow{E}_j^{adv} = \left[-C_j^{-1}\overrightarrow{R} + \sum_{i=0}^{j} \frac{(j-i)}{(j-i+1)!} C_i^{j-i} \overset{j-i}{\overrightarrow{v}}\right]. \tag{14}$$

Then,

$$\overrightarrow{E}_j^{adv} = (-1)^j \overrightarrow{E}_j^{ret} \tag{15}$$

Since in Sec. 3 we will be involved with terms which are proportional to R^{-2}, R^{-1} and R^0, we will only consider the particular case of $j = 0$, 1 and 2; then we have:

$$\overrightarrow{E}_0^{adv} = \overrightarrow{E}_0^{ret},\ \overrightarrow{E}_1^{adv} = -\overrightarrow{E}_1^{ret},\ \overrightarrow{E}_2^{adv} = \overrightarrow{E}_2^{ret}. \tag{16}$$

For the magnetic field, the result is similar; that is:

$$\overrightarrow{B}_0^{adv} = \overrightarrow{B}_0^{ret},\ \overrightarrow{B}_1^{adv} = -\overrightarrow{B}_1^{ret},\ \overrightarrow{B}_2^{adv} = \overrightarrow{B}_2^{ret}. \tag{17}$$

Other way for obtaining the advanced fields corresponds to consider that the particle is coming back from the future to the past and we have to interchange,

$$t \to -t,\ \overrightarrow{v} \to -\overrightarrow{v},\ \cos\theta \to -\cos\theta. \tag{18}$$

The result is similar to the one presented before. Therefore, the mean electromagnetic fields are described by

$$\begin{aligned}
\overrightarrow{E}_0 &= \frac{1}{2}\left[\overrightarrow{E}_0^{ret} + \overrightarrow{E}_0^{adv}\right] = \frac{1}{R^2}\widehat{n},\\
\overrightarrow{E}_1 &= \frac{1}{2}\left[\overrightarrow{E}_1^{ret} + \overrightarrow{E}_1^{adv}\right] = \overrightarrow{0},\\
\overrightarrow{E}_2 &= \frac{1}{2}\left[\overrightarrow{E}_2^{ret} + \overrightarrow{E}_2^{adv}\right] = -\frac{3\sin^2\theta}{2}\left(\frac{\dot{v}^2}{4} + \frac{v\ddot{v}}{3}\right)\widehat{n}\\
&+ \left(\frac{1}{3}(2+3v\cos\theta)\,\ddot{v} + \left(2v + \frac{3}{4}\cos\theta\right)\dot{v}^2\right)\widehat{k},\\
\overrightarrow{B}_0 &= \frac{1}{2}\left[\overrightarrow{B}_0^{ret} + \overrightarrow{B}_0^{adv}\right] = \frac{v}{R^2}\widehat{k}\times\widehat{n},\\
\overrightarrow{B}_1 &= \frac{1}{2}\left[\overrightarrow{B}_1^{ret} + \overrightarrow{B}_1^{adv}\right] = \overrightarrow{0},\\
\overrightarrow{B}_2 &= \frac{1}{2}\left[\overrightarrow{B}_2^{ret} + \overrightarrow{B}_2^{adv}\right] = -\left(\frac{\ddot{v}}{2} + \frac{15v\dot{v}^2}{8}\sin^2\theta\right)\widehat{k}\times\widehat{n}
\end{aligned} \tag{19}$$

These fields will be used in the next section in order to find the flux of energy through a sphere.

3. The flux of energy

It is a well-known result [25] that the rate of mechanical energy can be expressed as:

$$\frac{dE_{mech}}{dt} = -\int_V \left(\frac{\partial u}{\partial t} + \nabla \bullet \overrightarrow{s}\right) dV, \tag{20}$$

where $u = \frac{1}{2}\left(E^2 + B^2\right)$ represents the energy density and $\overrightarrow{s} = \frac{1}{4\pi}\overrightarrow{E}\times\overrightarrow{B}$ is the Poynting vector. We know that the rate of the energy of the field is:

$$\frac{dE_{field}}{dt} = \frac{d}{dt}\int_V u dV. \tag{21}$$

Let us consider a charged point particle moving with a speed $\overrightarrow{v}$, and a sphere of radius R with the charge at the center and moving along the path of the charge. By using a classical identity about the derivative with respect to the time of a volume integral with a volume with speed $\overrightarrow{v}$ [4], Eq. (21) may be expressed as:

$$\frac{dE_{field}}{dt} = \int_V \left(\frac{\partial u}{\partial t} + \nabla \bullet (u\overrightarrow{v})\right) dV. \tag{22}$$

By considering that the whole volume moves with the same velocity, we arrive at:

$$\frac{dE_{field}}{dt} = \int_V \frac{\partial u}{\partial t} dV + \overrightarrow{v} \bullet \int_V u d\overrightarrow{A}. \tag{23}$$

By adding Eqs. (20) and (23), we obtain that

$$\frac{dE_{mech}}{dt} + \frac{dE_{field}}{dt} = -\int_V \nabla \bullet \overrightarrow{s} dV + \overrightarrow{v} \bullet \oint u d\overrightarrow{A}. \tag{24}$$

By using Gauss theorem, we have:

$$\frac{dE_{mech}}{dt} + \frac{dE_{field}}{dt} = -\oint \overrightarrow{s} \bullet d\overrightarrow{A} + \overrightarrow{v} \bullet \oint u d\overrightarrow{A}. \tag{25}$$

Let us consider that the flux of energy will be evaluated at first order with the speed v of the particle. This means that we will neglect all the terms proportional to v^2. We are interested in calculating the flux of energy due to the particle. Therefore, we will consider the flux of energy through a sphere of radius R and then we will take the limit $R \to 0$. First of all, let us calculate the term $\frac{dE_{field}}{dt} = \frac{d}{dt}\int_V u dV$. As we have shown in Sec. 2, the electromagnetic fields can be expressed as $\overrightarrow{E}_i = \frac{1}{2}\left[\overrightarrow{E}_i^{ret} + \overrightarrow{E}_i^{adv}\right]$ and $\overrightarrow{B}_i = \frac{1}{2}\left[\overrightarrow{B}_i^{ret} + \overrightarrow{B}_i^{adv}\right]$ which are proportional to R^{i-2}. At the moment of taking the limit $R \to 0$, just the terms which are proportional to R^{-4}, R^{-3}, R^{-2}, R^{-1}, R^0 will survive. If we consider Eq. (19), we will just consider,

$$E^2 = q^2\left(E_0^2 + 2\overrightarrow{E}_0 \cdot \overrightarrow{E}_2\right). \tag{26}$$

If we integrate $2\overrightarrow{E}_0 \cdot \overrightarrow{E}_2$ which is proportional to R^{-2}, a R term appears and when the limit $R \to 0$ is taken it vanishes. On the other hand, E_0^2 doesn't depend on time, therefore the derivative vanishes. Therefore

$$\lim_{R\to 0}\frac{dE_{field}}{dt} = \lim_{R\to 0}\frac{d}{dt}\int_V u dV = 0. \tag{27}$$

Consequently

$$\lim_{R\to 0}\frac{dE_{mech}}{dt} = \lim_{R\to 0}\left[-\oint \overrightarrow{s} \bullet d\overrightarrow{A} + \overrightarrow{v} \bullet \int_V u d\overrightarrow{A}\right] \tag{28}$$

Let us define the functions G and W as:

$$W = \lim_{R\to 0} -\oint \overrightarrow{s} \bullet d\overrightarrow{A} \qquad \text{and} \qquad G = \lim_{R\to 0} \overrightarrow{v} \bullet \oint u d\overrightarrow{A}. \tag{29}$$

Then, let us begin by calculating W. We need to know the Poynting vector; that is:

$$\overrightarrow{E} \times \overrightarrow{B} = q^2\left[\overrightarrow{E}_0 \times \overrightarrow{B}_0 + \overrightarrow{E}_0 \times \overrightarrow{B}_2 + \overrightarrow{E}_2 \times \overrightarrow{B}_0\right].$$

W can be divided in two terms, $W = W_{-2} + W_0$, where W_i represents the term containing R^i;

$$W_{-2} = \lim_{R\to 0} -\frac{q^2}{4\pi}\int_0^{\pi}\int_0^{2\pi} \overrightarrow{E}_0 \times \overrightarrow{B}_0 \bullet \widehat{n} R^2 \sin\theta d\theta d\phi, \tag{30}$$

and

$$W_0 = \lim_{R\to 0} -\frac{q^2}{4\pi}\int_0^{\pi}\int_0^{2\pi} \left(\overrightarrow{E}_0 \times \overrightarrow{B}_2 + \overrightarrow{E}_2 \times \overrightarrow{B}_0\right) \bullet \widehat{n} R^2 \sin\theta d\theta d\phi,$$

Since

$$\widehat{n} \times \left(\widehat{k} \times \widehat{n}\right) \cdot \widehat{n} = 0,$$

and by using the Eq. (19), we obtain:

$$\begin{aligned} &\overrightarrow{E}_0 \times \overrightarrow{B}_0 \cdot \widehat{n} = 0, \\ &\overrightarrow{E}_0 \times \overrightarrow{B}_2 \cdot \widehat{n} = 0, \\ &\overrightarrow{E}_2 \times \overrightarrow{B}_0 \cdot \widehat{n} = -\frac{v}{R^2}\left(\frac{2}{3}\ddot{v} + \frac{3}{4}\cos\theta \dot{v}^2\right)\sin^2\theta. \end{aligned} \tag{31}$$

Therefore

$$\begin{aligned} W_{-2} &= 0, \\ W_0 &= \lim_{R\to 0} \frac{q^2}{4\pi}\int_0^{\pi}\int_0^{2\pi} \frac{v}{R^2}\left(\frac{2}{3}\ddot{v} + \frac{3}{4}\cos\theta \dot{v}^2\right) R^2 \sin^3\theta d\phi d\theta \end{aligned} \tag{32}$$

The result is

$$\begin{aligned} W_{-2} &= 0, \\ W_0 &= \frac{4q^2 v \ddot{v}}{9}. \end{aligned} \tag{33}$$

We arrive at

$$W = W_0 + W_{-2} = \frac{4q^2 v \ddot{v}}{9}, \tag{34}$$

On the other hand , in order to calculate $G = G_{-2} + G_0$ where G_i represents the term containing R^i and by using the Eq. (26), we have

$$\begin{aligned} G_{-2} &= \lim_{R\to 0} \overrightarrow{v} \bullet \int_0^{\pi}\int_0^{2\pi} \frac{E_0^2}{8\pi} \widehat{n} R^2 \sin\theta d\theta d\phi, \\ G_0 &= \lim_{R\to 0} \overrightarrow{v} \bullet \int_0^{\pi}\int_0^{2\pi} \frac{2\overrightarrow{E}_0 \cdot \overrightarrow{E}_2}{8\pi} \widehat{n} R^2 \sin\theta d\theta d\phi. \end{aligned} \tag{35}$$

Then, let us consider that the speed of the particle is $\overrightarrow{v} = v\widehat{k}$. we have

$$\begin{aligned} G_{-2} &= q^2 \lim_{R\to 0}\int_0^{\pi}\int_0^{2\pi} \frac{v}{8\pi R^2}\cos\theta\sin\theta d\theta d\phi, \\ G_0 &= q^2 \lim_{R\to 0}\int_0^{\pi}\int_0^{2\pi} \frac{1}{8\pi}\left[-\frac{6\sin^2\theta v}{2}\frac{\dot{v}^2}{4}\right. \\ &\quad \left. +2\left(\frac{2}{3}v\ddot{v} + \frac{3}{4}\cos\theta v\dot{v}^2\right)\cos\theta\right]\cos\theta\sin\theta d\theta d\phi. \end{aligned} \tag{36}$$

Integrating:

$$G_{-2} = 0,$$
$$G_0 = \frac{2q^2 v \ddot{v}}{9}. \tag{37}$$

G can be expressed by

$$G = G_{-2} + G_0 = \frac{2q^2 v \ddot{v}}{9}. \tag{38}$$

By Using Eqs. (28), (29), (34) and (38), we arrive at:

$$\frac{dE_{mech}}{dt} = \overrightarrow{f}_{rad} \bullet \overrightarrow{v} = \frac{2q^2 \ddot{v}}{3} v. \tag{39}$$

We can conclude that:

$$f_{rad} = \frac{2}{3} q^2 \ddot{v} + 0\left(v^2\right). \tag{40}$$

We obtain an expression for the reaction force without mass renormalization. This result has been obtained for parallel velocity, acceleration and hyperacceleration, nevertheless we can generalize to

$$\overrightarrow{f}_{rad} = \frac{2}{3} q^2 \ddot{\overrightarrow{v}} + 0\left(v^2\right) \tag{41}$$

This is the reaction force obtained by Abraham [A] [5]. Once, we have a nonrelativistic result, we can apply Pauli's anzats [7] and immediately we obtain:

$$f^{\mu}_{rad} = \frac{2q^2}{3}\left(\ddot{v}^{\mu} + a_{\nu} a^{\nu} v^{\mu}\right) \tag{42}$$

which is the LD equation of motion for spinless point charged particles in classical electrodynamics and it has been obtained without using the mass renormalization process. Even if there are many more elegant deduction of the LD equation by using the average technique, as we have mentioned, the Lagrange-Gordeyev method permit to analyze step by step the behavior of each R^n- term and the elimination of the renormalization process is explicitly showed.

4. Landau-Lifshitz equation of motion

Before deduction of the LL equation, it is convenient to consider certain relevant aspects. Let's start by quoting Hammond [22]: "it is not assumed a priori that a self-interactions exists, it is not put into the action principle from which the equations are derived, it is a consequence of the theory. This result, that self-interactions are not included in the basic formulation of theory, has been called a formal inconsistency in the theory,". Indeed, if we make a review of Eq. (20), it is obvious that everything is deduced by considering that the equation of motion of a spinless point particle is the Lorentz [L] equation. That is:

$$m\overrightarrow{a} = q\left[\overrightarrow{E} + \frac{\overrightarrow{v}}{c}\overrightarrow{B}\right]. \tag{43}$$

After some calculations, we end up with another equation of motion, that is the A equation :

$$m\overrightarrow{a} = q\left[\overrightarrow{E} + \frac{\overrightarrow{v}}{c}\overrightarrow{B}\right] + \frac{2}{3}q^2\overset{\bullet\bullet}{\overrightarrow{v}} + 0\left(v^2\right). \tag{44}$$

The question now is: which of the two equations, Eqs. (43) and (44), is the good? If we choose the L equation, Eq. (43), we will not have a reaction force as desired. But if we accept the A equation as the good one, we have two problem: firstly, we know that we begin by considering the L equation and we finish with another equation; secondly, if we repeat the same procedure beginning with the A equation, we will obtain another reaction force and so on. In both cases, we do not accomplish our goal.

However, let us consider that a spinless point particle moves according to the following equation:

$$m\overrightarrow{a} = q\left[\overrightarrow{E} + \frac{\overrightarrow{v}}{c}\overrightarrow{B}\right] + \overrightarrow{G} + \overrightarrow{f}_{rad}, \tag{45}$$

where $\overrightarrow{G}$ and $\overrightarrow{f}_{rad}$ represent a non-electromagnetic force and the reaction force, respectively. Consider now, that the charge is submitted to a force $\overrightarrow{G}$ which coincides with the unknown $-\overrightarrow{f}_{rad}$ at a point. Therefore, in this case, the particle will be driven by just the Lorentz force and consequently in this case the reaction force will be equal to $\frac{2}{3}q^2\overset{\bullet\bullet}{\overrightarrow{v}}$ ($\overrightarrow{G} = -\frac{2}{3}q^2\overset{\bullet\bullet}{\overrightarrow{v}}$). Thus, in this case,

$$m\overset{\bullet\bullet}{\overrightarrow{v}} = \frac{d}{dt}\left[q\left[\overrightarrow{E} + \frac{\overrightarrow{v}}{c}\overrightarrow{B}\right]\right], \tag{46}$$

or by putting

$$\overrightarrow{F} = q\left[\overrightarrow{E} + \frac{\overrightarrow{v}}{c}\overrightarrow{B}\right], \tag{47}$$

we can propose that when the external force does not exist, the radiation term is still

$$\overrightarrow{f}_{rad} = \frac{2}{3m}q^2\frac{d}{dt}\overrightarrow{F}. \tag{48}$$

Finally, the equation of motion is

$$m\overrightarrow{a} = \overrightarrow{F} + \frac{2}{3m}q^2\frac{d}{dt}\overrightarrow{F} = \overrightarrow{F} + \tau_o\frac{d}{dt}\overrightarrow{F}, \tag{49}$$

where $\tau_o = \frac{2}{3m}q^2$ represents the characteristic time of the charge. Eq. (49) is the Ford equation [26] which has been deduced in a different form by using some quantum considerations.

By applying Pauli's anzats [7], it is easy to obtain the LL equation [20]:

$$ma^\mu = \frac{q}{c}F^{\mu\nu}v_v + \tau_o\left[\left[\frac{q}{c}(\frac{\partial F^{\mu\nu}}{\partial x^\alpha}v^\alpha v_\nu - (q/cm)F^{\mu\nu}F_{\alpha v}v^\alpha) + (q^2/c^4m)F^2v^\mu\right]\right]. \tag{50}$$

Conclusions

Even if nowadays the LL equation is considered as the correct equation for a charged point particle [16], [17], [18], [20], it is always deduced by firstly obtaining the LD equation of motion. Indeed, for Spohn [17] and Rohrlich [18] the LD equation must be restricted to its critical surface yielding the LL equation and consequently, this last one represents the correct equation of motion for a spinless classical point charge. Therefore, the deduction of the Lorentz-Dirac equation without mass renormalization as we did in the paper, also supports the validity of the Landau-Lifshitz equation of motion. However, although many authors speak of the LL equation as exact or correct, some recent work than that cited has concluded that the LL equation can not be considered exact, but only a frequently useful approximation to the LD equation for a point charge.

However, this discussion would be clarified when a consistent closed theory of fields and particles will be found. Nowadays, some works has been successful by introducing some important concepts as the proper time [27] in order to begin to well-define the problem.

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