Implications of an Aether non Dragged by the Motion of Celestial Bodies on Optical Laws

Joseph Levy*

4 square Anatole France, 91250 St Germain lès Corbeil, France

Received 7 June 2011, Accepted 10 August 2011, Published 17 January 2012

Abstract: The conventional optical laws take for granted that the one-way phase velocity of light in a refractive medium at rest in the Earth frame is $C/n$. But if one assumes the existence of a fundamental reference frame and of an aether non-entrained by the motion of celestial bodies, then Hoek’s experiment shows that this velocity must be equal to $C/n - V/n^2$ in the direction of the Earth absolute motion, and $C/n + V/n^2$ in the opposite direction, where $V$ is the absolute speed of the Earth frame. It is important to draw the consequences of this data and to check whether it complies with well established laws of physics. Such an anisotropy implies that, according to non-entrained aether theory (NEAT), the ratio of the speed of light in vacuo to the speed of light in refractive media (i.e, the optical index) must vary as a function of the orientation of the light signal. This is indeed what the calculation shows. Therefore, if NEAT is exact, except for some orientations, $n$ is not the optical index in refractive media moving relative to the fundamental frame. However, as we shall see, NEAT does not preclude the fact that the Snell-Descartes’ law $\sin i = n \sin r$ applies to a high degree of accuracy whatever the orientation of the light signal. Thus, even if it exists, the anisotropy remains unapparent. It is worth noting that, while resorting to assumptions quite different from special relativity, non-entrained aether theory accounts for well established experimental results. Moreover, as will be checked, a thorough analysis of Fizeau’s experiment in light of Hoek’s studies establishes the need for an aether drift, providing a strong argument in support of aether theory.

© Electronic Journal of Theoretical Physics. All rights reserved.

Keywords: Non-entrained Aether, NEAT, Hoek, Fizeau, Fresnel, Snell-Descartes’ law, Light Speed Anisotropy, Refractive Media, Synchronization Procedures

PACS (2010): 03.30.+p; 95.10.Ce; 45.50.Pk; 42.15.-i

Email: levy.joseph@orange.fr
1. Introduction

It is a fact that, by definition, if the principle of relativity applies exactly, nothing can differentiate rest from rectilinear uniform motion, these two states being only relative depending on the point of observation. If so, no anisotropy of the physical laws can occur.

In contrast, if a reference frame in which an aether is at rest is assumed, (this aether being not entrained by the motion of celestial bodies), all the frames moving relative to this fundamental frame must be subjected to an aether drift responsible for an anisotropy of the laws of physics, which, we expect, should be observed.

This, we think, should be checked experimentally, and, in case this anisotropy was not observed, this might permit to disprove the theories which assume the existence of a fundamental frame and of an aether drift.

However, things are somewhat different. As the calculations show, insofar as experiments performed in vacuo require the measurement of time and lengths, even though the anisotropy exists, it cannot be easily highlighted. The reasons of this fact have been addressed in detail in previous publications [1]: if an aether non-entrained by the motion of celestial bodies (NEAT) is assumed, they result from the measurement distortions due to length contraction, clock retardation and arbitrary clock synchronization. (This fact explains why it is so difficult to differentiate Lorentz-Poincaré aether theory from special relativity. Both theories rest on the Lorentz transformations which imply that the measured laws of physics must remain invariant under a change of inertial frame. However, whereas for special relativity (at least in its most generally accepted interpretation) the measuring procedures are considered valid as such, this is not the case when the Lorentz aether is assumed, since, in this case,-to the extent that the determinations of length and/or time are required,- a thorough analysis shows that the invariance is only a consequence of the systematic measurement distortions, which prevent to highlight the effect of the aether drift on the physical laws).

For example, even though the one-way speed of light is anisotropic, this anisotropy could not be definitely proved, by the standard procedures, since all the measurements made by the classical authors [2] were measurements using clocks synchronized with light signals (Einstein-Poincaré procedure (E-P)) or by slow clock transport. But we know today that these methods are unable to determine the one-way speed of light without any doubt [3-12], and therefore they are unable to highlight a possible anisotropy.

However, theorists who assume the existence of an aether non-dragged by the motion of celestial bodies, or, at least, who expect to know the truth about its existence, try to demonstrate it using different original methods that deserve to be tested carefully. Such approaches, which follow the lines laid down by Lorentz, are also adopted by several contemporary authors [13-20].

Here, it is worth asking whether we have a chance to highlight the anisotropy in refractive media, since different optical tests do not require measurements of lengths and of time. We expect, a priori, that the optical experimental tests should permit to confirm the
anisotropy if it exists, or to disprove it if it does not. Yet, is it really the case? As we will show in chapter 8, when its evidence is sought using the Snell-Descartes’ law, even if it exists, the anisotropy remains unapparent. It is worth noting that, although it gives them an interpretation different from that of special relativity, NEAT accounts for the well established Snell-Descartes’ experimental laws.

In view of these results, it is fair to ask whether NEAT can be a constructive alternative to special relativity (SR). After all, if the experimental results they predict are considered mostly identical (as many physicists believe), is it really necessary to search for approaches different from SR and certainly more intricate? The prevailing view is that, between Lorentz aether theory and Special relativity “there is a stringent difference in philosophy”, but the difference does not concern the predictive power. Therefore the answer depends on the importance we assign to philosophy. This point of view was the one advocated by Bell in his essay “How to teach special relativity” [21], and might also extend to all theories which allow to predict the experimental data as well as the currently accepted theories [22].

However, the predictive power is not the only interesting thing in science whose role is more specifically to highlight the nature of the physical reality. As an example, it is not unimportant to know whether the one-way speed of light is isotropic (SR) or not (Aether theory), even if the predictions of the theories which assume these different postulates can be the same; all the more if the measurements can be affected by systematic measurement distortions [1].

It is also justified to wonder whether the predictions of the different theories will always be the same and there are good reasons to doubt this, as Michelson-Morley experiments in gas mode [23, 24] which are today reinterpreted, suggest. Different authors have recently explored these areas, and research is continuing in this direction [15-20].

Another line of investigation which suggests this, and that we shall follow in this text, is to conduct a reanalysis of Fizeau’s experiment in light of Hoek’s studies [25]. As we shall see, the conclusions drawn from this approach differ from special relativity, showing that no fringe shift can occur in the absence of an aether drift.

It is our purpose in this article, using methods which have not resulted in extensive researches until now, to address these issues and to highlight their importance for the development of physics.

Let us specify that in all the studies which follow, the refractive media through which light propagates are supposed to possess a molecular structure homogeneous and isotropic; which means that in the fundamental frame assumed by NEAT, their optical index should be \( n \) irrespective of the orientation.
2. One-way Light Speed in a Refractive Medium at Rest on Earth, Assuming the Existence of a Fundamental Aether Frame and of an Aether Drift

In 1868 Martinus Hoek performed an experiment simple but very significant [25]. Light from a source was split into two beams by a beam splitter BS. The beams travelled in opposite directions along a closed path, (one part of which consisting of air at atmospheric pressure, and another part consisting of a refractive medium), before being recombined to form an interference pattern in the detector [Figure 1]. The examination of the interference pattern did not reveal any difference as regards the time of propagation of light in the two opposite directions.

This result was interpreted by special relativity as a consequence of the light speed isotropy postulate. Indeed, in the absence of any aether drift nothing could differentiate the two opposite paths, so that:

\[ t_1 = t_2 = \frac{L}{C} + \frac{L}{C/n} + \frac{2\ell}{C}, \]

where \( C \) designates the speed of light in air and \( C/n \) the speed of light in the medium, both being isotropic. \( L \) denotes the longitudinal path and \( \ell \) the transversal one.

However Hoek could give an interpretation of the result in agreement with non-entrained aether theory, a fact that, as we shall see, has significant implications as regards the understanding of Fizeau’s experiment. Hoek’s experiment has been redone by different authors, yet, the facts suggest that the lessons of this experiment have not been sufficiently taken into account.

Although our approach differs slightly from that of Hoek, it is not contradictory and leads to the same mathematical results. However, the interpretation of \( V \) (see below) assumed by modern non-entrained aether theory, obviously differs, since, contrary to a widespread belief in Hoek’s time, it does not assign to the sun’s reference frame the location of the fundamental aether frame.

Let us now calculate the speed of light in the refractive medium according to non-entrained aether theory. We will first assume that the refractive cylinder is aligned in the direction of the Earth absolute velocity and that the speed of light in air is \( C - V \) in the forward direction and \( C + V \) in the opposite direction, (\( V \) being the absolute speed of the Earth frame). Denoting by \( L \) the length of the medium, and by \( C/n \) the speed of light in it \emph{when it is in a state of absolute rest and therefore not subject to an aether drift}, the time required, according to NEAT, for a rotation of the light signal in the circuit is:

\[ t_1 = \frac{L}{C + V} + \frac{L}{(C/n) - x} + \varepsilon, \quad (1) \]

in the clockwise direction, and:

\[ t_2 = \frac{L}{C - V} + \frac{L}{(C/n) + x} + \varepsilon. \quad (2) \]
in the counterclockwise direction.
In these expressions, \((C/n) - x\) and \((C/n) + x\) denote the speeds of light in opposite directions in the medium subject to absolute motion, (here the absolute motion of the Earth frame), \(x\) is the unknown expression to be determined, \(\varepsilon\) designates the time needed by the light signal to cover the orthogonal paths AB and CD.

Note that, if NEAT is exact, due to the motion of the Earth relative to the fundamental aether frame, the length of the refractive cylinder must be contracted. Denoting by \(L_0\) its value in the fundamental frame, its length on Earth must be:

\[
L = L_0 \sqrt{1 - V^2/C^2}
\]

**Fig. 1** Hoek’s experiment: Light from the source is split into two beams by the beam splitter BS. One of the beams follows the path ABCDA, and the other ADCBA. The beams are recombined in A to form an interference pattern in the detector. The segment BC is occupied by a refractive medium, while the other parts of the circuit consist of air at atmospheric pressure.

However, in the Earth frame the difference between \(L_0\) and \(L\) is of second order and can be generally ignored.

Since no fringe shift was recorded, \(t_1 = t_2\). So, from (1) and (2):

\[
\frac{L}{C + V} + \frac{L}{(C/n) - x} = \frac{L}{C - V} + \frac{L}{(C/n) + x}
\]

Solving this equation yields:

\[
x = \frac{V}{n^2}
\]

Therefore the speed of light in the medium is:

\[
\frac{C}{n} - \frac{V}{n^2}
\]

in the clockwise direction, and:

\[
\frac{C}{n} + \frac{V}{n^2}
\]
in the counterclockwise direction.  
(See the demonstration in appendix 1). 
This result which differentiates NEAT from Special relativity and from entrained aether theory is important: it applies to light signals travelling through refractive media at rest in the Earth frame, \( V \) being the Earth absolute velocity. It is different from the classical expression \( C/n \), which is identical in all directions. A priori this seems to indicate that NEAT should not take for granted the Snell Descartes’ law. But, as we will show in a subsequent chapter, contrary to appearances, this is not the case. 

3. Average two-way Speed of Light in a Refractive Medium at Rest in the Earth Frame

According to NEAT, the two-way transit time of light in the medium is: 

\[
\frac{L}{(C/n) - (V/n^2)} + \frac{L}{(C/n) + (V/n^2)} = \frac{2LC/n}{(C^2/n^2) - (V^2/n^4)} 
\]

Ignoring the second order term \( V^2/n^2 \) this expression reduces to \( 2L/v_n \). 
The average two-way speed of light in the medium is therefore equal, to first order, to \( C/n \). (This value is the exact value that the one-way speed of light would assume in isotropic refractive media if there was no aether drift, as is the case in the fundamental frame). 
(In contrast, for special relativity and entrained aether theory, \( C/n \) is regarded as the one-way light speed in refractive media at rest on Earth. In these theories the one-way speed of light in the Earth frame is considered isotropic).

4. Optical Indices in the Earth Frame

Indices of refractive media at rest on Earth according to non-entrained aether theory: As can be seen, due to the absolute motion of the Earth, these indices differ from the conventional value \( n \).

1. When the light rays, in air and in the medium, run parallel and propagate in the same direction as the Earth absolute velocity vector, the optical index is equal to:

\[
N = \frac{C - V}{(C/n) - (V/n^2)} 
\]

To first order this expression reduces to:

\[
n(1 - V/C) + V/C 
\]

2. When the light rays run in the opposite direction, the optical index is equal to:

\[
N' = \frac{C + V}{(C/n) + (V/n^2)} 
\]
which to first order yields:
\[ n(1 + V/C) - V/C \]
The difference between the two indices \( N' \) and \( N \) is:
\[ \Delta \approx 2(V/C)(n - 1) \]

According to NEAT, the two indices are therefore different from \( n \) and from each other. This result also reflects a conceptual difference between non-entrained aether theory, SR and entrained aether theory, as these other theories imply a value of \( \Delta \) equal to zero. But is this anisotropy perceptible using the law of refraction? The question will be addressed in subsequent chapters.

5. Interpretation of Fizeau’s Experiment According to Non-entrained Aether Theory [26]

Light from a source is split into two beams, which are sent toward the two branches of a pipe filled with water before being recombined to form an interference pattern (Figure 2). When the water starts to flow, the fringes are shifted. The fringe shift permits to deduce the variation of the speed of light due to the flow. For the sake of simplicity we will only consider in all the chapter 5, the case where the light beam runs in the direction of the Earth absolute velocity, the opposite case can be easily deduced.

From Hoek’s experiment, and if we assume light speed anisotropy on Earth, we can infer that if a refractive medium is moving with respect to the Earth frame at speed \( v \), then the speed of light measured from the frame of the medium will be:
\[ \frac{C}{n} - \frac{V + v}{n^2} \] (5)
when the medium moves in the direction of the light signal and of the Earth absolute velocity, and:
\[ \frac{C}{n} - \frac{V - v}{n^2} \] (6)
when the medium moves in the opposite direction.

These expressions can be easily deduced from the demonstration given in appendix I. To calculate the speed of light in the water, measured from the Earth frame, one usually supposes that this velocity in the water at rest on Earth is \( C/n \), and one adds to this speed the result deduced from the interference pattern due to the flow, which is \( \pm v(1 - 1/n^2) \). But, as we saw, according to NEAT \( C/n \) is the round trip velocity in a refractive medium at rest on Earth, and not the one-way speed of light.

The result derived from non-entrained aether theory differs in that, as we saw, the one-way speed of light in the frame of the moving water is given by the formulas (5) and (6). Therefore, measured from the Earth frame the speed of light in the water will be:
\[ \frac{C}{n} - \frac{V + v}{n^2} + v \]
Fig. 2 Fizeau’s experiment: Light from a source is split into two beams, which are sent toward the two branches of a pipe filled with water before being recombined to form an interference pattern. When the water starts to flow, the fringes are shifted. The fringe shift permits to deduce the variation of the speed of light due to the flow.

when the water flows in the direction of the light signal and of the Earth absolute velocity, and:

\[ \frac{C}{n} - V - \frac{v}{n^2} - v \]

when the water flows in the opposite direction.

Thus, if an aether drift exists, the formulas of Fizeau take a different form than usual. These modified Fizeau’s formulas can be written as:

\[ \frac{C}{n} - V + \frac{v}{n^2} + v\left(1 - \frac{1}{n^2}\right), \] \hspace{1cm} (7)

and:

\[ \frac{C}{n} - V - \frac{v}{n^2} - v\left(1 - \frac{1}{n^2}\right). \] \hspace{1cm} (8)

The general case will be studied in chapter 7.

Note

These expressions result from the application of the Galilean velocity addition law, which, according to NEAT, applies when no distortions alter the measurements [1C, Chapter 6]. (Of course this does not mean that the speeds of massive bodies can take any value between zero and infinity. Due to the increase of mass with velocity, which according to NEAT applies exactly when massive bodies move relative to the aether frame their sum cannot exceed the speed of light relative to this frame. [see Ref 12 chapter 3]).

Fizeau’s experiment lends support to aether theory

Hoek’s experiment shows that the speed of light in a medium at rest on Earth must take the value \( \frac{C}{n} - \frac{V}{n^2} \) only if the Earth frame is subject to an aether drift of speed \( V \) parallel and opposite to the direction of the light signal. This is what the analysis of the experiment teaches us; which means that a term having the structure \( \frac{speed}{n^2} \) derives from an aether drift. If the drift proves to be zero the formula is reduced to \( \frac{C}{n} \). Now, according to NEAT, for a refractive medium moving in the direction of the Earth absolute velocity at speed \( v \) relative to the Earth, the aether drift above the medium, amounts to \( V + v \), and Hoek’s formula, in the frame of the medium, becomes \( \frac{C}{n} - \frac{V + v}{n^2} \), from which one
can deduce the modified Fizeau’s formula given by expression (7). Here the term \( v/n^2 \) results from the additional aether drift due to the motion of the water. If there was no aether drift at all, the terms \( V/n^2 \), and \( v/n^2 \) would both disappear in contradiction with Fizeau’s experiment.

Therefore, Hoek’s formula allows us to infer that Fizeau’s interference pattern results from an aether drift, a fact that removes all doubts about the existence of the aether.

**Note**

One may nevertheless wonder whether the aether could be entrained by massive bodies but not by smaller structures placed on them? If this were the case, Fizeau’s device would be subjected to a small aether drift capable of explaining the result of the experiment, and the hypothesis of an aether entrained by celestial bodies could not be definitely ruled out.

6. Fresnel’s Formula

Fresnel’s explanation of his formula cannot be maintained, because it supposes that the amount of aether enclosed in a refractive medium depends on the wavelength of the light signal which travels through it.

However, assuming an aether non-entrained by the Earth motion and starting from Hoek’s experiment, which gives the value of the speed of light in the medium measured from the Earth frame where it is at rest, one can find a rational explanation of Fresnel’s formula: it is enough for that to add (or subtract) the absolute speed of the Earth \( V \) to the expressions (3) and (4) seen in chapter 2. We obtain:

\[
C' = \frac{C}{n} - \frac{V}{n^2} + V = \frac{C}{n} + V(1 - 1/n^2)
\]

when light travels in the direction of the Earth absolute velocity, and:

\[
C'' = \frac{C}{n} + \frac{V}{n^2} - V = \frac{C}{n} - V(1 - 1/n^2)
\]

when light travels in the opposite direction.

**Note 1**

Like for the expressions (7) and (8), the Galilean velocity addition law applies when no distortions alter the measurements [1C, Chapter 6].

**Note 2**

Hoek’s experiment, highlights the fact that Fresnel’s formula is only valid if the aether is not entrained by the motion of celestial bodies. Moreover, unlike what is often claimed, Fresnel’s formula is incompatible with special relativity for which no aether drift exists.
7. Generalization

7.1 Speed of Light in Directions Different from the Earth Absolute Velocity According to Non-entrained Aether Theory

**Speed of light in vacuo**

When the projection of the light velocity on the Earth absolute velocity vector $V$ is oriented in the same direction as the latter, the speed of light in vacuo is:

$$-V \cos \alpha + \sqrt{C^2 - V^2 \sin^2 \alpha},$$

where $\alpha$ is the angle separating the two vectors.

This result applies whatever the value of $\alpha$ between $-90^\circ$ and $+90^\circ$.

When the projection of the light signal is oriented in the direction opposite to the Earth absolute velocity vector, the speed of light in vacuo is equal to:

$$V \cos \alpha + \sqrt{C^2 - V^2 \sin^2 \alpha}.$$

See the demonstration in appendix 2.

Ignoring the minute terms of second and higher order in their series expansion, these expressions, are reduced to $C - V \cos \alpha$ and $C + V \cos \alpha$.

**Speed of light in a refractive medium.**

Denoting by $(C/n) - x$ and $(C/n) + x$ the light speeds in the medium, respectively in the clockwise and in the opposite direction, let us use these data to test Hoek's experiment in directions different from the Earth absolute velocity [Figure 1].

We will have:

$$t_1 = \frac{L}{C + V \cos \alpha} + \frac{L}{(C/n) - x} + \varepsilon$$

in the clockwise direction, and

$$t_2 = \frac{L}{C - V \cos \alpha} + \frac{L}{(C/n) + x} + \varepsilon$$

in the counterclockwise direction.

Hoek, who did not know the direction of the Earth absolute velocity but oriented his apparatus in the direction of revolution of the Earth around the sun which varies according to the season [25], did not mention any fringe shift depending on the orientation of the device.

Therefore $t_1 = t_2$ whatever the angle $\alpha$.

From formulas (9) and (10), we find for the speeds of light in the medium respectively in the clockwise and in the counterclockwise direction:

$$\frac{C}{n} - \frac{V \cos \alpha}{n^2},$$

and

$$\frac{C}{n} + \frac{V \cos \alpha}{n^2}.$$
Average speed of light in a round trip
For a travel to and fro through the medium the transit time of light is:

\[ T = \frac{L}{(C/n) - (V \cos \alpha/n^2)} + \frac{L}{(C/n) + (V \cos \alpha/n^2)} = \frac{2LC/n}{(C^2/n^2) - (V^2 \cos^2 \alpha/n^4)} \]

Ignoring the second order terms which depend on \( \alpha \) we obtain:

\[ T \approx \frac{2L}{C/n} \]

Therefore, according to NEAT, only the average two-way speed of light in a medium at rest on Earth is, to first order, independent of the orientation of the medium and equal to \( \frac{C}{n} \). In contrast, for special relativity and for entrained aether theory, given that no aether drift is assumed, \( \frac{C}{n} \) is regarded as the one-way speed of light, and, as a result, the optical index is found to be \( n \) whatever the direction of the light signal.

7.2 Interpretation of Fizeau’s Experiment According to NEAT in Directions Different from the Earth Absolute Velocity

In Fizeau’s experiment, the light signal is always parallel to the direction of the flow; therefore the dragging coefficient generated by the flow, does not show angular dependence.

Assuming that the projection of the light velocity on the Earth absolute velocity vector is oriented in the same direction as the latter, Hoek’s analysis shows that:

when the water flows in the direction of the light signal, the speed of light measured from the Earth frame will be:

\[ \frac{C}{n} - \frac{V \cos \alpha + v}{n^2} + v = \frac{C}{n} - \frac{V \cos \alpha}{n^2} + v(1 - 1/n^2) \]

and when the water flows in the opposite direction:

\[ \frac{C}{n} - \frac{V \cos \alpha - v}{n^2} - v = \frac{C}{n} - \frac{V \cos \alpha}{n^2} - v(1 - 1/n^2) \]

7.3 Indices Depending on the Angle According to Non-entrained Aether Theory

Unlike relativity and entrained aether theory, the optical indices, in NEAT, vary as a function of the orientation of the light rays and differ from \( n \). We assume here that the paths of the light rays in air and in the medium are aligned. For light signals, whose projection on the Earth absolute velocity vector is oriented in the same direction as the latter, the optical index is equal to:

\[ N = \frac{C - V \cos \alpha}{(C/n) - V \cos \alpha/n^2} \]
This result applies for any value of the angle $\alpha$ separating the light velocity from the Earth absolute velocity vector, between $-90^\circ$ and $+90^\circ$.

To first order this expression reduces to:

$$n(1 - V \cos \alpha/C) + V \cos \alpha/C$$

We see that for $\alpha=90^\circ$, the optical index is reduced to $n$.

For light signals running in the opposite direction, the optical index is:

$$N' = \frac{C + V \cos \alpha}{(C/n) + V \cos \alpha/n^2}$$

which to first order yields:

$$n(1 + V \cos \alpha/C) - V \cos \alpha/C$$

The difference between $N'$ and $N$ for a given value of $\alpha$ is:

$$\Delta \approx 2(V \cos \alpha/C)(n - 1)$$

8. Practical Example of Experimental Testing

Refractive of light in water

Let us determine, according to NEAT, the path taken by a light ray to cover the distance from a source of light S to a point O in a container of water (Figure 3).

We will assume first that the light ray runs in a plane perpendicular to the surface of the water and parallel to the Earth absolute velocity. Its projection on the Earth absolute velocity vector is oriented in the same direction as the latter.

We will consider the other cases later.

The time needed by the ray to cover the distance SO is equal to:

$$t = \frac{d}{C_i} + \frac{d'}{C_r},$$

where $C_i$ is the speed of light in air from S to the surface of the water, and $C_r$ the speed of light from the surface to point O.

According to NEAT, the value of the light speed in air is (see appendix 2):

$$C_i = C - V \cos(\frac{\pi}{2} - i) = C - V \sin i,$$

where $V$ is the Earth absolute velocity, and $\frac{\pi}{2} - i$ is the angle separating the light signal from the Earth absolute velocity vector.

The velocity of light in water can be deduced from Hoek’s experiment in accordance with formula (11).

$$C_r = \frac{C}{n} - \frac{V}{n^2} \cos(\frac{\pi}{2} - r) = \frac{C}{n} - \frac{V}{n^2} \sin r.$$
Therefore the time needed by the light ray to cover the distance from S to O is:

\[
\begin{align*}
t &= \frac{d}{C - V \sin i} + \frac{d'}{(C/n) - (V/n^2) \sin r} \\
    &= \frac{d}{C - V \frac{x}{d}} + \frac{d'}{(C/n) - (V/n^2) \frac{\ell - x}{d'}} \\
    &= \frac{d}{C \left(1 - \frac{Vx}{dC}\right)} + \frac{d'}{(C/n) \left(1 - \frac{V}{nC} \frac{\ell - x}{d'}\right)}.
\end{align*}
\]

Here it should be noted that all the experimental and theoretical works that were designed to evaluate the absolute velocity of the Earth [16-19, 23, 27], estimated that its value should be between 200 km/sec and 450 km/sec.

Therefore, in the series expansion of the expressions \(\frac{1}{1 - \frac{Vx}{dC}}\) and \(\frac{1}{1 - \frac{V}{nC} \frac{\ell - x}{d'}}\), the order of magnitude of the ratio of the third term to the second term, for each of these series, is between zero and \(10^{-3}\).

Ignoring all the minute terms from the third, the expression of \(t\) reduces to:

\[
\begin{align*}
\frac{d}{C} \left(1 + V \frac{x}{dC}\right) + \frac{d'}{(C/n) \left(1 + \frac{V}{nC} \frac{\ell - x}{d'}\right)},
\end{align*}
\]

which simplifies to:

\[
\frac{d + d'n}{C} + \frac{V \ell}{C^2}.
\]

Expressing this formula in terms of \(x\), we get:

\[
t = \frac{\sqrt{a^2 + x^2} + n \sqrt{b^2 + (\ell - x)^2}}{C} + \frac{V \ell}{C^2}.
\]

According to Fermat’s principle [28] this time must be the least time, required by the light ray to travel from S to O. Therefore the derivative of \(t\) relative to \(x\) must be equal to zero.
The derivative of $t$ relative to $x$ is:

$$t' = \frac{x}{C\sqrt{a^2 + x^2}} - \frac{n(\ell - x)}{C\sqrt{b^2 + (\ell - x)^2}}.$$  

Given that it cancels we obtain:

$$\frac{x}{\sqrt{a^2 + x^2}} = \frac{n(\ell - x)}{\sqrt{b^2 + (\ell - x)^2}},$$  

which yields

$$\frac{x}{\bar{d}} = n\frac{(\ell - x)}{d'},$$  

or, in other words:

$$\sin i = n \sin r.$$  

The same result is obtained with a light ray running symmetrically with respect to $d$ and $d'$ (relative to the vertical axis).

Of course according to NEAT, the ratio $C_i/C_r$ measured in this example differs from $n$ which, as we saw, is the optical index in the fundamental aether frame, and is also equal to the ratio of the two-way speed of light in air to the two-way speed of light in the medium.

For convenience let us call this ratio $\Gamma$. We have:

$$\Gamma = \frac{C_i}{C_r} = \frac{C - V \sin i}{(C/n) - (V/n^2) \sin r}.$$  

To first order, this ratio reduces to the following expression

$$n(1 - \frac{V}{C} \sin i) + \frac{V}{C} \sin r$$  

which, clearly highlights the angular dependence of $C_i/C_r$ according to NEAT.

However, even though, for non-entrained aether theory, $n$ is not the optical index in moving refractive media, the above calculation shows that NEAT permits to account for the Snell-Descartes’ relation $\sin i = n \sin r$, which proves that it does not disagree with the experimental facts.

(Thus, according to NEAT, instead of optical index, $n$ should be designated as refractive coefficient).

If the light ray is propagated symmetrically with respect to $d$ and $d'$ (relative to the vertical axis), the ratio of the speed of light in air to that in the medium (that for convenience we shall call $\Gamma'$) will be different, its value being:

$$\Gamma' = \frac{C'_i}{C'_r} = \frac{C + V \sin i}{(C/n) + (V/n^2) \sin r}.$$  

To first order this ratio reduces to:

$$n(1 + \frac{V}{C} \sin i) - \frac{V}{C} \sin r.$$
The difference between the expressions of $\Gamma'$ and $\Gamma$ yields:

$$\Delta \approx \frac{2V}{C}(n \sin i - \sin r).$$

Therefore

$$V \approx \frac{\Delta C}{2(n \sin i - \sin r)}.$$

If the light ray runs in a plane perpendicular to the Earth absolute velocity, the speed of the incident beam will be reduced to $C$, and the speed of the refracted beam will be reduced to $C/n$. Therefore the Snell-Descartes' law will apply exactly as in the conventional approaches.

All the intermediate cases are a combination of the two extreme cases we considered. Therefore the Snell-Descartes' law will apply in all of them, showing that NEAT complies with the experiment.

This study also shows that, if the Snell-Descartes' laws according to NEAT, on the one hand, and according to SR and aether drag theory, on the other hand, are identical to a high degree of accuracy, this does not exclude minute second order differences that could only be highlighted by experiments of extreme precision. If despite the difficulties these experiments could be performed, they could help to verify which theory better complies with facts.

**Conclusion**

This study shows that several physical data do not take the same value and are not subject to the same laws in NEAT and in special relativity. In addition to the speed of light which according to NEAT is anisotropic, this concerns in particular the optical indices which for non-entrained aether theory, depend on the angle and, also, the interpretation of Fizeau's experiment. Indeed, as a thorough analysis in light of Hoek’s studies shows, Fizeau's interference pattern results from an aether drift, a fact that removes all doubts about the existence of the aether. One might nevertheless wonder whether the differences between NEAT and SR could be objectified by experiments dealing with the refraction of light. This is not the case as the example of the refraction in water indicates. As we have shown, even though in the Earth frame $n$ is not systematically equal to the ratio of the speed of light in air to that in refractive media, the demonstration of the Snell-Descartes' law can be carried out on the basis of the assumptions of NEAT to a high degree of accuracy.

Finally, this study did not reveal any argument against the aether, and, instead, provided evidence for its support.
Appendix 1

Demonstration of Hoek’s Formula According to NEAT

We start from the expression seen in section 2:

\[
\frac{L}{C + V} + \frac{L}{(C/n) - x} = \frac{L}{C - V} + \frac{L}{(C/n) + x}
\]

From this expression we obtain successively:

\[
\left[\frac{C}{n} - x + C + V\right]\left(C - V\right)\left(\frac{C}{n} + x\right) = \left[\frac{C}{n} + x + C - V\right]\left(C + V\right)\left(\frac{C}{n} - x\right)
\]

and

\[
Vx^2 + \left(C^2 - V^2\right)x - VC^2/n^2 = 0
\]

Resolving the second degree equation yields:

\[
x = \frac{-\left(C^2 - V^2\right) \pm \sqrt{\left(C^2 - V^2\right)^2 + 4V^2C^2/n^2}}{2V}
\]

Retaining only the + sign which has only a physical meaning, and eliminating the minute second order terms in the development of the square root formula, this expression can be simplified so that:

\[
x \approx \frac{-\left(C^2 - V^2\right) + \left(C^2 - V^2\right)(1 + \frac{2V^2C^2}{n^2\left(C^2 - V^2\right)^2})}{2V}
\]

\[
= \frac{VC^2}{n^2\left(C^2 - V^2\right)}
\]

Given that \(V^2 << C^2\) this expression reduces to \(V/n^2\)

Therefore, to first order, the speed of light in refractive media at rest on Earth is:

in the direction of the Earth absolute velocity

\[
\frac{C}{n} - \frac{V}{n^2}
\]

and in the opposite direction:

\[
\frac{C}{n} + \frac{V}{n^2}
\]

Appendix 2

Speed of Light in any Direction of Space According to NEAT

Let us consider two co-ordinate systems, \(S_0\) and \(S\). \(S_0\) is at rest in the cosmic substratum (fundamental aether frame) and \(S\) is attached to a platform which moves with rectilinear
uniform motion at speed \( V \) along the \( x_0 \)-axis of the system \( S_0 \), and suppose that a rod \( MN \), making an angle \( \alpha \) with the \( x_0, x \)-axis, is at rest with respect to the system \( S \) [12, 29] (Figure 4).

At the two ends of the rod, let us place two mirrors facing one another by their reflecting surface, which is perpendicular to the axis of the rod \( \ell = MN \). At the initial instant, the two systems \( S_0 \) and \( S \) overlap. At this very instant a light signal is sent from the common origin and travels along the rod towards point \( N \). When the signal reaches this point the rod has been transferred to a distance equal to \( Vt \) and is referred to as \( M'N' \) where \( t \) is the time needed by the signal to cover the distance \( MN \). After reflection the signal reverses its travel.

We remark that the path of the light signal along the rod is related to the speed \( C_1 \) by the relation:

\[
C_1 = \frac{MN}{t}.
\]

In addition, when the signal reaches point \( N \), the system \( S \) has moved away from \( S_0 \) a distance \( MM' = Vt \), so that:

\[
V = \frac{MM'}{t}.
\]

![Fig. 4](image)

The speed of light is equal to \( C_1 \) from \( M' \) to \( N' \) and to \( C \) from \( M \) to \( N' \).

The same distance has been covered by point \( N \) which is transferred to \( N' \).

Now, from the point of view of an observer which is supposed at rest in \( S_0 \), the signal goes from point \( M \) to point \( N' \).

\( C \) being the speed of light in \( S_0 \), we have:

\[
\frac{MN'}{t} = C
\]

and hence, the projection along the \( x_0, x \)-axis of the speed of light \( C_1 \) relative to the system \( S \), will be equal to \( (C \cos \theta - V) \). So that:

\[
C \cos \theta - V = C_1 \cos \alpha.
\]
The three speeds, $C$, $C_1$ and $V$ being proportional to the three lengths $MN'$, $MN$ and $MM'$ with the same coefficient of proportionality, we have

$$C^2 = (C_1 \cos \alpha + V)^2 + C_1^2 \sin^2 \alpha.$$ 

Therefore:

$$C_1^2 + 2VC_1 \cos \alpha - (C^2 - V^2) = 0. \quad (13)$$

We must emphasize that the value of the three speeds $C$, $C_1$ and $V$ depend on the same time $t$ which according to NEAT is the real time (not affected by measurement distortions). Although this may be a challenge for the experiment, a theoretical investigation is possible.

Resolving the second degree equation, yields:

$$C_1 = -V \cos \alpha \pm \sqrt{C^2 - V^2 \sin^2 \alpha}.$$ 

The condition $C_1 = C$ when $V = 0$ compels us to consider only the + sign so:

$$C_1 = -V \cos \alpha + \sqrt{C^2 - V^2 \sin^2 \alpha}. \quad QED \quad (14)$$

Now, the return of light can be illustrated by the Figure 5 below:

![Figure 5](image)

**Fig. 5** The speed of light is equal to $C_2$ from $N''$ to $M''$ and to $C$ from $N'$ to $M''$. $\theta'$ is the angle separating $N'M''$ from the $x_0$,$x$-axis. (Not indicated in the figure)

From the point of view of an observer attached to the system $S$, the light comes back to its initial position with the speed $C_2$.

Therefore we can write:

$$C_2 = \frac{N''M''}{t'}$$

where $t'$ is the time of light transit from its final to its initial position.

For an observer which is supposed at rest relative to the system $S_0$, the light comes from $N'$ to $M''$ with the speed $C$, so that:

$$C = \frac{N'M''}{t'}.$$
During the light transfer, the system $S$ has moved from $M'$ to $M''$ with the speed $V$, therefore:

$$V = \frac{M'M''}{t'}.$$  

The projection of the speed of light relative to $S$ along the $xo,x$-axis will be:

$$C \cos \theta' + V = C_2 \cos \alpha$$

where $\theta'$ is the angle separating $N'M''$ from the $xo,x$-axis. We easily verify that:

$$(C_2 \cos \alpha - V)^2 + C_2^2 \sin^2 \alpha = C^2,$$

therefore,

$$C_2 = V \cos \alpha + \sqrt{C^2 - V^2 \sin^2 \alpha}.$$  \hspace{1cm} (15)

QED.

**Note**

The relations (14) and (15) have been demonstrated for a rod lying in the vertical plane, but it is also valid for any rod lying in a plane passing by the $xo,x$-axis, and, therefore, for any angle separating the light signal travelling along such a rod and the Earth absolute velocity vector.

**Acknowledgements**

I am grateful to Professor Gianfranco Spavieri for thorough comments and helpful exchange of views.

I would also like to thank Professor Maurizio Consoli, for the careful review he has done as reviewer of this article and for the pertinent remarks he did.

**References**


B. Aether theory clock retardation vs. special relativity time dilation, in *Ether space–time & cosmology*, Volume 2 (New insights into a key physical medium), Michael. C. Duffy and Joseph Levy, Editors, (Apeiron publ, Montreal, Canada, 2009).


http://openlibrary.org/books/OL13509851M/Einstein’s_theory_of_relativity


[28] P de Fermat’s principle of least time,

http://cnx.org/content/m12895/latest/
