

Magnetized Bianchi Type VI_0 Bulk Viscous Barotropic Massive String Universe with Decaying Vacuum Energy Density Λ

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Abstract: Bianchi type VI_0 bulk viscous massive string cosmological models using the technique given by Letelier (1983) with magnetic field are investigated. To get the deterministic models, we assume that the expansion (θ) in the model is proportional to the shear (σ) and also the fluid obeys the barotropic equation of state. The viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density. The value of the vacuum energy density Λ is observed to be small and positive at late time which is supported from recent supernovae Ia observations. The behaviour of the models from physical and geometrical aspects in presence and absence of magnetic field is also discussed.

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1. Introduction

The problem of the cosmological constant is one of the most salient and unsettled problems in cosmology. The smallness of the effective cosmological constant recently observed ($\Lambda_0 \leq 10^{-56} \text{cm}^{-2}$) constitutes the most difficult problems involving cosmology and elementary particle physics theory. To explain the striking cancellation between the “bare” cosmological constant and the ordinary vacuum energy contributions of the quantum fields, many mechanisms have been proposed during last few years [1]. The “cosmological constant problem” can be expressed as the discrepancy between the negligible value of Λ has for the present universe (as can be seen by the successes of Newton’s theory

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of gravitation [2]) and the values 10^{50} larger expected by the Glashow-Salam-Weinberg model [3] or by grand unified theory (GUT) where it should be 10^{107} larger [4]. The cosmological term Λ is then small at the present epoch. The problem in this approach is to determine the right dependence of Λ upon S or t . Recent observations of Type Ia supernovae (Perlmutter et al. [5], Riess et al. [6]) and measurements of the cosmic microwave background [7] suggest that the universe is an accelerating expansion phase [8].

Several ansätze have been proposed in which the Λ term decays with time (see Refs. Gasperini [9], Berman [10-12], Berman et al. [13-15], Freese et al. [16], Özer and Taha [17], Ratra and Peebles [18], Chen and Hu [19], Abdussattar and Vishwakarma [20], Gariel and Le Denmat [21], Pradhan et al. [22]). Of the special interest is the ansatz $\Lambda \propto S^{-2}$ (where S is the scale factor of the Robertson-Walker metric) by Chen and Wu [19], which has been considered/modified by several authors (Abdel-Rahaman [23], Carvalho et al. [24], Silveira and Waga [25], Vishwakarma [26]).

One of the outstanding problems in cosmology today is developing a more precise understanding of structure formation in the universe, that is, the origin of galaxies and other large-scale structures. Existing theories for the structure formation of the Universe fall into two categories, based either upon the amplification of quantum fluctuations in a scalar field during *inflation*, or upon symmetry breaking phase transition in the early Universe which leads to the formation of *topological defects* such as domain walls, cosmic strings, monopoles, textures and other 'hybrid' creatures. Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (see Refs. Zel'dovich et al. [27], Kibble [28, 29], Everett [30], Vilenkin [31]). It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies (Zel'dovich [32]). These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [33, 34] and Stachel [35].

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged. Several authors (Zeldovich [36], Harrison [37], Misner, Thorne and Wheeler [38], Asseo and Sol [39], Pudritz and Silk [40], Kim, Tribble, and Kronberg [41], Perley, and Taylor [42], Kronberg, Perry, and Zukowski [43], Wolfe, Lanzetta and Oren [44], Kulsrud, Cen, Ostriker and Ryu [45] and Barrow [46]) have pointed out the importance of magnetic field in different context. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The string cosmological models with a magnetic field are also discussed by Benerjee et al. [47], Chakraborty [48], Tikekar and Patel ([49], Patel and Maharaj [51] Singh and Singh [52]).

Recently, Bali et al. [53-57], Pradhan et al. [58-60], Yadav et al. [61] and Pradhan [62] have investigated Bianchi type I, II, III, V, IX and cylindrically symmetric magnetized string cosmological models in presence and absence of magnetic field. Pradhan and Bali [50] have investigated some solutions for Bianchi type VI_0 cosmology in presence and absence of magnetic field. In this paper we have derived some Bianchi type VI_0 string cosmological models for bulk viscous fluid distribution in presence and absence of magnetic field and discussed the variation of Λ with time. This paper is organized as follows: The metric and field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in presence of magnetic field. In Section 4, we have described the solution of the field equations in presence of bulk viscous fluid and some geometric and physical behaviour of the model. Section 5 includes the solution in absence of magnetic field. In Section 6, we have discussed the bulk viscous solution of the field equations in absence of magnetic field. In the last Section 7, concluding remarks are given.

2. The Metric and Field Equations

We consider the Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{2x}dy^2 + C^2(t)e^{-2x}dz^2. \quad (1)$$

The energy-momentum tensor for a cloud of strings in presence of magnetic field is taken into the form

$$T_{ik} = (\rho + p)v_iv_k + pg_{ik} - \lambda x_ix_k + [g^{lm}F_{il}F_{km} - \frac{1}{4}g_{ik}F_{lm}F^{lm}], \quad (2)$$

where v_i and x_i satisfy conditions

$$v^i v_i = -x^i x_i = -1, \quad v^i x_i = 0. \quad (3)$$

In equations (2), p is isotropic pressure, ρ is rest energy density for a cloud strings, λ is the string tension density, F_{ij} is the electromagnetic field tensor, x^i is a unit space-like vector representing the direction of string, and v^i is the four velocity vector satisfying the relation

$$g_{ij}v^i v^j = -1. \quad (4)$$

Here, the co-moving coordinates are taken to be $v^1 = 0 = v^2 = v^3$ and $v^4 = 1$ and $x^i = (\frac{1}{A}, 0, 0, 0)$. The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \quad (5)$$

$$F_{;k}^{ik} = 0, \quad (6)$$

are satisfied by

$$F_{23} = K(\text{say}) = \text{constant}, \quad (7)$$

where a semicolon (;) stands for covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$)

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j - \Lambda g_i^j, \quad (8)$$

for the line-element(1) lead to the following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{1}{A^2} = - \left[p - \lambda - \frac{K^2}{2B^2C^2} \right] - \Lambda, \quad (9)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} = - \left[p + \frac{K^2}{2B^2C^2} \right] - \Lambda, \quad (10)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} = - \left[p + \frac{K^2}{2B^2C^2} \right] - \Lambda, \quad (11)$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA} - \frac{1}{A^2} = \left[\rho + \frac{K^2}{2B^2C^2} \right] - \Lambda, \quad (12)$$

$$\frac{1}{A} \left[\frac{C_4}{C} - \frac{B_4}{B} \right] = 0, \quad (13)$$

where the sub indice 4 in A, B, C denotes ordinary differentiation with respect to t . The velocity field v^i is irrotational. The scalar expansion θ and components of shear σ_{ij} are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \quad (14)$$

$$\sigma_{11} = \frac{A^2}{3} \left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} \right], \quad (15)$$

$$\sigma_{22} = \frac{B^2}{3} \left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C} \right], \quad (16)$$

$$\sigma_{33} = \frac{C^2}{3} \left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B} \right], \quad (17)$$

$$\sigma_{44} = 0. \quad (18)$$

Therefore

$$\sigma^2 = \frac{1}{2} \left[(\sigma^1_1)^2 + (\sigma^2_2)^2 + (\sigma^3_3)^2 + (\sigma^4_4)^2 \right],$$

which leads to

$$\sigma^2 = \frac{1}{3} \left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} - \frac{C_4A_4}{CA} \right].$$

Above relation after using (13) reduces to

$$\sigma = \frac{1}{\sqrt{3}} \left(\frac{A_4}{A} - \frac{B_4}{B} \right). \quad (19)$$

3. Solutions of the Field Equations

We have revisited the solutions obtained by Pradhan and Bali [50]. The field equations (9)-(13) are a system of five equations with seven unknown parameters A , B , C , ρ , p , λ and Λ . We need two additional conditions to obtain explicit solutions of the system.

Equation (13) leads to

$$C = mB, \quad (20)$$

where m is an integrating constant.

We first assume that the expansion (θ) in the model is proportional to shear (σ). The motive behind assuming this condition is explained with reference to Thorne [63], the observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 per cent [64, 65]. To put more precisely, red-shift studies place the limit

$$\frac{\sigma}{H} \leq 0.3$$

on the ratio of shear, σ , to Hubble constant, H , in the neighbourhood of our Galaxy today. Collins et al. [66] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant. This condition and Eq. (20) lead to

$$\frac{1}{\sqrt{3}} \left(\frac{A_4}{A} - \frac{B_4}{B} \right) = l \left(\frac{A_4}{A} + \frac{2B_4}{B} \right) \quad (21)$$

which yields to

$$\frac{A_4}{A} = n \frac{B_4}{B}, \quad (22)$$

where $n = \frac{(2l\sqrt{3}+1)}{(1-l\sqrt{3})}$ and l are constants. Eq. (22), after integration, reduces to

$$A = \beta B^n, \quad (23)$$

where β is a constant of integration. Eqs. (10) and (12) lead to

$$p = -\frac{K^2}{2B^2C^2} - \left(\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} \right) - \Lambda, \quad (24)$$

and

$$\rho = \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA} - \frac{1}{A^2} - \frac{K^2}{2B^2C^2} + \Lambda, \quad (25)$$

respectively. Now let us consider that the fluid obeys the barotropic equation of state

$$p = \gamma\rho, \quad (26)$$

where $\gamma(\gamma \leq 0 \leq 1)$ is a constant. Eqs. (24) to (26) lead to

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + (1+\gamma)\frac{A_4C_4}{AC} + \gamma \left(\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} \right) - (1+\gamma)\frac{1}{A^2} +$$

$$(1 - \gamma) \frac{K^2}{2B^2 C^2} + (1 + \gamma)\Lambda = 0. \quad (27)$$

Eq. (27) with the help of (20) and (23) reduces to

$$2B_{44} + \frac{2(n^2 + 2\gamma n + \gamma) B_4^2}{(n + 1) B^2} = \frac{2(1 + \gamma)}{\beta^2 B^{2n-1}} + \frac{(1 - \gamma)K^2}{m^2 B^3} + 2l_0 B, \quad (28)$$

where $l_0 = (1 + \gamma)\Lambda$.

Let us consider $B_4 = f(B)$ and $f' = \frac{df}{dB}$. Hence Eq. (28) takes the form

$$\frac{d}{df}(f^2) + \frac{2\alpha}{B} f^2 = \frac{2(1 + \gamma)}{\beta^2 B^{2n-1}} + \frac{(1 - \gamma)K^2}{m^2 B^3} + 2l_0 B, \quad (29)$$

where $\alpha = \frac{(n^2 + 2n\gamma + \gamma)}{(n+1)}$. Eq. (29) after integrating reduces to

$$f^2 = \frac{2(1 + \gamma)B^{-2n+2}}{\beta^2(2\alpha - 2n + 2)} + \frac{(1 - \gamma)K^2}{2m^2(\alpha - 1)} + \frac{l_0 B^2}{(\alpha + 1)} + MB^{-2\alpha}, \quad \gamma \neq 1, \quad (30)$$

where M is an integrating constant. To get deterministic solution in terms of cosmic string t , we suppose $M = 0$ without any loss of generality. In this case Eq. (30) takes the form

$$f^2 = aB^{-2(n-1)} + bB^{-2} + kB^2, \quad (31)$$

where

$$a = \frac{2(1 + \gamma)}{\beta^2(2\alpha - 2n + 2)}, \quad b = \frac{(1 - \gamma)K^2}{2m^2(\alpha - 1)}, \quad k = \frac{(1 + \gamma)\Lambda}{(\alpha + 1)}.$$

Therefore, we have

$$\frac{dB}{\sqrt{aB^{-2(n-1)} + bB^{-2} + kB^2}} = dt. \quad (32)$$

To get deterministic solution, we assume $n = 2$. In this case integrating Eq. (32), we obtain

$$B^2 = \sqrt{(a + b)} \frac{\sinh(2\sqrt{kt})}{\sqrt{k}}. \quad (33)$$

Hence, we have

$$C^2 = m^2 \sqrt{(a + b)} \frac{\sinh(2\sqrt{kt})}{\sqrt{k}}, \quad (34)$$

$$A^2 = \beta^2(a + b) \frac{\sinh^2(2\sqrt{kt})}{k}, \quad (35)$$

where $k > 0$ without any loss of generality.

Therefore, the metric (1), in presence of magnetic field, reduces to the form

$$ds^2 = -dt^2 + \beta^2(a + b) \frac{\sinh^2(2\sqrt{kt})}{k} dx^2 +$$

$$\sqrt{(a+b)} \frac{\sinh(2\sqrt{kt})}{\sqrt{k}} e^{2x} dy^2 + m^2 \sqrt{(a+b)} \frac{\sinh(2\sqrt{kt})}{\sqrt{k}} e^{-2x} dz^2. \quad (36)$$

The expressions for the pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p) for the model (36) are given by

$$p = \left[\frac{k}{\beta^2(a+b)} - \frac{K^2 k}{2m^2(a+b)} \right] \coth^2(2\sqrt{kt}) + \left[\frac{K^2}{2m^2(a+b)} - \frac{1}{\beta^2(a+b)} - 8 \right] k - \Lambda, \quad (37)$$

$$\rho = \left[5k - \frac{k}{(a+b)} \left(\frac{K^2}{2m^2} + \frac{1}{\beta^2} \right) \right] \coth^2(2\sqrt{kt}) + \frac{k}{(a+b)} \left(\frac{K^2}{2m^2} + \frac{1}{\beta^2} \right) + \Lambda, \quad (38)$$

where $p = \gamma\rho$ is satisfied by (27).

$$\lambda = \left[\frac{2k}{\beta^2(a+b)} - \frac{K^2 k}{m^2(a+b)} - k \right] \coth^2(2\sqrt{kt}) + \left\{ \frac{K^2 k}{m^2(a+b)} - \frac{2k}{\beta^2(a+b)} - 4k \right\}, \quad (39)$$

$$\rho_p = \rho - \lambda = \left[\frac{K^2 k}{2m^2(a+b)} - \frac{3k}{\beta^2(a+b)} + k \right] \coth^2(2\sqrt{kt}) + 9k + \left\{ \frac{3k}{\beta^2(a+b)} - \frac{K^2}{2m^2(a+b)} \right\}, \quad (40)$$

4. Solutions for Bulk Viscous Fluid

Astronomical observations of large-scale distribution of galaxies of our universe show that the distribution of matter can be satisfactorily described by a perfect fluid. But large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyze dissipative effects in cosmology. Further, there are several processes which are expected to give rise to viscous effect. These are the decoupling of neutrinos during the radiation era and the recombination era [67], decay of massive super string modes into massless modes [68], gravitational string production [69, 70] and particle creation effect in grand unification era [71]. It is known that the introduction of bulk viscosity can avoid the big bang singularity. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [72] for a review on cosmological models with

bulk viscosity). A uniform cosmological model filled with fluid which possesses pressure and second (bulk) viscosity was developed by Murphy [73]. The solutions that he found exhibit an interesting feature that the big bang type singularity appears in the infinite past.

In presence of bulk viscous fluid distribution, we replace isotropic pressure p by effective pressure \bar{p} in Eq. (37) where

$$\bar{p} = p - \xi v_{;i}^i, \quad (41)$$

where ξ is the coefficient of bulk viscosity.

The scalar of expansion (θ) for the model (36) is given by

$$\theta = 4\sqrt{k} \coth(2\sqrt{kt}), \quad (42)$$

The expression for effective pressure \bar{p} for the model Eq. (36) is given by

$$\begin{aligned} \bar{p} = (p - \xi v_{;i}^i) = & \left[\frac{k}{\beta^2(a+b)} - \frac{K^2 k}{2m^2(a+b)} \right] \coth^2(2\sqrt{kt}) \\ & + \left[\frac{K^2}{2m^2(a+b)} - \frac{1}{\beta^2(a+b)} - 8 \right] k - \Lambda, \end{aligned} \quad (43)$$

Thus, for given $\xi(t)$ we can solve for the cosmological parameters. In most of the investigation involving bulk viscosity is assumed to be a simple power function of the energy density (Pavon [74], Maartens [75], Zimdahl [76], Santos [77])

$$\xi(t) = \xi_0 \rho^n, \quad (44)$$

where ξ_0 and n are constants. For small density, n may even be equal to unity as used in Murphy's work [52] for simplicity. If $n = 1$, Eq. (43) may correspond to a radiative fluid (Weinberg [2]). Near the big bang, $0 \leq n \leq \frac{1}{2}$ is a more appropriate assumption (Belinskii and Khalatnikov [78]) to obtain realistic models.

For simplicity sake and for realistic models of physical importance, we consider the following two cases ($n = 0, 1$):

4.1 Model I: When $n = 0$

When $n = 0$, Eq. (44) reduces to $\xi = \xi_0 = \text{constant}$. With the use of Eqs. (37), (26) and (42), Eq. (43) reduces to

$$(1 + \gamma)\rho = 4\xi_0\sqrt{k} \coth(2\sqrt{kt}) + (5k - k_1 + k_2) \coth^2(2\sqrt{kt}) + (k_1 - k_2), \quad (45)$$

where

$$k_1 = \frac{k}{(a+b)} \left(\frac{K^2}{2m^2} + \frac{1}{\beta^2} \right),$$

and

$$k_2 = \frac{k}{(a+b)} \left(\frac{1}{\beta^2} - \frac{K^2}{2m^2} \right).$$

Eliminating $\rho(t)$ between Eqs. (38) and (45), we obtain

$$(1 + \gamma)\Lambda = 4\xi_0\sqrt{k} \coth(2\sqrt{kt}) + [k_2 - (5k - k_1)\gamma] \coth^2(2\sqrt{kt}) - (k_1\gamma + k_2). \quad (46)$$

4.2 Model II: When $n = 1$

When $n = 1$, Eq. (44) reduces to $\xi = \xi_0\rho$. With the use of Eqs. (37), (26) and (42), Eq. (43) reduces to

$$\rho = \frac{(5k - k_1 + k_2) \coth(2\sqrt{kt}) + (k_1 - k_2)}{[1 + \gamma - 4\xi_0\sqrt{k} \coth(2\sqrt{kt})]}. \quad (47)$$

Eliminating $\rho(t)$ between Eqs. (38) and (47), we obtain

$$\Lambda = \frac{(5k - k_1 + k_2) \coth(2\sqrt{kt}) + (k_1 - k_2)}{[1 + \gamma - 4\xi_0\sqrt{k} \coth(2\sqrt{kt})]} - (5k - k_1) \coth(2\sqrt{kt}) - k_1. \quad (48)$$

From Eqs. (45) and (47), by choosing the appropriate values of constant quantities, we observe that $\rho(t)$ in both models are a decreasing function of time and $\rho > 0$ for all times. From Eqs. (46) and (48), we see that the cosmological terms Λ in both models are a decreasing function of time and they approach a small positive value at late time. Thus, our models are consistent with the results of recent observations (Perlmutter et al. [5], Riess et al. [6]).

The effect of bulk viscosity is to produce a change in perfect fluid and therefore exhibits essential influence on the character of the solution. A comparative inspection of ρ show apparent evolution of time due to perfect fluid and bulk viscous fluid. It is apparent that the vacuum energy density (ρ) decays much fast in later case. Also shows effect of uniform viscosity model and linear viscosity model. Even in these case decay of vacuum energy density is much faster than uniform. So the coupling parameter ξ_0 would be related with physical structure of the matter and provides mechanism to incorporate relevant property. In order to say more specific, detailed study would be needed which would be reported in future. Similar behaviour is observed for the cosmological constant Λ . We also observe here that Murphy's [73] conclusion about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid in general, is not true. The results obtained by Myung and Cho [68] also show that, it is not generally valid since for some cases big bang singularity occurs in finite past.

The shear tensor (σ) and the proper volume (V^3) for the model (36) are given by

$$\sigma = \sqrt{\frac{k}{3}} \coth(2\sqrt{kt}), \quad (49)$$

$$V^3 = \frac{\beta m(a+b)}{k} \sinh^2(2\sqrt{kt}). \quad (50)$$

From Eqs. (42) and (49), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \quad (51)$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -\left[\frac{\frac{8k}{3} - \frac{8k}{9} \coth^2(2\sqrt{kt})}{\frac{16k}{9} \coth^2(2\sqrt{kt})} \right]. \quad (52)$$

From (52), we observe that

$$q < 0 \quad \text{if} \quad \coth^2(2\sqrt{kt}) < 3$$

and

$$q > 0 \quad \text{if} \quad \coth^2(2\sqrt{kt}) > 3.$$

From (38), $\rho \geq 0$ implies that

$$\coth^2(2\sqrt{kt}) \leq \left[\frac{\frac{k}{(a+b)} \left(\frac{K^2}{2m^2} + \frac{1}{\beta^2} \right) + \Lambda}{\frac{k}{(a+b)} \left(\frac{K^2}{2m^2} + \frac{1}{\beta^2} \right) - 5k} \right]. \quad (53)$$

Also from (40), $\rho_p \geq 0$ implies that

$$\coth^2(2\sqrt{kt}) \leq \left[\frac{\frac{3k}{\beta^2(a+b)} - \frac{K^2}{2m^2(a+b)} + 9k}{\frac{3k}{\beta^2(a+b)} - \frac{K^2k}{2m^2(a+b)} - k} \right]. \quad (54)$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (53) and (54).

The model (36) starts with a big bang at $t = 0$. The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta} = \text{constant}$, hence the model does not approach isotropy. Since ρ , λ , θ , σ tend to infinity and $V^3 \rightarrow 0$ at initial epoch $t = 0$, therefore, the model (36) for massive string in presence of magnetic field has Line-singularity (Banerjee et al. [47]). For the condition $\coth^2(2\sqrt{kt}) < 3$, the solution gives accelerating model of the universe. It can be easily seen that when $\coth^2(2\sqrt{kt}) > 3$, our solution represents decelerating model of the universe.

5. Solutions in Absence of Magnetic Field

In absence of magnetic field, i.e. when $b \rightarrow 0$ i.e. $K \rightarrow 0$, we obtain

$$B^2 = 2\sqrt{2} \frac{\sinh(2\sqrt{kt})}{2\sqrt{k}},$$

$$C^2 = 2m^2 \sqrt{a} \frac{\sinh(2\sqrt{k}t)}{2\sqrt{k}},$$

$$A^2 = 4a\beta^2 \frac{\sinh^2(2\sqrt{k}t)}{4k}. \quad (55)$$

Hence, in this case, the geometry of the universe (36) reduces to

$$ds^2 = -dt^2 + 4\beta^2 a \frac{\sinh^2(2\sqrt{k}t)}{4k} dx^2 +$$

$$2\sqrt{2} \frac{\sinh(2\sqrt{k}t)}{2\sqrt{k}} e^{2x} dy^2 + 2m^2 \sqrt{a} \frac{\sinh(2\sqrt{k}t)}{2\sqrt{k}} e^{-2x} dz^2. \quad (56)$$

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ) for the model (56) are given by

$$p = \frac{k}{a\beta^2} \coth^2(2\sqrt{k}t) - \left(\frac{1}{a\beta^2} + 8 \right) k - \Lambda, \quad (57)$$

$$\rho = \left(5k - \frac{k}{a\beta^2} \right) \coth^2(2\sqrt{k}t) + \frac{k}{a\beta^2} + \Lambda, \quad (58)$$

$$\lambda = \left[\frac{2k}{a\beta^2} - k \right] \coth^2(2\sqrt{k}t) - \left\{ \frac{2k}{a\beta^2} + 4k \right\}, \quad (59)$$

$$\rho_p = \rho - \lambda = \left[k - \frac{3k}{a\beta^2} \right] \coth^2(2\sqrt{k}t) + 9k + \frac{3k}{\beta^2 a}, \quad (60)$$

$$\theta = 4\sqrt{k} \coth(2\sqrt{k}t), \quad (61)$$

6. Solutions for Bulk Viscous Fluid

The expression for effective pressure \bar{p} for the model Eq. (56) is given by

$$\bar{p} = (p - \xi v_{;i}^i) = \frac{k}{a\beta^2} \coth^2(2\sqrt{k}t) - \left(\frac{1}{a\beta^2} + 8 \right) k - \Lambda \quad (62)$$

Thus, for given $\xi(t)$ we can solve for the cosmological parameters. For simplicity sake and for realistic models of physical importance, we consider the following two cases ($n = 0, 1$):

6.1 Model I: When $n = 0$

When $n = 0$, Eq. (44) reduces to $\xi = \xi_0 = \text{constant}$. With the use of Eqs. (57), (26) and (61), Eq. (62) reduces to

$$(1 + \gamma)\rho = 4\xi_0 \sqrt{k} \coth(2\sqrt{k}t) + 5k \coth^2(2\sqrt{k}t) - 8k \quad (63)$$

Eliminating $\rho(t)$ between Eqs. (58) and (63), we obtain

$$(1 + \gamma)\Lambda = 4\xi_0 \sqrt{k} \coth(2\sqrt{k}t) - \left[5\gamma + \frac{(1 + \gamma)}{a\beta^2} \right] k \coth^2(2\sqrt{k}t) - \frac{(1 + \gamma)k}{a\beta^2} \quad (64)$$

6.2 Model II: When $n = 1$

When $n = 1$, Eq. (44) reduces to $\xi = \xi_0\rho$. With the use of Eqs. (57), (26) and (61), Eq. (62) reduces to

$$\rho = \frac{5k \coth(2\sqrt{kt}) + 8k}{[1 + \gamma + 4\xi_0\sqrt{k} \coth(2\sqrt{kt})]}. \quad (65)$$

Eliminating $\rho(t)$ between Eqs. (58) and (65), we obtain

$$\Lambda = \frac{5k \coth(2\sqrt{kt}) + 8k}{[1 + \gamma + 4\xi_0\sqrt{k} \coth(2\sqrt{kt})]} - [5 - \frac{1}{a\beta^2}]k \coth(2\sqrt{kt}) - \frac{k}{a\beta^2}. \quad (66)$$

From Eqs. (63) and (65), by choosing the proper values of constant quantities, we observe that $\rho(t)$ in both models are a decreasing function of time and $\rho > 0$ for all times. From Eqs. (64) and (66), we see that the cosmological terms Λ in both models are a decreasing function of time and they approach a small positive value at late time. Thus, our models are consistent with the results of recent observations (Perlmutter et al. [5], Riess et al. [6]).

The shear tensor (σ) and the proper volume (V^3) for the model (49) are given by

$$\sigma = \sqrt{\frac{k}{3}} \coth(2\sqrt{kt}), \quad (67)$$

$$V^3 = \frac{\beta ma}{k} \sinh^2(2\sqrt{kt}). \quad (68)$$

From Eqs. (61) and (67), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \quad (69)$$

From (58), $\rho \geq 0$ implies that

$$\coth^2(2\sqrt{kt}) \leq \left[\frac{\frac{k}{a\beta^2} + \Lambda}{\frac{k}{a\beta^2} - 5k} \right]. \quad (70)$$

Also from (60), $\rho_p \geq 0$ implies that

$$\coth^2(2\sqrt{kt}) \leq \left[\frac{\frac{3k}{a\beta^2} + ak}{\frac{3k}{a\beta^2} - k} \right]. \quad (71)$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (70) and (71).

The model (56) starts with a big bang at $t = 0$ and the expansion in the model decreases as time increases. The spatial volume of the model increases as time increases. Since $\frac{\sigma}{\theta} = \text{constant}$, hence the anisotropy is maintained throughout. Since ρ , λ , θ , σ tend to infinity and $V^3 \rightarrow 0$ at initial epoch $t = 0$, therefore, the model (56) for massive string in absence of magnetic field has Line-singularity [47].

Concluding Remarks

Some Bianchi type VI_0 massive string cosmological models with a bulk viscous fluid as the source of matter are obtained in presence and absence of magnetic field. Generally, the models are expanding, shearing and non-rotating. In presence of bulk viscous fluid it represents an accelerating universe during the span of time mentioned below equation (52) as decelerating factor $q < 0$ and it represents decelerating universe as $q > 0$. All the two massive string cosmological models obtained in the present study have Line-singularity (Banerjee et al. [47]) at the initial epoch $t = 0$. The variation of cosmological term in presence and absence of magnetic field is consistent with recent observations. To solve the age parameter and density parameter, one requires the cosmological constant to be positive or equivalently the deceleration parameter to be negative. The nature of the cosmological constant Λ and the energy density ρ have been examined.

We have also observed that the magnetic field gives positive contribution to expansion and shear which die out for large value of t at a slower rate than the corresponding quantities in the absence of magnetic field. In our derived models, in presence and absence of magnetic field, by proper choice of the constant quantities the cosmological term Λ are found to be a decreasing function of time and their values approach a small positive value at late time which is supported by recent results from the observations of Type Ia supernova explosion (SN Ia). Naturally a cosmological model is required to explain acceleration in the present universe. Thus, our theoretical models are consistent with the results of recent observations.

The effect of bulk viscosity is to produce a change in perfect fluid and therefore exhibits essential influence on the character of the solution. The effect is clearly visible on the p effective (see details in previous sections). We have observed regular well behaviour of energy density, cosmological term (Λ) and the expansion of the universe with parameter t in both presence and absence of magnetic field. Our solutions generalize the solutions recently obtained by Pradhan and Bali [50].

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