Some Bianchi type-I Cosmic Strings in a Scalar –Tensor Theory of Gravitation

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Abstract: The field equations are obtained in Sen–Dunn theory of gravitation with the help of LRS Bainchi type-I in the context of cosmic strings. We have solved the field equations when the shear σ is proportional to the scalar expansion θ. It is found that the cosmic do not exist with the scalar field except for some special cases and hence vacuum solutions are presented and discussed.

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1. Introduction

Brans and Dicke [1] have formulated a scalar-tensor theory of gravitation in which the tensor field alone is geometrised and the scalar field is alien to the geometry. Sen and Dunn [2] have proposed a new scalar-tensor theory of gravitation in which both the scalar and tensor fields have intrinsic geometrical significance. The scalar field in this theory is characterized by the function $\phi = \phi(x^i)$ where x^i are coordinates in the four – dimensional Lyra manifold and the tensor field is identified with the metric tensor $g_{ij}$of the manifold. The field equations given by Sen and Dunn for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R = \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \phi^{-2}T_{ij} \tag{1}$$

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where $\omega = \frac{3}{2}$, R_{ij} and R are respectively the usual Ricci-tensor and Riemann-curvature scalar (in our units $C = 8\pi G = 1$).

Sen-Dunn [3], Halford [4], Singh [5], Reddy [6, 7], Roy and Chatterjee [8, 9], and Reddy and Venkateswarlu [10]are some of the authors who have studied various aspects of this scalar-tensor theory of gravitation.

In recent years there has been lot of interest in the study of cosmic strings. Cosmic strings have received considerable attention as they are believed to have served in the structure formation in the early stages of the universe. Cosmic strings may have been created during phase transitions in the early era [11] and they act as a source of gravitational field [12]. It is also believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures of the universe.

The energy momentum tensor for a cloud of massive strings that can be written as

$$\mathrm{T}_{ij} = \rho u_i u_j - \lambda x_i x_j. \tag{2}$$

Here ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the tension density of the strings and $\rho = \rho_p + \lambda$, ρ_p being the energy density of the particles. The velocity u^i describes the 4 – velocity which has components (1, 0, 0, 0) for a cloud of particles and x^i represents the direction of string which will satisfy

$$u^i u_i = -x^i x_i = 1 \text{ and } u^i x_i = 0. \tag{3}$$

The study of cosmic strings in relativistic framework was initiated by Stachel [13] and Letelier [14]. Krori et.al. [15, 16], Raj Bali and Shuchi Dave [17], Bhattacharjee and Baruah [18], Mahanta and Abhijit Mukharjee [19], Rahaman et al. [20], Reddy [21], Pant and Oli [22], Venkateswarlu et al. [23] and Venkateswarlu and Pavan [24] are some of the authors who have studied various aspects of string cosmologies in general relativistic theory as well as in alternative theories of gravitation.

In this paper, we made an attempt to solve the LRS Bianchi type-I field equations in the context of cosmic strings in a new scalar-tensor theory of gravitation proposed by Sen and Dunn [2].

2. Metric and Field Equations

We consider the LRS Bianchi type –I metric as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2) \tag{4}$$

where A and B are functions of t only .

The field equation (1) for the metric (4) are given by

$$2\,\frac{B_{44}}{B} + \frac{B_4^2}{B^2} = \phi^{-2}\lambda + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 \tag{5}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 \tag{6}$$

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} = \phi^{-2}\rho - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 \tag{7}$$

where the subscript 4 denotes ordinary differentiation with respect to t.

3. Solutions to the Field Equations

The field equations (5) – (7) are a system of three equations with five unknown parameters A, B,ϕ, ρ and λ. We need two additional conditions to get a deterministic solution of the above system of equations. Thus we present the solutions of the field equations in the following physically meaningful cases:

***Case* 1:** We assume the following two conditions:

(i) The shear σ is proportional to the scalar expansion θ which leads to

$$B = A^m \tag{8}$$

where m is a constant and

(ii)

$$\rho = \lambda(\text{geometric strings}). \tag{9}$$

Now the field equations (5) - (7) together with (8) and (9) reduces to

$$\frac{A_{44}}{A} + 2m\frac{A_4^2}{A^2} = 0 \tag{10}$$

which on integration yields

$$A(t) = [(2m+1)(C_1t + C_2)]^{\frac{1}{(2m+1)}} \tag{11}$$

and from (8)

$$B(t) = [(2m+1)(C_1t + C_2)]^{\frac{m}{(2m+1)}} \tag{12}$$

where C_1 and C_2 are constants of integration.

From equation (6) the scalar field is given by

$$\phi = (C_1t + C_2)^k \tag{13}$$

where $k = \left[-\frac{2m(m+2)}{\omega(2m+1)^2}\right]^{1/2}$ and $m < -2$.

By making use of equations (11), (12) and (13) in equations (5) and (7), the string energy density ρ and the tension density λ become zero. Hence it is observed that cosmic geometric strings do not co-exist with the scalar field in this theory.

When $m = -1$, the vacuum solution of the field equations (5)-(7) can be written as (after suitably redefining the constants)

$$A(t) = \frac{1}{t} \tag{14}$$

$$B(t) = t \tag{15}$$

and the scalar field takes the form

$$\phi(t) = t^{\frac{\sqrt{2}}{\sqrt{\omega}}} \tag{16}$$

***Case* 2**: The p-strings or Takabayasi strings are represented by $\rho = (1+\xi)\,\lambda$, $\xi > 0$. In this case, the field equations (5) – (7) along with equation (8) reduced to

$$[\xi(1-m)+2]\,\frac{A_{44}}{A} + 2m[\xi(1-m)+2]\,\frac{A_4^2}{A^2} = 0 \tag{17}$$

Equation (17) is formerly similar to equation (10) and hence its solution is also similar to the one given by equations (11), (12) and (13). Thus in this case the p-strings do not co-exist with the Sen – Dunn's scalar field.

***Case* 3**: In this case we consider the condition given by equation (8) together with $\rho + \lambda = 0$. The solution of the field equations (5) – (7) together with (8) is similar to the solution obtained in case (1) and it is also observed that the non-existence of the scalar field in this theory.

***Case* 4**: To get a realistic solution here we assume that

$$A = A_0 t^m \text{ and } B = B_0 t^n \tag{18}$$

where A_0, B_0, m and n are arbitrary constants. Then from equation (6) we obtain

$$\phi = \phi_0 t^k \tag{19}$$

where ϕ_0 is an arbitrary constant and $k = \left(\frac{2(m^2+n^2+mn-m-n)}{\omega}\right)^{\frac{1}{2}}$.

Thus the LRS Bianchi type-I model can be written as (after suitably redefining the constants)

$$ds^2 = -dt^2 + t^{2m}\,dx^2 + t^{2n}(dy^2+dz^2). \tag{20}$$

The string energy density ρ, tension density λ, the particle density ρ_p, the scalar expansion θ, the shear scalar σ, spatial volume V and the deceleration parameter q are given by

$$\rho = \phi_0^2 \frac{(m+n)(m+2n-1)}{t^{2-2k}} \tag{21}$$

$$\lambda = \phi_0^2 \frac{(n-m)(m+2n-1)}{t^{2-2k}} \tag{22}$$

$$\rho_p = \rho - \lambda = \frac{2\phi_0^2 m(m+2n-1)}{t^{2-2k}}. \tag{23}$$

$$\theta = \frac{(m+2n)}{t} \tag{24}$$

$$\sigma = \frac{(m-n)}{\sqrt{3}\,t} \tag{25}$$

$$V = t^{(m+2n)} \tag{26}$$

$$q = \frac{(3-m-2n)}{(m+2n)} \tag{27}$$

The energy conditions viz.,$\rho > 0$, $\lambda > 0$ and $\rho_p > 0$ are identically satisfied for all $m > 0$ and $n > 0$. Since $\frac{\sigma}{\theta}$= constant, the model given by equation (20) does not approach isotropy at any stage. The spatial volume of the model increases with the increase in time. The model given by equation (20) for a cloud of cosmic strings possess a line singularity as ρ, λ, θ and σ tend to infinity and spatial volume tend to zero at initial epoch $t = 0$. When $m = n$, it is interesting to note that $\lambda = 0$ which corresponds to dust filled isotropic universe without strings.

Case 4.1: The case $\rho = \lambda$ refers to geometric strings. Here we have either $m = 0$ and $m = 1 - 2n$. If $m = 0$, the solution of the field equations (5)-(7) is given by

$$A = \text{Constant}, B = B_0 t^n \tag{28}$$

and the scalar field

$$\phi = \phi_0 \ t^{k_1} \tag{29}$$

where $k_1 = \left(\frac{2(n^2-n)}{\omega}\right)^{\frac{1}{2}}$.

The string energy density ρ and tension density λ are

$$\lambda = \rho = \phi_0^2 \ \frac{(2n^2-n)}{t^{2-2k}} \quad . \tag{30}$$

When $m = 1 - 2n$, the string energy density ρ and tension density λ become zero.

Case 4.2: Now we consider the case $\rho + \lambda = 0$ i.e., the sum of rest energy density and tension density for cloud of strings vanish. Then we have either $n = 0$ and $n = \frac{1-m}{2}$. When n = 0, the solution of the field equations (5)-(7) can be expressed as

$$A = A_0 t^m \text{ and } B = \text{Constant} \tag{31}$$

and the scalar field is given by

$$\phi = \phi_0 \, t^{k_2} \tag{32}$$

where $k_2 = \left(\frac{2(m^2-m)}{\omega}\right)^{\frac{1}{2}}$.

The string energy density ρ and tension density λ are

$$\lambda = -\rho = \phi_0^2 \ \frac{(m^2-m)}{t^{2-2k}}. \tag{33}$$

Again if $n = \frac{1-m}{2}$, $\lambda = \rho = 0$ this shows that the cosmic strings do not exist.

It is observed that the scalar field becomes a constant in both the cases i.e., when $m = 1$, $n = 0$ or $m = 0$, $n = 1$. These situations lead to the general relativity case.

Case 4.3: The p-strings or Takabayasi strings are represented by$\rho = (1+\xi)\,\lambda$, $\xi > 0$. Now the string energy density ρ, tension density λ, and the particle density ρ_p are given by

$$\rho = \phi_0^2 \frac{(m+n)(m+2n-1)}{t^{2-2k}} \tag{34}$$

$$\lambda = \phi_0^2 \frac{2m(m+2n-1)}{\xi \, t^{2-2k}} \tag{35}$$

$$\rho_p = \rho - \lambda = \frac{\phi_0^{\,2}(m+n-2m/\xi)(m+2n-1)}{t^{2-2k}}.$$

where the constants m, n and ξ are related by $n = \frac{2+\xi}{\xi}\ m$. It is evident that the energy conditions viz.,$\rho > 0,\ \lambda > 0$ and $\rho_p > 0$ are identically satisfied for all $m > 0$ and $n > 0$.

In this case we observed that the p-strings or Takabayasi strings do exist in a new scalar-tensor theory of gravitation proposed by Sen – Dunn.

Conclusions

We obtained the field equations of Sen – Dunn theory of gravitation with the help of LRS Bianchi type-I metric in the context of cosmic strings. The solutions of the field are discussed in various physically meaningful cases. It is observed that the cosmic strings do not co- exist when the shear scalar is proportional to the scalar of expansion. We also noticed that the cosmic strings do exists when the metric potentials are given by equation (17). Again for specific values of m and n, strings do not co – exist in Sen – Dunn theory of gravitation.

References

[1] Brans, C and Dicke ,R. H.: Phys.Rev.124,925 . (1961)

[2] Sen, D. K and Dunn, K. A J. Math. Phys.13, 557. (1971)

[3] Dunn K, A. J. Math. Phys .15, 2229. (1974)

[4] Halford W. D. J. Math. Phys.13, 1699 . (1972)

[5] Singh, T. J. Math. Phys.16, 2109 . (1975)

[6] Reddy, D. R. K. J. Phys. A: Math. Nucl. Gen .6, 1867. (1973)

[7] Reddy, D. R. K. J. Math. Phys. 20, 23. (1979)

[8] Roy A. R. and Chattterjee,B. Acta Phys. Hung. 48, 383. (1980)

[9] Roy A. R . and Chattterjee, B. Indian J. Pure Appl.Math. 12, 659. (1981)

[10] Reddy, D. R. K. and Venkateswarlu: Astrophys. Space Sci. **135**, 287 (1987)

[11] Kibble, T. W. B.: J. Phys. A **9**, 1387 (1976)

[12] Letelier, P. S.: Phys. Rev. D **28**, 2414 (1983)

[13] Stachel, J.: Phys. Rev. D **28**, 2171 (1990)

[14] Letelier, P. S.: Phys. Rev. D **20**, 1294 (1979)

[15] Krori, K. D., Choudhury, T., Mahanta, C. R.: Gen. Relativ. Gravit. **22**, 123 (1990)

[16] Krori, K. D., Choudhury, T., Mahanta, C. R.: Gen. Relativ. Gravit. **26**, 265 (1994)

[17] Bali, R., Dave, S.: Pramana, J. Phys. **56**, 4 (2001)

[18] Bhattacharjee, R., Baruah, K. K.: Indian J. Pure Appl. Math. **32**, 47 (2001)

[19] Mahanta, Mukharjee, A.: Indian J. Pure Appl. Math. **32**, 199 (2001)

[20] Rahaman, F., Chakraborty, S., Das, S., Hossain, M., Bera, J.: Pramana, J. Phys. **60**, 453 (2003)

[21] Reddy, D.R.K.: Astrophys. Space Sci. **286**, 359 (2003)

[22] Pant, D.N., Oli, S.: Pramana, J. Phys. **60**, 433 (2003)

[23] Venkateswarlu, R., Pavan Kumar, K., Rao, V.U.M.: Int. J. Theor. Phys. **47**, 640 (2008)

[24] Venkateswarlu, R., Pavan Kumar, K.,: Int. J. Theor. Phys. **49**, 1894(2010).