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Schwinger Mechanism for Quark-Antiquark Production in the Presence of Arbitrary Time Dependent Chromo-Electric Field

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Abstract: We study the Schwinger mechanism in QCD in the presence of an arbitrary time-dependent chromo-electric background field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3). We obtain an exact result for the non-perturbative quark (antiquark) production from an arbitrary $E^a(t)$ by directly evaluating the path integral. We find that the exact result is independent of all the time derivatives $\frac{d^nE^a(t)}{dt^n}$ where $n = 1, 2, ...\infty$. This result has the same functional dependence on two Casimir invariants $[E^a(t)E^a(t)]$ and $[d_{abc}E^a(t)E^b(t)E^c(t)]^2$ as the constant chromo-electric field E^a result with the replacement: $E^a \to E^a(t)$. This result relies crucially on the validity of the shift conjecture, which has not yet been established.
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In 1951 Schwinger derived an exact one-loop non-perturbative result for electron-positron pair production in QED from a constant electric field E by using the proper time method [1]. In QCD this result depends on two independent Casimir invariants in SU(3): $C_1 = [E^aE^a]$ and $C_2 = [d_{abc}E^aE^bE^c]^2$ where a, b, c=1,2,...8 [2, 3]. Recently we have studied the Schwinger mechanism for gluon pair production in the presence of arbitrary time dependent chromo-electric field in [4, 5]. This technique is also applied in [6] to study path integration in QCD in the presence of arbitrary space-dependent (one dimensional) static color potential. This result relies crucially on the validity of the shift conjecture [7], which has not yet been established.

In this paper we study the Schwinger mechanism for quark-antiquark production in the presence of an arbitrary time-dependent chromo-electric background field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3). We obtain an exact non-perturbative result

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for quark (antiquark) production from arbitrary $E^a(t)$ by directly evaluating the path integral.

We obtain the following exact non-perturbative result for the probability of quark (antiquark) production per unit time, per unit volume and per unit transverse momentum from an arbitrary time dependent chromo-electric field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3):

$$\frac{dW_{q(\bar{q})}}{dtd^3xd^2p_T} = -\frac{1}{4\pi^3}\sum_{j=1}^{3} |g\Lambda_j(t)| \ln[1 - e^{-\frac{\pi(p_T^2+m^2)}{|g\Lambda_j(t)|}}]. \tag{1}$$

In the above equation m is the mass of the quark and

$$\Lambda_1(t) = \sqrt{\frac{C_1(t)}{3}}\cos\theta(t);\ \Lambda_{2,3}(t) = \sqrt{\frac{C_1(t)}{3}}\cos(2\pi/3 \pm \theta(t));\ \cos^2 3\theta(t) = \frac{3C_2(t)}{C_1^3(t)}, \tag{2}$$

where

$$C_1(t) = [E^a(t)E^a(t)];\quad C_2(t) = [d_{abc}E^a(t)E^b(t)E^c(t)]^2 \tag{3}$$

are two independent time-dependent Casimir/gauge invariants in SU(3).

This result has the remarkable feature that it is independent of all the time derivatives $\frac{d^nE^a(t)}{dt^n}$ and has the same functional form as the constant chromo-electric field E^a result [2] with: $E^a \to E^a(t)$.

Now we will present a derivation of eq. (1).

The Lagrangian density for a quark (antiquark) in the presence of background chromo field $A_\mu^a(x)$ is given by

$$\mathcal{L} = \bar{\psi}^j(x)[\delta_{jk}\hat{\not p} - gT^a_{jk}\not A^a(x) - \delta_{jk}m]\psi^k(x) = \bar{\psi}^j(x)M_{jk}[A]\psi^k(x) \tag{4}$$

where a=1,2,...8 and j,k=1,2,3. The vacuum-to-vacuum transition amplitude is given by

$$<0|0>= \frac{Z[A]}{Z[0]} = \frac{\int[d\bar{\psi}][d\psi]e^{i\int d^4x\ \bar{\psi}^j(x)M_{jk}[A]\psi^k(x)}}{\int[d\bar{\psi}][d\psi]e^{i\int d^4x\ \bar{\psi}^j(x)M_{jk}[0]\psi^k(x)}} = \frac{\text{Det[M[A]]}}{\text{Det[M[0]]}} = e^{iS} \tag{5}$$

which gives

$$S = -i\text{Tr}\ \ln[\delta_{jk}\hat{\not p} - gT^a_{jk}\not A^a(x) - \delta_{jk}m] + i\text{Tr}\ \ln[\delta_{jk}\hat{\not p} - \delta_{jk}m]. \tag{6}$$

Since the trace is invariant under transposition we find

$$S = -i\text{Tr}\ \ln[\delta_{jk}\hat{\not p} - gT^a_{jk}\not A^a(x) + \delta_{jk}m] + i\text{Tr}\ \ln[\delta_{jk}\hat{\not p} + \delta_{jk}m]. \tag{7}$$

Adding eqs. (6) and (7) we find

$$S = -\frac{i}{2}\text{Tr}\ \ln[(\delta_{jk}\hat{\not p} - gT^a_{jk}\not A^a(x))^2 - \delta_{jk}m^2] + \frac{i}{2}\text{Tr}\ \ln[\delta_{jk}(\hat{p}^2 - m^2)] \tag{8}$$

where

$$\text{Tr}\mathcal{O} = \text{tr}_{\text{Dirac}}\text{tr}_{\text{color}}\int d^4x <x|\mathcal{O}|x> \tag{9}$$

Since it is convenient to work with the exponential of the trace we write

$$\ln(\frac{a}{b}) = \int_0^\infty \frac{ds}{s}[e^{-is(b-i\epsilon)} - e^{-is(a-i\epsilon)}]. \tag{10}$$

Hence we find from eq. (8)

$$S = \frac{i}{2}\mathrm{tr}_{\mathrm{Dirac}}\mathrm{tr}_{\mathrm{color}}\int d^4x$$
$$< x| \int_0^\infty \frac{ds}{s}[e^{-is[(\delta_{jk}\hat{p}-gT^a_{jk}A^a(x))^2+\frac{g}{2}\sigma^{\mu\nu}T^a_{jk}F^a_{\mu\nu}-\delta_{jk}m^2-i\epsilon]} - e^{-is[\delta_{jk}(\hat{p}^2-m^2)-i\epsilon]}]|x > . \tag{11}$$

We assume the arbitrary time dependent chromo-electric field $E^a(t)$ to be along the beam direction (say along the z-axis) and choose the axial gauge $A^a_3 = 0$ so that only

$$A^a_0(t,z) = -E^a(t)z \tag{12}$$

is non-vanishing. Using eq. (12) in (11) and evaluating the Dirac trace by using

$$(\gamma^0\gamma^3)_{\mathrm{eigenvalues}} = (\lambda_1,\lambda_2,\lambda_3,\lambda_4) = (1,1,-1,-1) \tag{13}$$

we find

$$S = \frac{i}{2}\sum_{l=1}^4 \mathrm{tr}_{\mathrm{color}} \int_0^\infty \frac{ds}{s}\int dt < t|\int dx < x|\int dy < y|\int dz < z|$$
$$e^{-is[(\frac{\delta_{jk}}{i}\frac{d}{dt}+gT^a_{jk}E^a(t)z)^2-\hat{p}_z^2-\hat{p}_T^2+ig\lambda_l T^a_{jk}E^a(t)-m^2-i\epsilon]} - e^{-is(\delta_{jk}(\hat{p}^2-m^2)-i\epsilon)}|z > |y > |x > |t > \tag{14}$$

We write this in the color matrix notation

$$S = \frac{i}{2}\sum_{l=1}^4 \mathrm{tr}_{\mathrm{color}} \; [\int_0^\infty \frac{ds}{s}\int dt < t|\int dx < x|\int dy < y|\int dz < z|$$
$$e^{-is[(\frac{1}{i}\frac{d}{dt}+gM(t)z)^2-\hat{p}_z^2-\hat{p}_T^2+ig\lambda_l M(t)-m^2-i\epsilon]} - e^{-is((\hat{p}^2-m^2)-i\epsilon)}|z > |y > |x > |t >]_{jk} \tag{15}$$

where
$$M_{jk}(t) = T^a_{jk}E^a(t). \tag{16}$$

Inserting complete set of $|p_T >$ states (using $\int d^2p_T \; |p_T >< p_T| = 1$) we find from the above equation

$$S^{(1)} = \frac{i}{2(2\pi)^2}\sum_{l=1}^4 \mathrm{tr}_{\mathrm{color}}[\int_0^\infty \frac{ds}{s}\int d^2x_T\int d^2p_T e^{is(p_T^2+m^2+i\epsilon)}$$
$$[\int_{-\infty}^{+\infty} dt < t|\int_{-\infty}^{+\infty} dz < z|e^{-is[(\frac{1}{i}\frac{d}{dt}+gM(t)z)^2-\hat{p}_z^2+ig\lambda_l M(t)]}|z > |t > -\int dt\int dz\frac{1}{4\pi s}]]_{jk} \tag{17}$$

where we have used the normalization $< q|p >= \frac{1}{\sqrt{2\pi}}e^{iqp}$. At this stage we use the shift theorem [7] and find

$$S^{(1)} = \frac{i}{2(2\pi)^2}\sum_{l=1}^{4}\text{tr}_{\text{color}}[\int_0^\infty \frac{ds}{s}\int d^2x_T\int d^2p_T e^{is(p_T^2+m^2+i\epsilon)}[\int_{-\infty}^{+\infty} dt < t|\int_{-\infty}^{+\infty} dz$$
$$< z+\frac{i}{gM(t)}\frac{d}{dt}|e^{-is[g^2M^2(t)z^2-\hat{p}_z^2+ig\lambda_l M(t)]}|z+\frac{i}{gM(t)}\frac{d}{dt} > |t> - \int dt\int dz\frac{1}{4\pi s}]]_{jk} \tag{18}$$

where the z integration must be performed from $-\infty$ to $+\infty$ for the shift theorem to be applicable.

Note that a state vector $|z+\frac{i}{a(t)}\frac{d}{dt} >$ which contains a derivative operator is not familiar in physics. However, the state vector $|z+\frac{i}{a(t)}\frac{d}{dt} >$ contains the derivative $\frac{d}{dt}$ not $\frac{d}{dz}$. Hence the state vector is defined in the z-space with $\frac{d}{dt}$ acting as a c-number shift in z-coordinate (not a c-number shift in t-coordinate). To see how one operates with such state vector we find

$$< z+\frac{i}{a(t)}\frac{d}{dt}|p_z > f(t) = \frac{1}{\sqrt{2\pi}}e^{i(z+\frac{i}{a(t)}\frac{d}{dt})p_z}f(t) = \frac{1}{\sqrt{2\pi}}e^{izp_z}e^{-\frac{p_z}{a(t)}\frac{d}{dt}}f(t). \tag{19}$$

Inserting complete sets of $|p_z>$ states (using $\int dp_z\ |p_z >< p_z| = 1$) in eq. (18) we find

$$S^{(1)} = \frac{i}{2(2\pi)^2}\sum_{l=1}^{4}\int_0^\infty \frac{ds}{s}\int d^2x_T\int d^2p_T e^{is(p_T^2+m^2+i\epsilon)}[F_l(s) - \int dt\int dz\ \frac{3}{4\pi s}] \tag{20}$$

where

$$F_l(s) = \frac{1}{(2\pi)}\text{tr}_{\text{color}}\ [\int_{-\infty}^{+\infty} dt < t|\int_{-\infty}^{+\infty} dz\int dp_z\int dp'_z\ e^{izp_z}e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}$$
$$< p_z|e^{is[-g^2M^2(t)z^2+\hat{p}_z^2-i\lambda_l gM(t)]}|p'_z > e^{\frac{1}{gM(t)}\frac{d}{dt}p'_z}e^{-izp'_z}|t>]_{jk}. \tag{21}$$

It can be seen that the exponential $e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}$ contains the derivative $\frac{d}{dt}$ which operates on $< p_z|e^{is[-g^2M^2(t)z^2+\hat{p}_z^2-i\lambda_l gM(t)]}|p'_z >$ hence we can not move $e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}$ to the right. We insert more complete set of t states to find

$$F_l(s) = \frac{1}{(2\pi)}\text{tr}_{\text{color}}\ [\int_{-\infty}^{+\infty} dt\int_{-\infty}^{+\infty} dz\int dp_z\int dp'_z\ \int dt'\int dt'' < t|e^{izp_z}e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}|t' >$$
$$< t'| < p_z|e^{is[-g^2M^2(t)z^2+\hat{p}_z^2-i\lambda_l gM(t)]}|p'_z > |t'' >< t''|e^{\frac{1}{gM(t)}\frac{d}{dt}p'_z}e^{-izp'_z}|t>]_{jk}$$
$$= \frac{1}{(2\pi)}\text{tr}_{\text{color}}\ [\int_{-\infty}^{+\infty} dt\int_{-\infty}^{+\infty} dz\int dp_z\int dp'_z\ \int dt' < t|e^{izp_z}e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}|t' >$$
$$< p_z|e^{is[-g^2M^2(t')z^2+\hat{p}_z^2-i\lambda_l gM(t')]}|p'_z >< t'|e^{\frac{1}{gM(t)}\frac{d}{dt}p'_z}e^{-izp'_z}|t>]_{jk}. \tag{22}$$

Inserting more complete sets of states as appropriate we find

$$F_l(s) = \frac{1}{(2\pi)}\text{tr}_{\text{color}}\,[\int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dz \int dp_z \int dp'_z \int dt' \int dp_0 \int dp'_0 \int dz' \int dz''$$
$$\int dp''_0 \int dp'''_0 < t|p_0 > e^{izp_z} < p_0|e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}|p'_0 >< p'_0|t' >< p_z|z' >< z'|$$
$$e^{is[-g^2M^2(t')z^2+\hat{p}_z^2-i\lambda_l gM(t')]}|z'' >< z''|p'_z >< t'|p''_0 >< p''_0|e^{\frac{1}{gM(t)}\frac{d}{dt}p'_z}|p'''_0 > e^{-izp'_z} < p'''_0|t >]_{jk}$$
$$= \frac{1}{(2\pi)^4}\text{tr}_{\text{color}}\,[\int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dz \int dp_z \int dp'_z \int dt' \int dp_0 \int dp'_0 \int dz' \int dz''$$
$$\int dp''_0 \int dp'''_0\; e^{itp_0} e^{izp_z} < p_0|e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}|p'_0 > e^{-it'p'_0} e^{-iz'p_z}$$
$$< z'|e^{is[-g^2M^2(t')z^2+\hat{p}_z^2-i\lambda_l gM(t')]}|z'' > e^{iz''p'_z} e^{it'p''_0} < p''_0|e^{\frac{1}{gM(t)}\frac{d}{dt}p'_z}|p'''_0 > e^{-izp'_z} e^{-itp'''_0}]_{jk}. \quad (23)$$

It can be seen that all the expressions in the above equation are independent of t except $e^{it(p_0-p'''_0)}$. This can be seen as follows

$$< p_0|f(t)\frac{d}{dt}|p'_0 >= \int dt' \int dt'' \int dp''''_0 < p_0|t' >< t'|f(t)|t'' >< t''|p''''_0 >< p''''_0|\frac{d}{dt}|p'_0 >$$
$$= \int dt' \int dt'' \int dp''''_0 e^{-it'p_0}\; \delta(t'-t'')f(t'')e^{it''p''''_0}\; ip'_0\; \delta(p''''_0 - p'_0) = ip'_0 \int dt'\; e^{-it'(p_0-p'_0)} f(t') \quad (24)$$

which is independent of t and $\frac{d}{dt}$. Hence by using the cyclic property of trace we can take the matrix $[< p''_0|e^{\frac{1}{gM(t)}\frac{d}{dt}p_z}|p'''_0 >]_{jk}$ to the left. The t integration is now easy ($\int_{-\infty}^{+\infty} dt e^{it(p_0-p'''_0)} = 2\pi\delta(p_0 - p'''_0)$) which gives

$$F_l(s) = \frac{1}{(2\pi)^3}\text{tr}_{\text{color}}\,[\int_{-\infty}^{+\infty} dz \int dp_z \int dp'_z \int dt' \int dp_0 \int dp'_0 \int dz' \int dz'' \int dp''_0\; e^{izp_z}$$
$$< p''_0|e^{\frac{1}{gM(t)}\frac{d}{dt}p'_z}|p_0 >< p_0|e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}|p'_0 > e^{-iz'p_z} e^{-it'p'_0} < z'|e^{is[-g^2M^2(t')z^2+\hat{p}_z^2-i\lambda_l gM(t')]}|z'' >$$
$$e^{it'p''_0} e^{iz''p'_z} e^{-izp'_z}]_{jk}. \quad (25)$$

As advertised earlier we must integrate over z from $-\infty$ to $+\infty$ for the shift theorem to be applicable [7]. The matrix element $< z'|e^{-is[-g^2M^2(t)z^2+\hat{p}_z^2-i\lambda_l gM(t)]}|z'' >$ is independent of the z variable (it depends on z' and z'' variables). Hence we can perform the z integration easily by using $\int_{-\infty}^{+\infty} dz e^{iz(p_z-p'_z)} = 2\pi\delta(p_z - p'_z)$ to find

$$F_l(s) = \frac{1}{(2\pi)^2}\text{tr}_{\text{color}}\,[\int dp_z \int dt' \int dp_0 \int dp'_0 \int dz' \int dz'' \int dp''_0$$
$$< p''_0|e^{\frac{1}{gM(t)}\frac{d}{dt}p_z}|p_0 >< p_0 e^{-\frac{1}{gM(t)}\frac{d}{dt}p_z}|p'_0 > e^{-iz'p_z} e^{-ip'_0 t'} < z'|e^{is[-g^2M^2(t')z^2+\hat{p}_z^2-i\lambda_l gM(t')]}|z'' >$$
$$e^{ip''_0 t'} e^{iz''p_z}]_{jk}. \quad (26)$$

Using the completeness relation $\int dp_0|p_0 >< p_0| = 1$ we obtain

$$F_l(s) = \frac{1}{(2\pi)^2}\text{tr}_{\text{color}}\,[\int dp_z \int dt' \int dp'_0 \int dz' \int dz''$$
$$e^{-iz'p_z} < z'|e^{is[-g^2M^2(t')z^2+\hat{p}_z^2-i\lambda_l gM(t')]}|z'' > e^{iz''p_z}]_{jk}. \quad (27)$$

Since the color matrix $M_{jk}(t) = T^a_{jk}E^a(t)$ is antisymmetric it can be diagonalized [2] by an orthogonal matrix $U_{jk}(t)$. The eigenvalues

$$[M_{jk}(t)]_{\rm eigenvalues} = [T^a_{jk}E^a(t)]_{\rm eigenvalues} = (\Lambda_1(t),\ \Lambda_2(t),\ \Lambda_3(t)) \tag{28}$$

can be found by evaluating the traces of $M(t)$, $M^2(t)$ and Det$[M(t)]$ respectively:

$$\Lambda_1(t)+\Lambda_2(t)+\Lambda_3(t)=0; \qquad \Lambda_1^2(t)+\Lambda_2^2(t)+\Lambda_3^2(t)=\frac{E^a(t)E^a(t)}{2},$$
$$\Lambda_1(t)\Lambda_2(t)\Lambda_3(t)=\frac{1}{12}[d_{abc}E^a(t)E^b(t)E^c(t)] \tag{29}$$

the solution of which is given by eq. (2).

Using these eigen values we perform the color trace in eq. (27) to find

$$F_l(s)=\frac{1}{(2\pi)^2}\sum_{j=1}^{3}\int dp_z\int dt'\int dp_0'\int dz'\int dz'' e^{-iz'p_z} < z'|e^{is[-g^2\Lambda_j^2(t')z^2+\hat{p}_z^2-i\lambda_l g\Lambda_j(t')]}|z''> e^{iz''p_z}. \tag{30}$$

The above equation boils down to usual harmonic oscillator, $\omega^2(t)z^2+\hat{p}_z^2$, with the constant frequency ω replaced by time dependent frequency $\omega(t)$. The harmonic oscillator wave function

$$< z|n_t >= \psi_n(z) = (\frac{\omega(t)}{\pi})^{1/4}\frac{1}{(2^n n!)^{1/2}}H_n(z\sqrt{\omega(t)})e^{-\frac{\omega(t)}{2}z^2} \tag{31}$$

(H_n being the Hermite polynomial) is normalized

$$\int dz|< z|n_t >|^2 = 1. \tag{32}$$

Inserting a complete set of harmonic oscillator states (by using $\sum_n |n_t >< n_t| = 1$) in eq. (30) we find

$$F_l(s)=\frac{1}{(2\pi)^2}\sum_n\sum_{j=1}^{3}\int dp_z\int dt'\int dp_0'\int dz'\int dz'' e^{-iz'p_z} < z'|n_{t'}> e^{(-sg\Lambda_j(t')(2n+1)+s\lambda_l g\Lambda_j(t'))}$$
$$< n_{t'}|z''> e^{iz''p_z} = \frac{1}{(2\pi)}\sum_n\sum_{j=1}^{3}\int dt\int dp_0\int dz\ |< z|n_t >|^2 e^{(-sg\Lambda_j(t)(2n+1)+s\lambda_l g\Lambda_j(t))}$$
$$=\frac{1}{(2\pi)}\sum_n {\rm tr_{color}}\ [\int dt\int dp_0\ e^{(-s(2n+1)gM(t)+sg\lambda_l M(t))}]_{jk} \tag{33}$$

where we have used eq. (32). The Lorentz force equation in color space, $\delta_{jk}dp_\mu = gT^a_{jk}F^a_{\mu\nu}dx^\nu$, gives (when the chromo-electric field is along the z-axis, eq. (12)), $\delta_{jk}dp_0 = gT^a_{jk}E^a(t)dz = gM_{jk}(t)dz$. Using this in eq. (33) we obtain

$$F_l(s)=\frac{1}{(2\pi)}\sum_{j=1}^{3}\int dt\int dz\ g\Lambda_j(t)\frac{e^{sg\lambda_l\Lambda_j(t)}}{2{\rm sinh}({\rm sg}\Lambda_{\rm j}({\rm t}))}. \tag{34}$$

Using this expression of $F_l(s)$ in eq. (20) and summing over l (by using the eigen values of the Dirac matrix from eq. (13)) we find

$$S = \frac{i}{8\pi^3} \sum_{j=1}^{3} \int_0^\infty \frac{ds}{s} \int d^4x \int d^2p_T e^{is(p_T^2+m^2+i\epsilon)} [g\Lambda_j(t)\ \frac{\cosh(sg\Lambda_j(t))}{\sinh(sg\Lambda_j(t))} - \frac{1}{s}]. \quad (35)$$

The imaginary part of the above effective action gives real particle pair production. The s-contour integration is straight forward [1, 2, 3, 8]. Using the series expansion

$$\frac{1}{\text{sinh}x} = \frac{1}{x} + 2x \sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^2 n^2 + x^2} \quad (36)$$

we perform the s-contour integration around the pole $s = \frac{in\pi}{|g\Lambda_j(t)|}$ to find

$$W = 2\text{Im}S = \frac{1}{4\pi^3} \sum_{j=1}^{3} \sum_{n=1}^{\infty} \frac{1}{n} \int d^4x \int d^2p_T |g\Lambda_j(t)| e^{-n\pi \frac{(p_T^2+m^2)}{|g\Lambda_j(t)|}}. \quad (37)$$

Hence the probability of non-perturbative quark (antiquark) production per unit time, per unit volume and per unit transverse momentum from an arbitrary time dependent chromo-electric field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3) is given by

$$\frac{dW}{dt d^3x d^2p_T} = -\frac{1}{4\pi^3} \sum_{j=1}^{3} |g\Lambda_j(t)|\ \ln[1 - e^{-\frac{\pi(p_T^2+m^2)}{|g\Lambda_j(t)|}}], \quad (38)$$

which reproduces eq. (1). The expressions for gauge invariant $\Lambda_j(t)$'s are given in eq. (2).

To conclude we have obtained an exact non-perturbative result for quark-antiquark production from arbitrary time-dependent chromo-electric field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3) via the Schwinger mechanism by directly evaluating the path integral. This result relies crucially on the validity of the shift conjecture, which has not yet been established. We have found that the exact non-perturbative result is independent of all the time derivatives $\frac{d^nE^a(t)}{dt^n}$ where $n = 1, 2, ...\infty$ and has the same functional dependence on two casimir invariants $[E^a(t)E^a(t)]$ and $[d_{abc}E^a(t)E^b(t)E^c(t)]^2$ as the constant chromo-electric field E^a result [2] with the replacement: $E^a \rightarrow E^a(t)$.

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