

Canonical Relational Quantum Mechanics from Information Theory

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Abstract: In this paper we construct a theory of quantum mechanics based on Shannon information theory. We define a few principles regarding information-based frames of reference, including explicitly the concept of information covariance, and show how an ensemble of all possible physical states can be setup on the basis of the accessible information in the local frame of reference. In the next step the Bayesian principle of maximum entropy is utilized in order to constrain the dynamics. We then show, with the aid of Lisi’s universal action reservoir approach, that the dynamics is equivalent to that of quantum mechanics. Thereby we show that quantum mechanics emerges when classical physics is subject to incomplete information. We also show that the proposed theory is relational and that it in fact is a path integral version of Rovelli’s relational quantum mechanics. Furthermore we give a discussion on the relation between the proposed theory and quantum mechanics, in particular the role of observation and correspondence to classical physics is addressed. In addition to this we derive a general form of entropy associated with the information covariance of the local reference frame. Finally we give a discussion and some open problems.

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“Information is the resolution of uncertainty.”

- Claude Shannon

1. Introduction

Quantum mechanics constitutes a conceptual challenge as it defies many pivotal classical concepts of physics. Despite this the theoretical and experimental success of quantum

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mechanics is unparallel [15]. The intuitive construction of classical mechanics and the perhaps counter-intuitive quantum mechanical formulation of reality has thus amounted to a problem of interpretation of quantum mechanics. The list of interpretations of quantum mechanics is long, but perhaps the most common interpretations are: Copenhagen [2], consistent histories [7], many worlds [5] and relational [15]. Many of these approaches share most central features of quantum mechanics, the difference is mainly the philosophical context in which they are situated. The canonical problem in these approaches is perhaps the counter-classic, and in many peoples views, counter-intuitive principle that physics is fundamentally based on probability. The perhaps strongest opponent of a probability-based theory of physics was Einstein who constructed several unsuccessful thought experiments and theories in order to disprove the commonly accepted view of quantum mechanics [4]. As a matter of fact all approaches to create a non-probabilistic version of quantum mechanics have failed [3]. It has also been proven, via for example Bell's theorems, that the construction of such a deterministic or "local hidden variable theory" is impossible [3]. Thus one has no choice but to conclude that physics, inevitably, has quantum properties.

1.0.1 Relational Quantum Mechanics

It was Einstein's revised concepts of simultaneity and frame of reference that inspired Rovelli to formulate a theory of mechanics as a relational theory; *relational quantum mechanics*. In this seminal approach frames of reference were utilized and only relations between systems had meaning. This setup gave interesting solutions to several quantum-related conceptual problems such as the EPR-paradox [15]. It may be concluded that Rovelli recast physics in the local frame of reference in such a way that the interactions between systems amounted to the observed quantum phenomena. A particular facet of Rovelli's approach was that any physical system could be observed in different states by different observers "simultaneously". This property can be interpreted as an extension of the concept of simultaneity in special relativity [15]. The theory of relational quantum mechanics was based on the hypothesis that quantum mechanics ultimately arose when there was a lack of information of investigated systems.

1.0.2 Universal Action Reservoir

In a recent paper Lisi gave an interesting approach to quantum mechanics based on information theory and entropy [12]. He showed that given a *universal action reservoir* and a principle for maximizing entropy quantum mechanics could be obtained. In his paper the origin of the universal action reservoir was postulated as a principle and was given no deeper explanation. This was addressed in recent papers by Lee [10, 11] where he suggested that it was related to information theory coupled to causal horizons.

1.0.3 Relational Quantum Mechanics and the Universal Action Reservoir

In this paper we shall recast Rovelli's relational approach to quantum mechanics on the concept of information covariance and connect it to Lisi's canonical information theoretic approach. In the process of this we show that the universal action reservoir is an inevitable consequence of incomplete information in physics. This theory is, in spirit of Rovelli's approach, a generalization of the special relativistic concept regarding frames of reference.

2. Physics with Information Covariance

2.1 Principles for Information-based Frames of Reference

Let us assume that we have a set of observations of certain physical quantities of a physical system. This is the obtained information regarding the system. For all other events that has not been observed we only know that *something* possible happened. If we knew with certainty *what* happened then we as observers would be in a frame of reference based on complete information regarding the explored system. However such a theory would require some form of a conservation law which would require that a physical system has a predetermined state when not observed. Let us instead consider the opposite: Assume that no observation is made of the system, then information regarding that system is not accessible and thus not inferable without more observations. It is then reasonable that *anything physically possible* could have occurred when it was not observed. If one can base the laws of physics on the premise of only what is known in the frame of reference, in terms of information, regarding any physical system in connection with the observer and her frame then one has attained a high form of generality in the formulation of the laws of physics. This amounts to a seemingly tautological yet powerful principle:

If a system in physics is not observed it is in any physically possible state.

We shall call this principle *the principle of physical ignorance*. In one sense this principle is intuitive and trivial: the unknown is not known. In another more intricate sense it violates a number of basic principles of physics such as many of the laws of classical physics. The law of inertia is one such principle for example; the inertia of a body does not necessarily hold when we do not observe it. However one cannot assume that Newton's laws or any other classical laws hold in a system for which limited information is known. For all we know regarding classical physics is that it holds in a certain classical limit. The definition of "known" here is what information has become accessible in ones frame of reference obtained through interaction with another system. The laws of physics should thus be formulated in such a way that they hold regardless of information content in the local frame of reference; the formulation of the laws of physics should be invariant with respect to the information content. This amounts to a principle for the formulation of the laws of physics:

The laws of physics are defined on the basis of the information in the frame of reference.

This principle shall be call the *principle of information covariance*. It's scope of generality is similar to the *principle of general covariance*, which is the natural generalization

of the frames of reference used in special relativity. The *principle of information covariance* suggests that the frame of reference is entirely based on information; thus any new observations will alter the information content and thus re-constrain the dynamics of the studied system "projected" within the frame of reference. One should also keep in mind that physics cannot, in a frame of incomplete information, "project" any information through physical interactions that is not inferable from the accessible information. This creates a form of locality of physics: Physics only exists in every frame of reference. Thus there exists no such thing as an objective observer. It should be noted here that there exists a similar notion of frames of reference in traditional relational quantum mechanics [15]. The dynamics is derived from a different set of principles and formal setup but arrives at a similar conclusions [15].

2.2 Formal Setup

Let us assume that we have a set of observations A_n regarding physical quantities of a system C from a frame of reference K . The available information regarding the system C in K is given by the information in the observed physical quantities A_n and what can be inferred from them, the rest of the properties of the system are by *the principle of physical ignorance* unknown. Formally according to *the principle of physical ignorance* we may conclude that the possible configurations of the explored physical system C in the frame of reference K has to form a set of all possible physical configurations under the constraint of the set of observations A_n for the system. Consider a set of configuration parameters for each possible physical configuration or path in configuration space $path = q(t)$. The set of configurations parameters is parameterized by one or more parameters t . Then for each physical system we may associate an action $S[path] = S[q(t)]$ defined on the basis of the configuration space path $q(t)$. The structure of the action is here assumed to be something of a universal quantity of information inherent to every frame of reference, it is the paths in the configuration space of objects that is unknown to every observer until observed. The action of a system is defined classically as $S = \int L(q, \dot{q})dt$, where $L(q, \dot{q})$ is the Lagrangian for the system [17]. Traditionally with the aid of a variation principle the expected action of a classical system is used to derive the dynamics of the system [17]. In quantum mechanics a probabilistic setup is performed, ideally via a path integral formulation. In our situation we shall instead look for all possible configurations under condition of the observations A_n in accordance with the previously defined *principle of physical ignorance*. The possible actions, based on the possible configurations, are each associated with a probability of occurring, $p[path] = p[q(t)]$. The sum of these probabilities need to satisfy the ubiquitous normalization criterion:

$$1 = \sum_{paths} p[path] = \int Dq p[q], \quad (1)$$

Along with this we may conclude that any functional, or observable quantity Q , has an expected value according to the expression [12]:

$$\langle Q \rangle = \sum_{paths} p[path] Q[path] = \int Dqp[q] Q[q]. \quad (2)$$

We have the expected action $\langle S \rangle$ according to [12]:

$$\langle S \rangle = \sum_{paths} p[path] S[path] = \int Dqp[q] S[q]. \quad (3)$$

As a measure of the information content, or rather lack thereof, we can construct the entropy of the system according to:

$$H = - \sum_{paths} p[path] \log p[path] = - \int Dqp[q] \log p[q]. \quad (4)$$

So far we have merely utilized information theoretic concepts, we shall deal with its physical consequences further on in this paper. Although the probabilities for the events in system C observed from reference system K are yet undefined we have a set of possible configurations that could occur for a system and we have associated a probability of occurrence with each based on the ensemble setup above. In order to deduce the probability associated with each possible event we need some form of restriction on the ensemble of possibilities. In a Bayesian theory of inference there is a maximization principle regarding the entropy of a system called the *Principle of maximum entropy* [8] which postulates the following:

Subject to known constraints, the probability distribution which best represents the current state of knowledge is the one with largest entropy.

This principle is utilized in several fields of study, in particular thermodynamics where it serves as the second fundamental law [8]. We shall assume that this principle holds and we shall utilize it as a restriction on our framework. In [12] Lisi performed the following derivation which is worth repeating here. By employing Lagrange multipliers, $\lambda \in C$ and $\alpha \in C$, the entropy (4) is maximized by:

$$H' = - \int Dqp[q] \log p[q] + \lambda \left(1 - \int Dqp[q] \right) + \alpha \left(\langle S \rangle - \int Dqp[q] S[q] \right), \quad (5)$$

which simplified becomes:

$$H' = \lambda + \alpha \langle S \rangle - \int Dq (p[q] \log p[q] + \lambda p[q] + \alpha p[q] S[q]). \quad (6)$$

If we perform variation on the probability distribution we get:

$$\delta H' = - \int Dq (\delta p[q]) (\log p[q] + 1 + \lambda + \alpha S[q]) \quad (7)$$

which is extremized when $\delta H' = 0$ which corresponds to the probability distribution:

$$p[q] = e^{-1-\lambda} e^{-\alpha S[q]} = \frac{1}{Z} e^{-\alpha S[q]} \quad (8)$$

which is compatible with the knowledge constraints [12]. By varying the Lagrange multipliers we enforce the two constraints, giving λ and α . Especially one gets: $e^{-1-\lambda} = \frac{1}{Z}$ where Z is the partition function on the form:

$$Z = \sum_{paths} e^{-\alpha S[path]} = \int Dq e^{-\alpha S[q]} \quad (9)$$

and the parameter α is determined by solving:

$$\langle S \rangle = \int Dq S[q] p[q] = \frac{1}{Z} \int Dq S[q] e^{-\alpha S[q]} = -\frac{\partial}{\partial \alpha} \log Z. \quad (10)$$

In order to fit the purpose Lisi concluded that the Lagrange multiplier value; $\alpha \equiv \frac{1}{i\hbar}$. Lisi concluded that this multiplier value was an intrinsic quantum variable directly related to the average path action $\langle S \rangle$ of what he called the universal action reservoir. In similarity with Lisi's approach we shall also assume that the arbitrary scaling-part of the constant α is in fact $1/\hbar$. Lisi also noted that Planck's constant in α is analogous to the thermodynamic temperature of a canonical ensemble, $i\hbar \leftrightarrow k_B T$; being constant reflects its universal nature - analogous to an isothermal canonical ensemble [12]. This assumption along with (9) brings us to the following partition function:

$$Z = \sum_{paths} e^{i \frac{S[path]}{\hbar}} = \int Dq e^{i \frac{S[q]}{\hbar}}. \quad (11)$$

By inserting (11) into (2) we arrive at the following expectation value for any physical quantity Q :

$$\langle Q \rangle = \sum_{paths} p[path] Q[path] = \int Dq Q[q] p[q] = \frac{1}{Z} \int Dq Q[q] e^{i \frac{S[q]}{\hbar}}, \quad (12)$$

This suggests that a consequence of the incomplete information regarding the studied system is that physics is inevitably based on a probabilistic framework. Conversely, had physics not been probabilistic in the situation of incomplete information then information of the system could be inferred. But a process of inferring results from existing limited information does not provide more information regarding that system than the limited information had already provided. That would have required, as we previously argued, an additional principle of perfect information. Instead it is only interaction that can provide new information. We may conclude that by the *principle of information covariance* physics is local and based only on the available information in the local information-based frame of reference. In turn this this creates an ensemble of possible states with a definite and assigned expectation value for each physical quantity in the studied system according to (12). This formalism, which might be called *information covariant*, is then directly compatible with the general principle of relativity wherein *All systems of reference are equivalent with respect to the formulation of the fundamental laws of physics*.

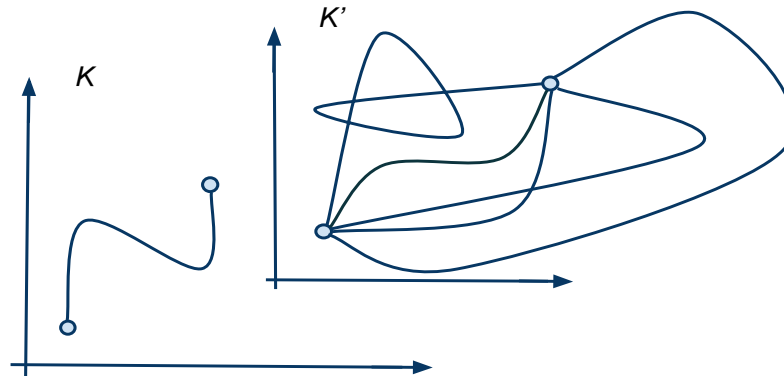


Fig. 1 This illustration shows the path of a particle from one point to another in a complete information frame of reference K (essentially a particle that is observed along its path). It also shows some of the possible paths a particle takes in the incomplete information frame of reference K' .

3. Connections to Quantum Mechanics

3.1 Path Integral Formulation

The path integral formulation, originally proposed by Dirac but rigorously developed by Feynman [6], is perhaps the best foundational approach to quantum mechanics available [12]. It shows that quantum mechanics can be obtained from the following three postulates assuming a quantum evolution between two fixed endpoints [6]:

1. The probability for an event is given by the squared length of a complex number called the probability amplitude.
2. The probability amplitude is given by adding together the contributions of all the histories in configuration space.
3. The contribution of a history to the amplitude is proportional to $e^{iS/\hbar}$, and can be set equal to 1 by choice of units, while S is the action of that history, given by the time integral of the Lagrangian L along the corresponding path.

In order to find the overall probability amplitude for a given process then one adds up (or integrates) the amplitudes over postulate 3 [6]. In an attempt to link the concept of information-based frames of reference - developed in this paper - to quantum mechanics we shall utilize Lisi's approach wherein the probability for the system to be on a specific path is evaluated according to the following setup (see [12] for more information). The probability for the system to be on a specific path in a set of possible paths is:

$$p(\text{set}) = \sum_{\text{paths}} \delta_{\text{path}}^{\text{set}} p[\text{path}] = \int Dq \delta(\text{set} - q) p[q]. \quad (13)$$

Here Lisi assumed that the action typically reverses sign under inversion of the parameters of integration limits:

$$S^{t'} = \int^{t'} dt L(q, \dot{q}) = - \int_{t'} dt L(q, \dot{q}) = -S_{t'}. \quad (14)$$

This implies that the probability for the system to pass through configuration q' at parameter value t' is:

$$p(q', t') = \int Dq \delta(q(t') - q) p[q] = \left(\int^{q(t')=q'} Dq p^{t'}[q] \right) \left(\int_{q(t')=q'} Dq p_{t'}[q] \right) = \psi(q', t') \psi^\dagger(q', t'), \quad (15)$$

in which we can identify the quantum wave function:

$$\psi = \int^{q(t')=q'} Dq p^{t'}[q] = \frac{1}{\sqrt{Z}} \int^{q(t')=q'} Dq e^{-\alpha S^{t'}} = \frac{1}{\sqrt{Z}} \int^{q(t')=q'} Dq e^{i \frac{S^{t'}}{\hbar}}. \quad (16)$$

The quantum wave function $\psi(q', t)$ defined here is valid for paths $t < t'$ meeting at q' while its complex conjugate $\psi^\dagger(q', t')$ is the amplitude of paths with $t > t'$ leaving from q' . Multiplied together they bring the probability amplitudes that gives the probability of the system passing through $q'(t')$, as is seen in (15). However, just as Lisi points out [12], this quantum wave function in quantum mechanics is subordinate to the partition function formulation since it only works when t' is a physical parameter and the system is t' symmetric, providing a real partition function Z . Indeed, the postulate of an information covariant setup on the laws of physics according to the previous section suggests that physics is ruled by the general complex partition function (9):

$$Z = \sum_{paths} e^{i \frac{S[path]}{\hbar}} = \int Dq e^{i \frac{S[q]}{\hbar}}. \quad (17)$$

How does this relate to the path integral formulation? The sum in the partition function (17) is a sum over paths. Let us take the common situation when the path is that of a particle between two points. We can then conclude that each term is on the form $e^{iS[path]/\hbar}$ which is equivalent to postulate 3. Furthermore all paths are added, thus postulate 2 is also checked. Also, at least for the situation where $p(q', t') = \psi(q', t') \psi^\dagger(q', t')$ the sum adds up to the probability density, checking postulate 1 as well. Thus we may conclude that the information covariant approach is equivalent to the canonical *path integral formulation* of quantum mechanics under the circumstances provided for it.

3.2 Quantum Properties

The path integral formulation is canonical for quantum mechanics and covers the wide variety of special features inherent to quantum mechanics [6, 12]. Since the approach in this paper is equivalent to the path integral formulation in most aspects, some properties are be worth discussing. A pivotal component of quantum mechanics is the canonical commutation relation which gives rise to the Heisenberg uncertainty principle [3, 6]. For

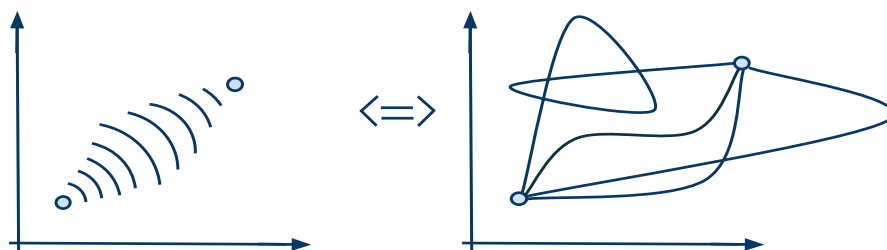


Fig. 2 This illustration shows on the left hand side the uncertainty of path of a particle from one point to another and on the right hand side that the particle takes all possible paths from one point to another. These two interpretations are equivalent under the general interpretation that information is incomplete. Uncertainty in path means in practice that it takes any possible path until we observe it, a superposition of states is inevitable when information is incomplete.

example the famous commutation relation between position x and momentum p of a particle is defined as:

$$[x, p] = i\hbar. \quad (18)$$

This can be obtained through the path integral formulation by assuming a random walk of the particle from starting point to end point [6]. This works with this theory as well under the same considerations since a random walk is equivalent to a walk with no information about direction. In the path integral formulation it is also possible to show that for a particle with classical non-relativistic action (where where m is mass and x is position):

$$S = \int \frac{m\dot{x}^2}{2} dt, \quad (19)$$

that the partition function Z in the path integral formulation turns out to satisfy the following equation [6]:

$$i\hbar \frac{\partial Z}{\partial t} = \left[-\frac{1}{2}\nabla^2 + V(x) \right] Z. \quad (20)$$

This is the Schrödinger equation for $Z = \psi$ and where $V(x)$ is a potential [3]. It is also possible to show the conservation of probability from the Schrödinger equation (20) [3]. Here we can see that the traditional usage of operators on a Hilbert space in quantum mechanics is a useful tool when information is incomplete. Another interesting aspect of quantum mechanics is the *superposition principle* which states that a particle occupies all possible quantum states simultaneously [3]. That the dynamics of a system is fundamentally unknown or occupying all states simultaneously are both parts of the same concept that information is incomplete regarding the system. The popular quantum superposition thought experiment *Schrödinger's cat* in which the alive/dead state of a cat in a hazardous closed box is also evidently based on the lack of information regarding the state of the cat. The superposition is intuitively equivalent to the lack of information. The resolution of this problem in this theory is that the state of the cat is fundamentally

unknown in our system of reference until we open the box and observe it, thus obtaining information. The situation is also relational because even when information is obtained the information is only inherent to our frame of reference which might not necessarily be the same for any other frame of reference. For a complete verification that quantum mechanics can be interpreted as a relational theory see [15]. The proof that quantum mechanics can be interpreted as relational is constructed with Bra-kets in Hilbert spaces and is directly related to a linearity inherent to quantum mechanics. Another interesting topic worth mentioning here is the famous double slit experiment. The setup is as follows: one has two slits and behind them a detection screen is setup. Some form of beam of particles is then sent through the slits and a statistical pattern is shown on the detection screen [3]. The result is an interference pattern equivalent to that described by the path integral formulation of quantum mechanics. Such a pattern is not expected in classical mechanics. The interpretation from the theory developed in this paper goes as follows: due to the lack of information in the local information-based frame of reference a particle takes any possible path, a similar interpretation was given by Lee [10]. This situation is equivalent to the path integral formulation. The particle is, in our frame of reference, wave-like until we observe it. This shows how the ubiquitous wave-particle dualism arises under the lack of information. As far as observation goes, we will discuss that in section 3.5 below. Quantum entanglement is also a particular feature of quantum mechanics that has spurred interpretational complications regarding quantum mechanics. Quantum entanglement implies, among other things, that information can travel at a superluminal speeds in most interpretations of quantum mechanics [4]. This violates the principle of invariant speed of light inherent to special relativity [4]. However it has been solved for relational quantum mechanics by postulating that physics is local, and then it can be shown that no superluminal transfer of information occurs [11]. Since the theory presented in this paper by design is relational the same conceptual solution holds. We will discuss the connection between the theory developed in this paper and Rovelli's version of relational quantum mechanics in section 3.3 below. Since the theory developed in this paper does not violate quantum mechanics it ought also to be completely compatible with the Bohm-De Broglie pilot-wave approach [1] to quantum mechanics.

3.3 Connections to Relational Quantum Mechanics

Relational quantum mechanics is a theory of quantum mechanics based on the notion that only systems in relation have meaning [15]. The observer and the partially observed system makes out such a system typically. This addresses the *problem of the third person* or *Wigner's friend* as it is also called in which an observer observes another observer observing. The problem is solved by assuming that the two observers may "simultaneously" observe different states regarding the same object under observation. This is shown to be a legal construction in quantum mechanics and is, as Rovelli points out, non-antagonistic towards the most common formulations of quantum mechanics [15]. Furthermore, in his seminal paper introducing relational quantum mechanics Rovelli postulates that quantum

mechanics arises from the lack of information in classical physics. Given these facts one might ask the following question: what is the similarity and what is the difference between Rovelli's approach and the theory developed in this paper? First of all our approach is relational and based on a local frame of reference, which is similar to the concept used by Rovelli. Second, this theory of quantum mechanics is based on information which is similar to that used in Rovelli's approach. There are two main differences. First of all this theory is based on a few principles - different than those used by Rovelli - that sets the foundation for a information covariant relational theory of physics. Second, it utilizes an ensemble setup of Shannon information theory, developed by Lisi, which equips relational quantum mechanics with an information-based path integral formulation.

3.4 Correspondence Principle

The correspondence from quantum mechanics, or any quantum field theory, to classical physics is when $\hbar \rightarrow 0$ or more generally when $S \gg \hbar$. Since this theory is equivalent to the canonical path integral approach to quantum mechanics under reasonable considerations we may state tautologically that the correspondence to classical physics follows the same limits as for regular quantum mechanics. The meaning of the correspondence is also reasonable: If \hbar descends to zero the partition function will "collapse" and give only one expected value for each quantity: The most expected one which by the Ehrenfest theorem is the classical [3]. If the action is very large ($S \gg \hbar$) the situation is the same; the larger the ratio between the classical action and Planck's reduced constant is the more likely the classical outcome is.

3.5 Observations and Wave Function Collapse

Observation is by definition obtaining information from a studied object [13, 12]. In order to obtain information regarding a system one has to interact with it. This suggests that observation of a system in practice is interacting with it, a view of observation that is also held within the field of relational quantum mechanics [15]. In quantum mechanical terms when an observation is performed then a wave function collapse occurs [3]. In the theory developed in this paper the probability for a specific path (or state) becomes one. Naturally the quantum expectation value for a quantity Q simply becomes the one for the observed path A :

$$\langle Q \rangle = \sum_{paths} p[path]Q[path] = \int Dqp[q]Q[q] = \frac{1}{Z} \int DqQ[q]e^{i\frac{S[q]}{\hbar}} = Q[A]. \quad (21)$$

Observation of a system limits the possibilities of that system by obtaining information about it. The kinematics of a system, as viewed from our frame of reference, is based on the local information about it by *the principle of information covariance*.

4. Notes on Relativistic Invariance

The theory developed in this paper is by definition implicitly relativistic. The relativistic kinematics inherent to special relativity should hold in this theory under at least the same conditions for which the canonical path integral formulation is relativistic. In the situation of complete information (or $S \gg \hbar$) the laws of (special) relativity hold in the classical sense. We shall not detail the relativistic concepts further in this paper, nor shall we attempt at constructing a general relativistic, or a *general information covariant* theory of gravitation for the information-based frames of reference. Instead, this is left for future investigation. However one might presume that such an approach might share certain properties with the relational approach to quantum gravity [15].

5. Entropy

A great deal of this paper has consisted of meshing together mathematical structures derived by previous authors under a new set of principles. In contrast to this we shall here provide an explicit calculation of the entropy associated with an information-based frame of reference. We defined the entropy as follows (4):

$$H = - \sum_{paths} p[path] \log p[path] = - \int Dqp[q] \log p[q]. \quad (22)$$

The entropy (22) is based purely on information theory and has thus no obvious direct connection to physical quantities. Let us for this sake allow a scaling constant between the information entropy H and the thermodynamical entropy \mathcal{H} :

$$\mathcal{H} \equiv kH. \quad (23)$$

It was shown that after maximizing the entropy the probability of a specific path becomes (8):

$$p[path] = \frac{1}{Z} e^{i \frac{S[path]}{\hbar}}. \quad (24)$$

Despite the fact that the probability (24) is a complex entity and thus ill-defined in traditional probability theories it might still have meaning when used to calculate the entropy. Insert (24) in (22):

$$\mathcal{H} = -k \sum_{paths} p[path] \left(i \frac{S[path]}{\hbar} - \log Z \right) \quad (25)$$

We also have the normalization (1):

$$1 = \sum_{paths} p[path], \quad (26)$$

and the expression for the expected action (3):

$$\langle S \rangle = \sum_{paths} p[path] S[path] \quad (27)$$

Together (25), (26), (27) and the fact that the partition function Z is path-independent brings the following general expression for the entropy of the system:

$$\mathcal{H} = -k \left(\sum_{paths} p[path] i \frac{S[path]}{\hbar} - \sum_{paths} p[path] \log Z \right) = -k \left(i \frac{\langle S \rangle}{\hbar} - \log Z \right). \quad (28)$$

Let us now further assume the special case when the following identity holds:

$$\psi = Z. \quad (29)$$

This identity holds at least when $\psi = \psi(q', t')$ and t' is a symmetric physical parameter, just as described in section 3.1. Let us also assume that the structure of the wave function is as follows:

$$\psi = R e^{i \frac{S_c}{\hbar}} \quad (30)$$

where $R = |\psi|$ and S_c is the classical action [3]. This brings the following expression:

$$\log \psi = \log |\psi| + i \frac{S_c}{\hbar}. \quad (31)$$

Together (28) and (31) amounts to the following special case of the entropy:

$$\mathcal{H} = -k \left(i \frac{\langle S \rangle}{\hbar} - i \frac{S_c}{\hbar} - \log |\psi| \right). \quad (32)$$

If we assume the equivalence between the classical action S_C and the expected action $\langle S \rangle$, which is in accordance with the Ehrenfest theorem [3], then we get the following expression for entropy:

$$\mathcal{H} = k \cdot \log |\psi|. \quad (33)$$

An expression similar to (33) was suggested as a basis for the holographic approach to gravity [18] in a somewhat more speculative paper recently [13]. In that approach the constant was suggested to be $k = -2k_B$, where k_B was Boltzmann's constant. The expression for entropy (33) is strikingly similar to Boltzmann's formula for entropy in thermodynamics:

$$\mathcal{H} = k_B \cdot \log(W), \quad (34)$$

where \mathcal{H} is the entropy of an ideal gas for the number W of equiprobable microstates [14]. The suggested entropy (33) and its more general version (28) are, up to a scalable constant, measures of the lack of information in the information-based frame of reference.

6. Discussion

This paper proposes a conceptual foundation for quantum mechanics based on information brought on by the concept of information covariance. This approach supports the notion that physics is largely based on information, a concept that among others Wheeler strongly endorsed [19]. The suggested framework in this paper, building on Lisi's universal action reservoir and Rovelli's relational quantum mechanics, gives an intuitive description of physics; Physics in the quantum realm is a consequence of the incompleteness of information in the local frame of reference. By setting up an information covariant foundation in the local frame of reference by means of a maximization of Shannon entropy on the possible paths of a system we managed to - by using Lisi's theorem - establish a canonical formulation of relational quantum mechanics. This implies that Lisi's proposed universal action reservoir is the inevitable result of the observer ignorance of a system. We also explicitly calculated the entropy associated with any quantum mechanical system.

6.1 Open Problems

This theory primarily serves as a conceptual framework for quantum mechanics. However it also brings new concepts like for example the particular entropy (33) of a quantum mechanical system. The role and the application of the new entropy is not yet fully investigated. It could, for example, perhaps be related to holographic theories of gravitation [13, 18]. In addition to this the theory might give interesting effects in quantum statistical mechanics. Another open issue is how to construct a general relativistic approach to this theory. Such a theory ought to arrive at some similar problems that many quantum gravity theories have stumbled upon because this theory is merely a relational version of the canonical path integral formulation of quantum mechanics.

6.2 Final Comments

When cast in a local frame of reference physics has only a limited amount of information with which to function. When physics is fundamentally bound by a limited amount of information probabilistic effects will occur. By maximizing entropy probabilistic effects of quantum physics arise. The result of subjecting classical mechanics to incomplete information is quantum mechanics.

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