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C-field Barotropic Fluid Cosmological Model with Variable G in FRW space-time

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Abstract: C-field cosmological model with variable G for barotropic perfect fluid distribution in flat FRW (Friedmann-Robertson-Walker) space-time is investigated. To get the deterministic model of the universe, we assume that G = R^n where R is scale factor and n is a constant. We find that the creation field (C) increases with time, G and ρ (matter density) decreases with time and $\left|\frac{\dot{G}}{G}\right| = H(t)$ where H is the Hubble parameter. These results match with the observations.
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1. Introduction

In Einstein's General Theory of Relativity, the gravitational constant (G) plays the role of coupling constant between geometry and matter. Therefore, in an evolving universe, it is natural to look at this constant which depends on time based on different arguments proposed in last few decades. Motivated by the occurrence of large numbers hypothesis, Dirac [1] proposed a theory with a variable gravitational constant. Subsequently, mathematically well posed alternative theories of gravity were developed to generalize Einstein's general theory of relativity by including variable G and satisfying conservation equation. The scalar tensor theory of gravity was first formulated by Jordan [2]. To achieve possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity, many attempts (Brans and Dicke [3], Hoyle and Narlikar [4,5], Canuto et al. [6]) have been proposed for the possible extensions of

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Einstein's general relativity with time dependent G.

The astronomical observations reveal that the predictions of Friedmann-Robertson-Walker (FRW) type models do not meet our requirements as was developed earlier (Smoot et al. [7]). Thus alternative theories were proposed from time to time. The most well known theory is the Steady State Theory by Bondi and Gold [8]. In this theory, the universe does not have any singular beginning nor an end on the cosmic time scale. For the maintenance of uniformity of mass density, they envisaged a very slow but continuous creation of matter in contrast to the explosive creation at t = 0 of the standard model. However, it suffers from the serious disqualifications by not giving any physical justifications in the form of any dynamical theory for continuous creation of matter. To overcome this difficulty, Hoyle and Narlikar [9] adopted a field theoretic approach introducing a massless and chargeless scalar field to account for creation of matter. In C-field theory, there is no big-bang type singularity as in the steady-state theory of Bondi and Gold [8]. Narlikar [10] has explained that matter creation is accomplished at the expense of negative energy C-field. He also emphasized that if overall energy conservation is to be maintained then primary matter creation must be accompanied by the release of negative energy and the repulsive nature of this negative reservoir will be sufficient to prevent the singularity. Narlikar and Padmanabhan [11] investigated the solution of Einstein's field equation which admits radiation and negative energy massless scalar creation field as a source. Vishwakarma and Narlikar [12] discussed modeling repulsive gravity with creation. Recently Bali and Tikekar [13] have investigated C-field cosmological model for dust distribution with variable G in the frame work of flat Friedmann-Robertson-Walker space-time and the results match with the observations.

In this paper, we have investigated C-field barotropic perfect fluid cosmological model with variable G using flat FRW space-time. In particular, we obtain the same result for dust distribution for C-field cosmological model with variable G in flat FRW model as obtained by Bali and Tikekar[13]. The physical aspects of the model have been discussed and the results match with the observations.

2. Formation of Line-Element

We consider the flat FRW space-time as

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2\, d\theta^2 + r^2 sin^2\theta d\phi^2] \tag{1}$$

Einstein's modified field equations by the introduction of C-field are given by (Narlikar [15]) as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G({}^m T_i^j + {}^C T_i^j) \tag{2}$$

where

$${}^m T_i^j = (\rho + p)\, v_i\, v^j - p g_i^j \tag{3}$$

and

$${}^C T_i^j = -f \left(C_i\, C^j - \frac{1}{2} g_i^j\, C^\alpha\, C_\alpha \right) \tag{4}$$

with f > 0 and $C_i = \frac{dC}{dx^i}$.

The modified field equation (2) for the metric (1) for variable G(t) leads to

$$\frac{3\dot{R}^2}{R^2} = 8\,\pi\,G(t)\left(\rho - \frac{1}{2}\,f\dot{C}^2\right) \tag{5}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = 8\,\pi\,G(t)\left(\frac{1}{2}\,f\dot{C}^2 - p\right) \tag{6}$$

The conservation equation

$$(8\,\pi\,GT_i^j)_{;j} = 0$$

leads to

$$8\,\pi\,\dot{G}\left(\rho - \frac{1}{2}\,f\dot{C}^2\right) + 8\,\pi\,G\left(\dot{\rho} - f\dot{C}\,\ddot{C} - 3f\dot{C}^2\,\frac{\dot{R}}{R} + 3\,(\rho + p)\frac{\dot{R}}{R}\right) = 0 \tag{7}$$

which yields $\dot{C} = 1$when used in the source equation.

Using $\dot{C} = 1$ in equation (5), we have

$$8\,\pi\,G\rho = \frac{3\dot{R}^2}{R^2} + 4\,\pi\,Gf \tag{8}$$

We assume that universe is filled with barotropic perfect fluid i.e. $p = \gamma\rho(0 \leq \gamma \leq 1)$,p being the isotropic pressure, ρ the matter density. Now using p = $\gamma\rho$ and $\dot{C} = 1$ in equation (6), we have

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = 4\,\pi\,Gf - 8\,\pi G\gamma\rho \tag{9}$$

Equations (8) and (9) lead to

$$\frac{2\ddot{R}}{R} + (1 + 3\gamma)\frac{\dot{R}^2}{R^2} = (1 - \gamma)4\pi G(t)f \tag{10}$$

To get the deterministic solution, we assume that

$$G = R^n \tag{11}$$

where R is the scale factor and n is a constant.

Using the condition (11) in (10), we have

$$2\ddot{R} + (1 + 3\gamma)\frac{\dot{R}^2}{R} = (1 - \gamma)\,KR^{n+1} \tag{12}$$

where

$$K = 4\,\pi\,f \tag{13}$$

Let us assume $\dot{R} = F(R)$. Thus equation (12) leads to

$$\frac{d}{dR}(F^2) + \left(\frac{1 + 3\gamma}{R}\right)F^2 = (1 - \gamma)KR^{n+1} \tag{14}$$

where

$$\ddot{R} = FF', F' = \frac{dF}{dR}.$$

From equation (14), we have

$$F^2 = \left(\frac{dR}{dt}\right)^2 = \frac{K(1-\gamma)R^{n+2}}{n+3\gamma+3} + \frac{L}{R^{3\gamma+1}} \tag{15}$$

where L is the constant of integration. Equation (15) leads to

$$\frac{R^{\frac{3\gamma+1}{2}} dR}{\sqrt{K(1-\gamma)\, R^{n+3\gamma+3} + L(n+3\gamma+3)}} = \frac{dt}{\sqrt{n+3\gamma+3}} \tag{16}$$

where L is the constant of integration.

To obtain the deterministic value of R in terms of cosmic time t, we assume that

$$n = -\left(\frac{3\gamma+3}{2}\right) \tag{17}$$

Using condition (17) in (16), we have

$$\frac{R^{\frac{3\gamma+1}{2}} dR}{\sqrt{R^{\frac{3\gamma+3}{2}} + \frac{L(3\gamma+3)}{2K(1-\gamma)}}} = \sqrt{\frac{2K(1-\gamma)}{3\gamma+3}} dt \tag{18}$$

Equation (18) leads to

$$R^{\frac{3\gamma+3}{2}} = \left[\left\{\frac{1}{2}\sqrt{\frac{K(1-\gamma)(3\gamma+3)}{2}} t + \left(\frac{3\gamma+3}{4}\right) N\right\}^2 - \frac{L(3\gamma+3}{2K(1-\gamma)}\right] \tag{19}$$

where N is the constant of integration. Therefore

$$G = R^n = R^{-\frac{3\gamma+3}{2}} = \left[\left\{\frac{1}{2}\sqrt{\frac{K(1-\gamma)(3\gamma+3)}{2}} t + \left(\frac{3\gamma+3}{4}\right) N\right\}^2 - \frac{L(3\gamma+3)}{2K(1-\gamma)}\right]^{-1} \tag{20}$$

From equations (8), (19) and (20), we have

$$8\,\pi\rho = \frac{\left\{\frac{3K(1-\gamma)(3\gamma+3)}{2} + K\left(\frac{3\gamma+3}{2}\right)^2\right\}\left\{\frac{\sqrt{K(1-\gamma)(3\gamma+3)}}{2\sqrt{2}} t + \frac{N(3\gamma+3)}{4}\right\}^2 - \frac{L}{8}\frac{(3\gamma+3)^3}{(1-\gamma)}}{\left(\frac{3\gamma+3}{2}\right)^2 \left[\left\{\frac{\sqrt{3K(1-\gamma^2)}}{2\sqrt{2}} t + \frac{3(1+\gamma)N}{4}\right\} - \frac{3L(1+\gamma)}{2K(1-\gamma)}\right]} \tag{21}$$

Using barotropic equation p = $\rho\gamma$ in equation (7), we have

$$8\,\pi\,(G\dot{\rho}+\dot{G}\rho)-\ 4\pi\dot{G}f\dot{C}^{2}--8\,\pi Gf\dot{C}\ddot{C}$$

$$-24\pi G\rho\frac{\dot{R}}{R}f\dot{C}^{2}+\ 24\pi G\rho\frac{\dot{R}}{R}(1+\gamma)=0 \tag{22}$$

Equation (22) after using (19), (20) and (21) leads to

$$\dot{C}^{2}t^{\frac{6-2\gamma}{1+\gamma}}=\int\frac{\left[\frac{9(1-\gamma^{2})}{2}+\frac{9(1+\gamma)^{2}}{4}\right]}{\frac{9(1+\gamma)^{2}}{8}}t^{\frac{5-3\gamma}{1+\gamma}}dt \tag{23}$$

where we have set the constants of integration L = 0, N = 0 to get the deterministic value of $\dot{C}$. Now equation (23) leads to

$$\dot{C}^{2}.t^{\frac{6-2\gamma}{1+\gamma}}=\frac{36(1-\gamma^{2})+18(1+\gamma)^{2}}{9(1+\gamma)}\frac{t^{\frac{6-2\gamma}{1+\gamma}}}{6-2\gamma}+M$$

$= \frac{4(1-\gamma)+2(1+\gamma)}{2(3-\gamma)}\,t^{\frac{6-2\gamma}{1+\gamma}}$ (setting M = 0)

$$=\frac{6-2\gamma}{6-2\gamma}t^{\frac{6-2\gamma}{1+\gamma}} \tag{24}$$

Thus, we have

$$\dot{C}^{2}=1 \tag{25}$$

which leads to

$$C=t \tag{26}$$

We find that $\dot{C}=1$ which agrees with the value used in source equation. The creation field (C) increases with time. The metric (1) after using the value of R, leads to

$$ds^{2}=dt^{2}-\left\{\frac{\sqrt{3K(1-\gamma^{2})}}{2\,\sqrt{2}}t\right\}^{\frac{8}{3\gamma+3}}[dr^{2}+r^{2}d\theta^{2}+r^{2}sin^{2}\theta d\phi^{2}] \tag{27}$$

where $\gamma\neq 1$.

Special Cases

i) $\gamma=0$ gives that FRW model for dust distribution with variable G obtained by Bali and Tikekar [13].

ii) $\gamma=1/3$ i.e. $\rho=3$p gives radiation dominated universe for flat FRW model with variable G.

iii) $\gamma=1$ leads to stiff fluid universe but in our case flat FRW model with variable G is not possible for $\gamma=1$.

Discussion

The homogeneous mass density (ρ), the gravitational constant G, the spatial volume (R^3), the deceleration parameter (q), the Hubble constant (H) and $\left|\frac{\dot{G}}{G}\right|$ for the case L=0, N=0 are given by

$$8\pi\rho = \text{constant} \tag{28}$$

$$G = \frac{8}{3K(1-\gamma^2)t^2} \tag{29}$$

$$R^3 = \left[\frac{3K(1-\gamma^2)}{8}t^2\right]^{\frac{2}{3\gamma+3}} \tag{30}$$

$$q = \frac{3\gamma - 1}{4}$$

$$\left|\frac{\dot{G}}{G}\right| = \frac{1}{t} = H$$

The reality condition $\rho \quad > 0$ leads to

$$3 - \gamma^2 + 2\gamma > 0$$

We find that when t $\rightarrow$ 0, G $\rightarrow \infty$. The spatial volume increases as t increases. The deceleration parameter q $<$ 0 for $\gamma = 0$. Thus for dust filled universe, the model (27) represents accelerating universe. For $\gamma = 1/3$ i.e. for radiation dominated universe,

q = 0 which gives Milne universe (Narlikar [16]). The creation field (C) increases as time increases. These results match with the observations. The homogeneous mass density $\rho =$ constant. This can be interpreted as the matter is supposed to move along the geodesic normal to the surface (t = constant). When the matter moves further apart, it is assumed that more matter is created continuously to maintain the density at constant value (Hoyle and Narlikar [4], Hawking and Ellis [14]).

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