EJTP 8, No. 25 (2011) 319–326 *Electronic Journal of Theoretical Physics*

Two-Fluid Cosmological Models in Bianchi Type-III Space-Time

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Received 10 July 2010, Accepted 16 March 2011, Published 25 May 2011

Abstract: In this paper we have studied anisotropic, homogeneous two-fluid cosmological models in a Bianchi type III space-time. Here one fluid represents the matter content of the universe and another fluid is chosen to model the CMB radiation. These cosmological models depict two different scenarios of cosmic history i.e. one when the radiation and matter content of the universe are in interactive phase and another when the two are in non-interacting phase.

Keywords: Bianchi III space time; Two fluid
PACS (2010): 04.20jb; 98.80 hw

1. Introduction

The present stage of the expanding universe is modeled by the isotropic and homogeneous space time given by Friedman, Robertson and Walker. Many authors (Davidson 1962; McIntosh 1968; Coley and Tupper 1986) were motivated to investigate FRW models with a two fluid source after the discovery of 2.73K isotropic cosmic microwave background radiation (CMBR). The isotropy of the CMB models was one of the problems of standard cosmology and inflationary models were deemed to solve these(problems) difficulties with non credible evidences.

The photons were released when the universe was cooled sufficiently to form atomic hydrogen in the recombination epoch. These photons are moved freely in the universe forming the presently observed CMB. The COBE (1992) discovered temperature variations in the CMB level of 1 part in 100,000. These small anisotropies are believed to have the information about the geometry and the content of the early universe. The investigation of microwave Anisotropy Probe (MAP) and COBRAS-SAMBA (Planck Surveyor)

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satellites gives details of CMBR anisotropy. These observed CMBR anisotropies at various angular scales gives different directions in investigation of two fluid models which exhibit anisotropy.

Bianchi type space-times are spatially homogeneous and isotropic models of universe. Bianchi type VI_0 model with a two-fluid source has been investigated by Coley and Dunn (1990). Pant and Oli (2002) examined two fluid cosmological models in Bianchi type II space time. Two fluid Bianchi type I models are studied by Oli (2008 a,b) with and without variable G and Λ . Here we have investigated physically sound co-moving two-fluid models in Bianchi type-III space-time.

2. Field Equations

Bianchi type III space time is given by

$$ds^2 = -dt^2 + A^2\, dx^2 + B^2\, e^{2x}\, dy^2 + C^2\, dz^2 \tag{1}$$

The Einstein's field equations for a two fluid source in natural unit (or in gravitational units) $((8\pi G = 1,\, c = 1)$are

$$G^i_j = -\,(\,T^{i(m)}_j + T^{i(r)}_j\,) \quad . \tag{2}$$

Where $G^i_j = R^i_j - \frac{1}{2}\,\delta^i_j\, R$is the Einstein tensor. $T^{i\,(m)}_j$ is the energy momentum tensor for matter field and $T^{i\,(r)}_j$is the energy momentum tensor for radiation field.

As given by Coley and Dunn (1990) these are

$$T^{i\,(m)}_j = (p_m + \rho_m)\, u^m_i\, u^m_j - p_m g_{ij} \tag{3}$$

$$T^{i\,(r)}_j = {}^4\!/_3\, \rho_r\, u^r_i\, u^r_j - {}^1\!/_3\, \rho_r\, g_{ij} \tag{4}$$

with

$$g^{ij}\ u^m_i\, u^m_j = 1 \;,\; g^{ij}\ u^r_i\, u^r_j = 1 \tag{5}$$

and

$$u^{(m)}_i = (0,0,0,1) \qquad u^{(r)}_i = (0,0,0,1) \tag{6}$$

Using, (1), (3), (4) and (6) the field equations (2) reduce to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -(-p_m - \frac{\rho_r}{3}) \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -(-p_m - \frac{\rho_r}{3}) \tag{8}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{B}\dot{A}}{BA} = -(-p_m - \frac{\rho_r}{3}) \tag{9}$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\rho_m + \rho_r) \tag{10}$$

$$\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = 0 \quad . \tag{11}$$

These are five equations in six $(A, B, C, p_m\,,\ \rho_r, \rho_m)$unknowns. Thus, to obtain a solution we require additional relations. These relations may be taken by involving field variables as well as physical variables.

Now we have two different situations as:

(1) The two fluids are in the interactive phase and the matter distribution obeys the –law of equation of state. This situation is most relevant to describe the scenario before the recombination epoch when the photons were bound to the matter.

(2) The two fluids are in non-interacting phase. The situation is generally considered to model the post recombination era when the photons got themselves free to from the CMB being observed presently.

In order to evaluate arbitrary constants, we shall assume that the present age of the universe is $t_0 = 5 \times 10^{17}\, s$, which is estimated from the ages of low luminosity population II stars in globular clusters(Harrison 2000). The density of CMB at the present epoch is assumed to be given by $\rho_{r_0} = 4.67 \times 10^{-34}\, gcm^{-3}$ which corresponds to the blackbody radiation at 2.73K.

3. Solutions of the Field Equations

We first assume the relation between pressure and energy density of matter field through the "gamma-law" equation of state

$$p_m = (\gamma - 1)\,\rho_m\ ,\ \ 0 \leq \gamma \leq 2$$

From (11) , we get

$$B = A \tag{12}$$

Substituting (12) in equations (8) & (9), we get

$$-\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}^2}{A^2} + \frac{1}{A^2} = 0 \tag{13}$$

As number of above equation is one and there are two unknowns, we require one additional relation. We use here the physical condition that the expansion scalar is proportional to the shear scalar. According to Throne(1967), observations of velocity red shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within about 30% range approximately [Kantowaski & Sachs (1966); Kristian & Sachs (1966)] and red shift studies place the limit $\frac{\sigma}{H} \leq 0.30$, where σ is the shear and H is Hubble constant. Collins (1979) discussed the physical significance of this condition for perfect fluid and barotropic equation of state in a more general case. In many papers Roy & Singh (1985), Roy & Banerjee(1988), Bali et al. (2008) have proposed this condition to find exact solutions of cosmological models.

So we use the condition as

$$A = C^n. \tag{14}$$

From equations (13) & (14), we get

$$\frac{\ddot{C}}{C} + 2n\,\frac{\dot{C}^2}{C^2} = \frac{1}{n-1}\frac{1}{C^{2n}} \tag{15}$$

Let

$$\dot{C} = f(C) \tag{16}$$

$$\ddot{C} = f f' \; where \, f' = \frac{df}{dC} \quad .$$

With the help of (16), equation (15) becomes

$$f^2 = \frac{1}{1-n}C^{2-2n} + K_1 C^{-2n} \tag{17}$$

where K_1 is constant of integration.

But

$$\dot{C} = f(C) \tag{18}$$

Using (18), equation (17) becomes

$$\frac{C^{2n-1}}{\sqrt{\frac{C^{2n}}{2n-1-n^2} + K_1\, C^{2n-2}}}\,dC = dt \tag{19}$$

To get determinate solution, we assume $K_1 = 0$

Equation (19) reduces to

$$\frac{2n-1-n^2}{n^2}\,C^{2n} = (t+K_2)^2$$

$$C = \left[\frac{n^2}{2n-1-n^2}\,(t+K_2)^2\right]^{1/2n} \tag{20}$$

Hence , we get $A = B = \left(\left[\frac{n^2}{2n-1-n^2}\,(t+K_2)^2\right]^{1/2n}\right)^n$

i.e.

$$A = B = \left[\frac{n^2}{2n-1-n^2}\right]^{1/2}(t+K_2) \tag{21}$$

$$\rho_m = \frac{1}{4-3\gamma}\left[\frac{8n^2 + 8n + 4}{n^2}\,\frac{1}{(t+K_2)^2} + \frac{1}{t^{1/2}\,(t+K_2)^{3/2}}\right] \tag{22}$$

$$\rho_r = \frac{-6n^2\gamma + 6n\gamma - 3\gamma}{(4-3\gamma)\,n^2(t+K_2)^2} + \frac{3-3\gamma}{(4-3\gamma)t^{1/2}\,(t+K_2)^{3/2}} \tag{23}$$

In order to investigate the physical behavior of the fluid parameters we consider the particular case of dust i.e. when $\gamma = 1$.

The cosmological parameters are (Ellis et al 1997):

$$l_1 = l_2 = \left[\frac{n^2}{2n-1-n^2}\right]^{1/2}(t+K_2), \quad l_3 = \left[\frac{n^2}{2n-1-n^2}\right]^{1/2n}(t+K_2)^{1/n}$$

$$l = t^{-n}\left[\left(\frac{n^2}{2n-1-n^2}\right)^{\frac{2n+1}{2n}}(t+K_2)^{\frac{2n+1}{n}}\right]^{1/3}$$

$$\theta = 3H = \frac{4t^{1/2} + (t+K_2)^{1/2}}{2t^{1/2}(t+K_2)} \tag{24}$$

$$\sigma^2 = \frac{2}{3}\left[\frac{2t^{1/2} - (t+K_2)^{1/2}}{2t^{1/2}(t+K_2)}\right]^2$$

$$q = -\left[\frac{8t + 7t^{1/2}(t+K_2)^{1/2} + (t+K_2) - (t+K_2)^{3/2}t^{-1/2}}{16t + 8t^{1/2}(t+K_2)^{1/2} + (t+K_2)}\right]$$

$$\Omega_m = \frac{12(8n^2-8n+4)t^{1/2} + 3n^2 t(t+K_2)^{1/2}}{n^2\left[4t^{1/2} + (t+K_2)^{1/2}\right]^2}$$

$$\Omega_r = \frac{-18(8n^2-8n+4)t^{1/2}(t+K_2)^{-1}}{n^2\left[4t^{1/2} + (t+K_2)^{1/2}\right]^2}$$

For $\frac{1}{2} < n < 1$, $l_1, l_2, l_3 \to 0$ as $t \to 0$ thus the singularity at $t = 0$ is point type.

The shear scalar θ & σ^2 are positive for all values of t.

The negative value of deceleration parameter (q) indicates that universe is accelerating which is consistent with the present day observations.

The density parameter Ω_m increases with t, whereas Ω_r decreases with t.

From (23) we obtain

$$K_2 = -t_0 \pm \frac{\sqrt{U_0^2(t_0^2-1) - U_0(6n^2-6n+3)}}{U_0} \tag{25}$$

Where

$$U_0 = n^2\rho_{r0}t_0^2 \tag{26}$$

4. Models with Non-Interacting Matter and Radiation

In this case, we get

$$A = B = \left[\frac{n^2}{2n-1-n^2}\right]^{1/2}(t+K_2)$$

Table 1 : March of the matter density , radiation density, matter-radiation density ratio and density parameter in the dust model for $n = {}^{2}\!/_{3} \approx 0.66, t_0 = 5 \times 10^{17}\, s, \rho_{r_0} = 4.67 \times 10^{-34}\, gcm^{-3}$, and here $K_2 = -4.87754 \times 10^{17}$

Sr.No.	t (sec)	$\rho_m\,(g\,cm^{-3})$	$\rho_r\,(g\,cm^{-3})$	${}^{\rho_m}\!/_{\rho_r}$
1	10^{18}	$2.02017249 \times 10^{-35}$	$-1.446723 \times 10^{-35}$	-1.396378
2	2.5×10^{18}	$3.4657132 \times 10^{-36}$	$-9.3751973 \times 10^{-37}$	-3.6966829
3	5×10^{18}	$7.1517666 \times 10^{-37}$	$-1.8644769 \times 10^{-37}$	-3.8358033
4	7.5×10^{18}	$2.9958127 \times 10^{-37}$	$-7.7201951 \times 10^{-38}$	-3.8804883
5	10^{19}	$1.6372828 \times 10^{-37}$	$-4.1954294 \times 10^{-38}$	-3.9025393
6	2.5×10^{19}	$1.0071926 \times 10^{-38}$	-6.31795×10^{-39}	-1.5941763
7	5×10^{19}	$6.1239477 \times 10^{-39}$	$-1.5485214 \times 10^{-39}$	-3.9547065
8	7.5×10^{19}	$2.7069076 \times 10^{-39}$	$-6.8373406 \times 10^{-40}$	-3.9590065

Table 2 : March of the matter density , radiation density, matter-radiation density ratio and density parameter in the dust model for $n = 0.9, t_0 = 5 \times 10^{17}\, s, \rho_{r_0} = 4.67 \times 10^{-34}\, gcm^{-3}$, and here $K_2 = -1.0574458 \times 10^{10}$

Sr.No.	t (sec)	$\rho_m\,(g\,cm^{-3})$	$\rho_r\,(g\,cm^{-3})$	$\rho_r + \rho_m$
1	3.5×10^{17}	$4.4847777 \times 10^{-35}$	$-2.4792281 \times 10^{-35}$	$2.4792281 \times 10^{-35}$
2	4×10^{17}	$3.4337823 \times 10^{-35}$	$-1.8982431 \times 10^{-35}$	$1.5355392 \times 10^{-35}$
3	4.5×10^{17}	$2.7130108 \times 10^{-35}$	$-1.4997778 \times 10^{-35}$	1.213233×10^{-35}
4	5×10^{17}	$2.1975380 \times 10^{-35}$	$-1.2148196 \times 10^{-35}$	$0.9827184 \times 10^{-35}$
5	5.5×10^{17}	$1.8161465 \times 10^{-35}$	$-1.0039828 \times 10^{-35}$	$0.8121637 \times 10^{-35}$
6	7.5×10^{17}	9.766803×10^{-36}	$-5.3991769 \times 10^{-36}$	$0.4367626 \times 10^{-35}$
7	10^{18}	$5.4938288 \times 10^{-36}$	$-3.0370376 \times 10^{-36}$	$0.2456791 \times 10^{-35}$
8	2.5×10^{18}	$2.3190129 \times 10^{-36}$	$-4.859263 \times 10^{-37}$	$0.1833086 \times 10^{-35}$
9	3×10^{18}	$1.1610427 \times 10^{-35}$	$-3.3745079 \times 10^{-37}$	$1.1272976 \times 10^{-35}$
10	3.5×10^{18}	$1.1831698 \times 10^{-36}$	$-2.4792153 \times 10^{-37}$	$.93524827 \times 10^{-36}$

$$C = \left[\frac{n^2}{2n-1-n^2}\right]^{1/2n} (t + K_2)^{1/n} \tag{27}$$

Here the two fluids are non-interacting i.e. they obey separate conservation laws. The conservation of radiation $T^{ij\,(r)}_{;j} = 0$ leads to

$$\rho_r = \frac{K_3}{\left(\frac{n^2}{2n-1-n^2}\right)^{\frac{4n+2}{3n}} (t + K_2)^{\frac{10}{3}}} \tag{28}$$

Where K_3 is constant of integration.

The expression for matter density and pressure are

$$\rho_m = \frac{3n^2 - 2n + 1}{n^2 (t + K_2)^2} - \rho_r \tag{29}$$

$$p_m = \frac{1}{3} \left\{ \frac{n^2 t^{-3/2} (t + K_2)^{3/2} - 2n^2 (t + K_2) + 2(2n - 1 - n^2)}{2n^2 (t + K_2)^2} - \rho_r \right\} \tag{30}$$

And

$$\frac{\rho_m}{\rho_r} = \frac{(3n^2 - 2n + 1)\,(n^2)^{\frac{n+2}{3n}}\,(t + K_2)^{\frac{4}{3}}}{(2n - 1 - n^2)^{\frac{4n+2}{3n}} K_3} - 1 \tag{31}$$

All the cosmological parameters in this case are given by (24), except and which are given by

$$\Omega_m = \frac{12t\,(3n^2 - 2n + 1)}{n^2 \left[4t^{1/2} + (t + K_2)^{1/2}\right]^2} - \frac{12t\,(t + K_2)^{-4/3} K_3}{\left(\frac{n^2}{2n-1-n^2}\right)^{\frac{4n+2}{3n}} \left[4t^{1/2} + (t + K_2)^{1/2}\right]^2} \tag{32}$$

$$\Omega_r = \frac{12t\,(t + K_2)^{-4/3} K_3}{\left(\frac{n^2}{2n-1-n^2}\right)^{\frac{4n+2}{3n}} \left[4t^{1/2} + (t + K_2)^{1/2}\right]^2} \tag{33}$$

$$\Omega_0 = \frac{12t\,(3n^2 - 2n + 1)}{n^2 \left[4t^{1/2} + (t + K_2)^{1/2}\right]^2} \tag{34}$$

Conclusion

We have standard observations of anisotropy of the CMB. Here, we have presented power law solutions of general relativistic two fluid cosmological field equations in Bianchi type III space time. In both cases we get, point singularity.

We have examined dust ($\gamma = 1$) model in absence of cosmological constant and when the two fluids are in interacting phase obeying the γ-law equation of state for the matter field. It is observed that this dust model isotropizes as $t \to \infty$. We have obtained model in which the deceleration parameter(q) becomes negative for an intermediate period of time t.

The models presented here are physically meaningful as the associated parameters behave responsibly, however, they do not reflect the general picture. These are particular and simple models, nevertheless, our investigation demonstrates the space-time anisotropy may be an interesting factor to be introduced and investigated to arrive at a rigorous and reasonable cosmic picture.

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