EJTP 8, No. 25 (2011) 137–152 *Electronic Journal of Theoretical Physics*

Market Fluctuations – the Thermodynamics Approach

S. Prabakaran*

College of Business Administration, Kharj,
King Saud University - Riyadh,
Kingdom Saudi Arabia

Received 6 July 2010, Accepted 10 February 2011, Published 25 May 2011

Abstract: A thermodynamic analogy in economics is older than the idea of von Neumann to look for market entropy in liquidity, advice that was not taken in any thermodynamic analogy presented so far in the literature. In this paper, we go further and use a standard approach in market fluctuation and develop a set of equations which are a simple model for market fluctuation in a hypothetical financial market.In the past decade or so, physicists have begun to do academic research in economics. Perhaps people are now actively involved in an emerging field often called Econophysics. The scope of this paper is to present a phenomenological analysis for Market Fluctuations through Thermodynamics approach The main ambition of this study is fourfold: 1) First we begin our description with how market parameters vary with time by using of simplest example. 2) To extend that the market fluctuations appears with the enforced changes of macro parameters of the market and land speculations with non existence. 3) Next we derived the equation for how market fluctuates with respect to time in an equilibrium state. 4) Finally we analyze the how the fluctuations affects the perceptions of the market agents on the future. And this paper end with conclusion.

Keywords: Thermodynamics; Econophysics; Market Equilibrium and Market Fluctuation
PACS (2010): 05.70.-a; 89.65.Gh; 89.65.Gh

1. Introduction

Attempts at neo-classical equilibrium economic analogies with thermodynamics go back to Guilluame [1] and Samuelson [2]. Von Neumann apparently believed that thermodynamic formalism could potentially be useful in computer theory, for formulating a description of intelligience, and was interested in the possibility of a thermodynamics

* jopraba@gmail.com

of economics. But presented with Guillaume's work, he criticized it on the basis of the misidentification of a quantity as entropy [3]. Much more recently, Smith and Foley [4] have presented a much more careful mapping of neo-classical equilibrium theory onto an apparently formal thermodynamics. The neo-classical equilibrium analog of the zeroth law requires identifying price as an intensive variable (not necessarily as temperature), and quantities of assets held are treated as extensive variables. Starting with utility maximization as the fundamental principle, analogs of thermodynamic potentials were defined purely formally by constructing Legendre transforms. A quantity that they identified as 'entropy' was constructed as a Legendre transform on utility, with utility maximization being interpreted as analogous to entropy maximization for a closed mechanical system in thermodynamic equilibrium (entropy maximization is not the second law, but is a deduction from the second law).

A word before going further. Equilibrium is not always essential for the identification of an abstract, formal thermodynamics, and neither is a heat bath, but a correct identification of entropy as disorder *is* necessary. E.g., entropy and formal thermodynamics exist [5] and have been constructed mathematically for the symbolic dynamics representing chaotic dynamical systems with generating partitions [6], but in this case the entropy is simply the Boltzmann entropy of the symbol sequences corresponding to a single Liapunov exponent, and therefore correctly describes disorder in the usual Boltzmann-Gibbs sense [7]. The underlying chaotic dynamical system is driven-dissipative and is far from equilibrium (one can illustrate the formal thermodynamics via a tent, logistic or Henon map, e.g.), but the entropy and thermodynamics are based on time-independent quantities and therefore do not contradict the nonequilibrium nature of the underlying dynamics that gives rise to the invariant symbolic dynamics. Another way to say it is that the formal thermodynamics and entropy are based on the topological invariants and generating partition of the dynamical system [6,7], and not at all on the time-evolution from initial conditions. In what follows, as in [4], we address the question of trying to use an economic (finance) model 'directly'to construct a thermodynamics, because topological invariants (and generating partitions) do not exist in financial data in particular, or in economic phenomena more generally. That is, we simply apply empirically-based market dynamics to the same variables treated statically by Smith and Foley, but without the unnecessary introduction of empirically unobservable quantities like utility.

If we observe how market parameters vary with time for example, consider prices or the amount of goods sold we observe certain fluctuations of these parameters about their equilibrium states. The reasons for these fluctuations may be various but for us it is necessary to differentiate two important types of fluctuations.

The first type of fluctuations is the result of the fact that markets can rarely if ever be considered isolated. As a rule, markets are parts of larger markets and even if the system as a whole, i.e., not only the directly observed part but other connected with it are in equilibrium certain random deviations are possible which will be the larger the smaller system is being considered.

The second type of fluctuations is fluctuations due to speculations. The values of the mean quadratic deviation are inversely proportional to the square root of the number of the economic agents of the market. In what follows we will show what kind of theory can be used for calculating mean quadratic fluctuations.

This theory is again identical to the fluctuation theory known from the statistical physics. But in our case — being applied to economics — the fluctuation theory is especially viable because market fluctuations is a quite accessible procedure and we will see in what follows the mean value of market fluctuations become related with thermodynamic parameters of the markets. This means that the measurements of the mean values of fluctuations can be used to determine thermodynamic parameters of the markets in particular to define the temperature.

It is hardly needed to explain how important this may be for the construction of a theory.

We obtain at last in our hands the measurement tool that can replace special experimental conditions. Nevertheless, everything said about the artificial reality of the experiment remains valid. In addition to purely probabilistic factors related with the peculiarities of the market structure the fluctuations of the quantities to be measured such as prices and volumes of the goods will be affected by other non-market factors such as social, political, demographic and so on.

Our thermodynamic model of the market ignores these extra facts whereas in reality we cannot get rid of them at best reduce their influence to a minimum by selecting specific moments for the measurements or excluding certain data. As before we face the same dilemma. In order to actually verify a theory one needs to create an artificial reality or, at least, select particular cases which will be close to an artificial reality so that the influence of the factors not included in the model will be reduced to a minimum.

In this sense to measure fluctuations in the given systems at hand is much more economic way to study economic systems than special constructing of experimental situations which among other things can hardly be possible because of an incredibly high price of such experiments.

2. Values Of Fluctuations

It is natural to assume in complete agreement with the man postulate of the statistical theory that the probability of the system to be in a state described by a macro-parameter X is proportional to the number of microstates corresponding to this value of the macroparameters. In other words, the probability $W(X)$ of the system to be in the macro-state X is proportional to the statistical weight $g(X)$ of this state. Recall that these basic principles of the statistical theory are never proved. We define the elementary microstates to be equally probable. This easily implies that the probability of the system to be in the macro-state X is proportional1 to the exponent of the entropy of this macro-state just due to the definition of the entropy as the logarithm of the statistical weight:

$$W(x) = g(x) = e^{Ing(z)} = e^{S(x)}. \tag{1}$$

We see therefore that if we are only interested in the relation between macro and micro parameters of the system, there is no difference between physical and economic systems.

If we consider a system in an equilibrium state X_0we can expand the entropy $S\left(X_0+\Delta X\right)$in the series with respect to$X$. Since, by definition of the equilibrium state, the entropy is maximum at we have

$$\left.\frac{\partial S}{\partial X}\right|_{X_0}=0,\quad \left.\frac{\partial^2 S}{\partial X^2}\right|_{X_0}<0, \tag{2}$$

Again the properties of the singular point of a map enable us to construct a theory. We see that the probability $W\left(x\right)$ of the system to possess the value $X$of the macro-parameter that differs by $\Delta X$from the equilibrium state X_0 is proportional to

$$W\left(X_0+\Delta X\right)=e^{S_0-\frac{1}{2}\frac{\partial^2 S}{\partial X^2}\Delta X^2} \tag{3}$$

Since the exponent decreases very rapidly as the argument grows, the role of $W\left(X_0+\Delta X\right)$for large values of $W\left(x\right)$ is insufficient actually and we can with good accuracy obtain the normalized constant for the probability distribution integrating Walong $\Delta X-\infty$to ∞

Thus we obtain

$$\int_{-\infty}^{\infty}dW\left(X_0+\Delta X\right)=A\int_{-\infty}^{\infty}e^{-\frac{\Delta X^2}{2}}\left.\frac{\partial^2 S}{\partial X^2}\right|_{X_0}d\Delta X=1 \tag{4}$$

or

$$W\left(X_0+\Delta X\right)=\sqrt{\frac{1}{2\pi}\frac{\partial^2 S}{\partial X^2}}\Bigg|_{X_0}e^{-\frac{\Delta X^2}{2}\frac{\partial^2 S}{\partial X^2}\Delta X^2\big|_{X_0}}. \tag{5}$$

It is not difficult to deduce from here the mean square of the fluctuation:

$$\overline{\Delta X^2}=\sqrt{\frac{1}{2\pi}\left.\frac{\partial^2 S}{\partial X^2}\right|_{X_0}}\int_{-\infty}^{\infty}\left(\Delta X\right)^2e^{-\frac{\partial^2 S}{\partial X^2}\Delta X}d\Delta X=\frac{1}{\left.\frac{\partial^2 S}{\partial X^2}\right|_{X_0}} \tag{6}$$

Therefore, for the probability of system deviation from the equilibrium state, we obtain the Gauss distribution:

$$W\left(X\right)=\frac{1}{\sqrt{2\pi\overline{\Delta X^2}}}e^{-\frac{\Delta X^2}{2\overline{\Delta X^2}}} \tag{7}$$

Since $\Delta X$is small and the probability steeply drops as Xgrows, we can simply find the mean square of any function $f\left(X\right)$by expanding it into the Taylor series and confining to the first term:

$$\overline{\Delta f^2}=\left(\left.\frac{\partial f}{\partial X}\right|_{X=X_0}\right)^2\overline{\Delta X^2}. \tag{8}$$

To compute the mean of the product of the fluctuations of thermodynamic quantities, observe that the mean of the fluctuation vanishes thanks to the symmetry of the distribution function relative to the point $\Delta X=0$. The mean of the product of the fluctuations of independent values $\overline{\Delta\alpha\Delta b}$ also vanishes since for the independent values we have $\overline{\Delta\alpha\Delta b}=\overline{\Delta\alpha}\cdot\overline{\Delta b}=0$.

Now consider the mean of the product of fluctuations, which are not independent. We will need approximately the same technique of dealing with thermodynamic quantities that we have already used to derive thermodynamic inequalities. If a fluctuation occurs in a portion of the market, which is in equilibrium, this means that we can assume the temperature of the universum and the price constants in the first approximation (for a fluctuation). This in turn means that the deviations of the flow of money from the equilibrium will be given by thermodynamic potential in accordance with arguments [8]

$$\Delta\Phi = \Delta E - T_0\Delta S + P_0\Delta V \tag{9}$$

where T_0- is the equilibrium temperature and P_0- is the equilibrium price.

Indeed, the entropy of the market is a function on the money flow. If we perform a modification of this flow in a part of the system then the entropy of the system as a whole will change:

$$\Delta S = \frac{\partial S}{\partial E} \cdot \Delta E|_{P_0 T_0} = T_0\Delta\Phi. \tag{10}$$

The changes of Φcan be found by expanding Einto the series with respect to δS and δV.Observe here that the distribution function depends on the total change of the entropy of the system under the fluctuation whereas δS and δV- are the changes of entropy and the goods flow only for the separated part of the system.

In the same way as above we have

$$\begin{aligned} \Delta\Phi &= \Delta E - T_0\delta S + P_0\delta V \\ &= \tfrac{1}{2}\left(\tfrac{\partial^2 E}{\partial S^2}\delta S^2 + 2\tfrac{\partial^2 E}{\partial S\partial V}\delta S\delta V + \tfrac{\partial^2 E}{\partial V^2}\delta V^2\right) \\ &= \tfrac{1}{2}\left(\delta S\delta\left(\tfrac{\partial E}{\partial V}\big|_S\right)\right) = \tfrac{1}{2}\left(\delta S\delta T - \delta V\delta P\right) \end{aligned} \tag{11}$$

Here we see the mathematical meaning of the variation of the thermodynamics potential $\Delta\Phi$.It shows how much the money flow deviates from the tangent plane to the surface of state $E = E(V,T)$.

Now we can express this change of $\Delta\Phi$in varies coordinate system. Having selected for example, the variables $\delta V, \delta T$ we can express δS and δP in terms of $\delta V, \delta T$. After implications we obtain

$$\delta P\delta V - \delta T\delta S = -\frac{C_V}{T}\delta T^2 + \left.\frac{\partial P}{\partial V}\right|_T \delta V^2 \tag{12}$$

The probability of fluctuation under the deviation of the systems from equilibrium is accordingly proportional to the product of two factors depending on δV and δT

$$W(\delta T, \delta V) \approx e^{-\frac{1}{2T}\left(-\frac{C_V}{T}\delta T^2 + \frac{\partial P}{\partial V}\big|_T \delta V^2\right)} \tag{13}$$

i.e. $\overline{\delta V\delta T = 0}$

It is not difficult to compute the mean quadratic values of the fluctuations by comparing $W(\delta T, \delta V)$with the Gauss distribution formula [9]

$$\begin{aligned} \overline{(\delta T)^2} &= \tfrac{T^2}{C_V}, \\ \overline{(\delta T)^2} &= -T\tfrac{\partial V}{\partial P}\big|_T \end{aligned} \tag{14}$$

This gives for the mean value of following value:

$$\overline{\delta P \delta T} = \overline{\left(\left.\frac{\partial P}{\partial V}\right|_T \delta V - \left.\frac{\partial P}{\partial T}\right|_V \delta T \right) \delta T} = \left.\frac{\partial P}{\partial V}\right|_T \overline{\delta V^2} = -T \tag{15}$$

This relation enables us to measure the temperature of the market by computing the mean of the product of the fluctuation of the market by the fluctuation of the flow of goods. Since the averaging over the ensemble can be replaced by averaging over time, we obtain the following formula for the empirical computation of the market's temperature:

$$T = \frac{1}{2T} \lim \int_0^T \left(V(t) - \bar{V} \right) \left(P(t) - \bar{P} \right) dt. \tag{16}$$

Both the dependence of the price on time and the dependence of the flow of goods on time are accessible, in principle, data, say, for the stock market. Therefore if we assume that no external factors not determined by the structure of the market as such influence the prices and the flows of goods then we have a means to measure thermodynamic parameters of markets.

3. Fluctuation in Time

Here we consider the dependence of fluctuations on time in the system led out of the equilibrium state. Observe here that we may only consider the not too large deviations from equilibrium states but, on the other hand, not too small ones [10]

If the initial deviation of the equilibrium state is very tiny, the dynamics of the fluctuations will not differ from the chaotic spontaneous fluctuations. If, on the other hand, the initial deviation is very large, one has to take into account the non-linear effects on the dependence on the speed of the deviation of the quantity under the study on the value of the initial deviation.

Therefore we will confine ourselves to a linear case that is we will assume that in the dependence of the speed with which the quantity returns to the equilibrium state on the deviation we can ignore all the in the Taylor series expansion except the first one.

Speaking about practical applications of such an approach for prediction of behavior of time series, say of the price on the share and stock markets this means that reasonable predictions can be only made for short periods of time when the prices are still capable to return to the equilibrium state but the time spans are still larger than the value of dispersion.

In order to purely thermodynamic approach to work it is necessary that the "shadow of future" does not affect too much the behavior of the market agents and the existence of a certain symmetry between the sellers and the buyers. In other words, the thermodynamic approach will hardly be effective for stock exchange, where the fulfillment of both of the above requirements is hard to imagine but it can certainly be applicable for the commodity markets.

In order to construct the theory of time fluctuations we have to introduce an important value called autocorrelation function. It is defined as the average over the ensemble of the product of the values of the quantities studied separated by a fixed time interval t_0 and the time segments $[t_{i_n}, t_{i_k}]$(the subscripts n and k stand for the first letters of Russian words "beginning" and "end" are the periods during which a prediction can be significant. One can determine the value of dispersion of ΔX by means of the arguments from the preceding section.

$$\phi\left(t_0\right) = \left\langle \Delta X\left(t\right) \Delta X\left(t+t_0\right)\right\rangle . \tag{17}$$

This quantity enables to determine the spectral density of the fluctuations, that is the probability that the "frequency" of the fluctuation belongs to a certain interval. Here speaking about "frequency" of fluctuations we use a metaphorical language because in actual fact we have in mind the existence of processes with a certain characteristic relaxation time t_0.

The frequency is inversely proportional to the relaxation time:

$$\omega = \frac{1}{t_0} \tag{18}$$

In a more formal presentation, the arguments on a relation of the frequency of fluctuations and the relaxation time are as follows. Consider the Fourier transform of$\Delta X\left(t\right)$:

$$\Delta X_\omega = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \Delta X\left(t\right) e^{i\omega t} dt. \tag{19}$$

Then $\Delta X\left(t\right)$ can be considered as the inverse Fourier transform of ΔX_ω :

$$\Delta X = \int\limits_{-\infty}^{\infty} \Delta X_\omega e^{-i\omega t} d\omega \tag{20}$$

This expression for $\Delta X\left(t\right)$ can be substituted into the definition of the autocorrelation function for $\Delta X\left(t\right)$:

$$\phi\left(t_0\right) = \left\langle \int \int\limits_{-\infty}^{\infty} \Delta X_\omega \Delta X_{\omega'} e^{-i(\omega+\omega')t} e^{-i\omega t_0} d\omega d\omega' \right\rangle \tag{21}$$

Now observe that, in accordance with the main principles of statistical thermodynamics, the average over the ensemble can be replaced by averaging over time.

$$\phi\left(t_0\right) = \int \int\limits_{-\infty}^{\infty} \Delta X_\omega \Delta X_{\omega'} \left(\frac{1}{T} \int\limits_{-T}^{T} e^{-i(\omega+\omega')t} dt \right) e^{-i\omega t_0} d\omega d\omega' \tag{22}$$

In the limit as $T \to \infty$the integral in parentheses become an expression for the delta function$\delta\left(\omega{+}\omega'\right)$. Therefore for the autocorrelation function $\phi\left(t_0\right)$we get the expression

$$\phi\left(t_0\right) = \int\limits_{-\infty}^{\infty} \Delta X_\omega^2 e^{-i\omega t_0} d\omega. \tag{23}$$

or, by performing the Fourier transformation, we obtain an expression for the spectral density

$$X_\omega^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi\left(t_0\right) e^{i\omega t_0} dt_0 \tag{24}$$

This expression is known as the Wiener-Kinchin Theorem [11 & 12]

Now we can better understand how the frequencies of the fluctuations and the relaxation time are related.

Let the speed with the variable ΔXreturns to the equilibrium position (i.e. to 0) only depends on the values of this variables itself

$$\frac{d\Delta X\left(t\right)}{dt} = f\left(\Delta X\right) \tag{25}$$

Expanding $f\left(X\right)$into the Taylor series and taking into account that $f\left(0\right) = 0$(i.e., in equilibrium the rate of change is equal to 0) and selecting all the terms of the expansion except the liner one we obtain

$$\frac{d\Delta X\left(t\right)}{dt} = -\lambda \Delta X, \tag{26}$$

where $\lambda > 0$, *i.e.*, $\Delta X\left(t\right) = \Delta X\left(0\right) e^{-\lambda t}$.

Substituting this expression for $\Delta X\left(t\right)$into the formula for the autocorrelation function we get

$$\phi\left(t_0\right) = \left\langle \Delta X^2 \right\rangle e^{-\lambda t}. \tag{27}$$

We have to recall here that we

(1) Neglect the higher terms in the expression of $\Delta X\left(t\right)$with respect to ΔX

(2) Consider the values of ΔXgreater than typical involuntary fluctuations, i.e., $|\Delta X| < \sqrt{D}$, where D is the dispersion of the thermodynamics fluctuations of ΔX

Now we can determine λ if we know the autocorrelation function:

$$\int_{-\infty}^{\infty} \phi\left(|t_0|\right) dt_0 = \left\langle \Delta X^2 \right\rangle 2 \int_{0}^{\infty} e^{-\lambda t_0} dt_0 = \left\langle X^2 \right\rangle \frac{2}{\lambda} \tag{28}$$

$Thismeansthatifwehaveatimeseries \Delta X\left(t\right)_{wecanhavecomputedtheautocorrelationfunction}$

$$\phi\left(t_0\right) = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T} \Delta X\left(t\right) \Delta X\left(t + t_0\right) dt. \tag{29}$$

and integrating it with respect to time obtain the weight with which the variable ΔX returns to equilibrium, i.e., obtain the constant

$$\lambda = \frac{2\left\langle X_2 \right\rangle}{\int_0^\infty \phi\left(t_0\right) dt_0} = \frac{2\pi \left\langle X^2 \right\rangle}{X_\omega X^2\left(0\right)}. \tag{30}$$

where X_ω^2 is the spectral density at zero frequency in accordance with the Wiener - Kinchin theorem.

Observe that for the exponential autocorrelation function the spectral density is as follows

$$X_\omega^2 = \frac{\lambda}{\pi\left(\omega^2 + \lambda^2\right)}\left\langle X^2\right\rangle . \tag{31}$$

For frequencies smaller thanλ, the spectral density is approximately equal to $\frac{1}{\pi\lambda}\left\langle X^2\right\rangle$, i.e., for the frequencies smaller than the inverse relaxation time of the system, we see that the probabilities for ΔX to have such frequency component are approximately equal.

Observe that to apply the above theory of thermodynamic fluctuations to the study of price relaxations on markets should be implemented with utmost caution for the reasons that we consider in the next section.

4. The Collective Behavior at Market

Now we have to consider a very important question directly related with the study of market fluctuations. How do fluctuations affect the perceptions of the market agents on the future?

Generally speaking, the market agents have different information on the situation, it is difficult to expect any coordinated behavior of the market agents anywhere, in particular, in a neighborhood of the equilibrium. Indeed, the very lack of such a coordinated behavior characterizes the "extended order" F. Hayek [13-15], wrote about. The one who discovers new possibilities, new types of behavior gets an advantage and the multitude of such possibilities is unlimited. Moreover it is unknown.

In the case when a certain stereotype of behavior starts to dominate one should not expect the growth of "order". Contrariwise, one should expect its destruction. This is precisely what happens when a certain idea becomes common for a considerable majority of market agents: for example to invest into a particular type of activity or company. In this case, the shares quickly become overvalued and this sooner or later (usually relatively soon) becomes clear thus influencing a new wave of spontaneously coordinated behavior, this time to withdraw money from the corresponding activity.

Such situations lead not just to market fluctuations, but to considerable oscillations and sometimes to a total transition of the market. Examples of this type are quite numerous. It suffices to recall economic catastrophes in Mexico and South East Asia during the 1990s.

A decisive role in such spontaneously coordinated behavior of the market agents is played by the "shadow of future" that is, perceptions on a possible development of the situation. The result turns out unexpected for the participants because their collective behavior leads precisely to the very result that each of them tries to avoid.

A similar effect is well known in so-called non-cooperative games and is best studied with an example of the game called "prisoner's dilemma". The innumerable literature is devoted to this topic and here is not the place to discuss this problem, still observe that it is precisely in non-cooperative games (though in a somewhat different sense) the radically important role of the “shadow of the future” in the molding of spontaneous patterns of collective behavior have been singled out [16,17]

In this case the following problem becomes of interest: how are the small fluctuations of the system related to patterns of spontaneous behavior that totally change the market situation? In other words: what is the role of spontaneously formed collective behavior in the problem of stability of market economy?

If small fluctuations of market parameters help to form spontaneous collective behavior that destroys the market equilibrium then the market becomes evolutionary unstable despite of the fact that in “neoclassical” sense such a market should possess equilibrium.

Lately similar questions are in the center of attention of researchers that try to leave “neoclassical” orthodoxy and extend the frameworks of economic studies in particular in connection with the study of the influence of technical innovations to economics [18]. Here we will confine ourselves to the simple model of “speculative behavior” which shows under what conditions the market fluctuations can be considered as thermodynamic ones.

The study of the process of molding of the stock market price is of particular interest both for creating forecasting models and from purely theoretical point of view since this price is a good example illustrating how a directed activity of a multitude of people based on individual forecasts and decision making leads to a formation of a certain collective variable.

The problem of forecasting stock market prices requires a very detailed study of the concrete situation and discovery of a number of factors not only of economic but also of political character. An attempt of construction in mathematical model taking into account all these factors is doomed to failure. Nevertheless observe that many of external factors and also a number of economic factors (for example the level of actual demand of an item of goods) may remain constant during a sufficiently long time though the prices are subject to constant fluctuations. The reason for these fluctuations is a speculative activity. Analysis of dependence of stock market prices on time shows us that for sufficiently short lapses of time the nature of fluctuations often possesses certain common peculiarities. This hints to study speculative oscillations provided the “long-ranged” factors are constant. This makes it possible to model the modification of prices making use of the difference in the “time scale” for price fluctuations caused by speculative activities and oscillations resulting by “long-ranged” factors.

Let us abstract from the real conditions of stop market functioning making several simplifications.

Define the “mean price” $\bar{X}(t)$as the ratio of the mean amount of money $\bar{P}(t)$ spent for the purchase of the goods per unit of time to the mean value of goods $\bar{Q}(t)$sold at the same time:

$$\bar{X}(T) = \frac{\bar{P}(t)}{Q(t)} \tag{32}$$

In the free market this ratio coincides with the marginal price that determines the market equilibrium, where $\bar{P}(t)$ and $\bar{Q}(t)$are slowly changing quantities whose value is determined by the productive powers and other slowly changing factors.

Speculative activities lead to the change of both $P(t)$and $Q(t)$ and this in turn leads to the change of price. The "instant" price $X(t)$depends on $\Delta P(t)$and $\Delta Q(t)$:

$$X(t) = \frac{\bar{P}(t) + \Delta P(t)}{\bar{Q}(t) + \Delta Q(t)} \tag{33}$$

For short lapses of time we assume that we may assume that $\bar{Q}$and $\bar{P}$are time – independent.

The idea of the "shadow of future" discussed above suggests that the models of molding the market price should radically differ in structure from traditional mechanical models of equilibrium. The mechanical models of equilibrium describe a future state on the base of our knowledge of the past. Contrariwise, the market price is essentially formed as a result of interaction of goal-minded systems (in other words as a result of correlation of models of the future by people taking decisions to purchase a certain amount of goods). The market agents act on the base of predictions they have and therefore the market price depends on the predictions that the market agents that take decisions stake to. These predictions may depend not only on the price value in the past and present but also on their evaluation of the direction of development of long-ranged factors, on political situation and various other factors..

It is precisely the fact that the market price is molded as a result of forecasts which leads to a certain unpredictability of the market prices. Indeed, in order to predict the price one has to predict the forecasts of each separate market agent.

If we confine ourselves to a simple assumption that certain extra amount of goods and money appearing on the market depends on a possible profit we can express the market by means of the following equation

$$X(t) = \frac{\bar{P} + \sum_l f_l\left(X(t), \tilde{X}_l(t+T)\right)}{\bar{Q} + \sum_k \phi_k\left(X(t), \tilde{X}(t+T)\right)}, \tag{34}$$

Where $l$is the index that characterizes the buyers $k$is the index characterizing the sellers, $\tilde{X}(t+T)$is the predicted price for the period $T$and ϕ_k and $f_1$are the functions that characterizes the relation of an additional flow of goods and money on the instant and predicted prices.

In this form the equation is too general and not fit for investigations but it can serve as a starting point for further simplifications leading to more tangible equations. Our main problem will be investigation of the conditions for which we observe an equilibrium type of price fluctuations — oscillations about a certain mean value which slowly varies perhaps together with the volume of goods Q and the volume of money P

Let us simplify as follows:

(1) Assume that all the buyers use the same forecast and all the sellers use the same forecast (though these forecasts are not necessarily identical);
(2) The increase of the offer is proportional to a possible (predicted) profit per unit of goods;

(3) The increase of demand is also proportional to a possible profit.

Under these assumptions equation (34) takes the form

$$X\left(t\right)=\frac{\bar{P}+\alpha\left(\tilde{X}_{P}\left(t+T\right)-X\left(t\right)\right)}{\bar{Q}+\beta\left(\tilde{X}_{Q}\left(t+\theta\right)-X\left(t\right)\right)}, \tag{35}$$

Where $\tilde{X}_P\left(t+T\right)$is the buyer's prediction that use basis prediction time $T$and $\tilde{X}_Q\left(t+\theta\right)$is the seller's prediction that use the basis prediction timeθ

We should expect that $T$and θmay be rather different, i.e., the market is, generally speaking, asymmetric [19].

Let us simplify further. It is natural to assume that the prediction is determined by the expansion of the price $X\left(t\right)$ in the Taylor series with respect to time and terms higher second order are neglected. It is difficult to conceive the influence of the derivatives of the price greater than the second one on human perception: the eye usually catches the first and second derivatives from the form of the curve. Thus the equation (35) takes the form

$$X\left(t\right)=\frac{\bar{P}+\alpha\left(T\dot{X}+\frac{T^{2}}{2}\ddot{X}\right)}{\bar{Q}+\beta\left(\theta\dot{X}+\frac{\theta^{2}}{2}\ddot{X}\right)}. \tag{36}$$

We have the following alternatives:

(1) We may confine ourselves to the first derivatives thus obtaining the equation

$$X\left(t\right)=\frac{\bar{P}+\alpha T\dot{X}}{Q+\beta\theta\dot{X}}; \tag{37}$$

(1) We may study the more complicated equation (36)

Case (a) is rather simple. Resolving (37)

$$\dot{X}=\frac{\bar{P}-\bar{Q}X}{BX-A}, \tag{38}$$

where $B=\beta\theta$ and $A=\alpha T$.

Certainly one can integrated this equation but we will study it in a simply way. The change of variables $Z=\frac{B}{A}X$leads us to

$$F\dot{Z}=\frac{\Phi-Z}{Z-1}, \tag{39}$$

Where $F=\frac{A}{Q},\quad \Phi=\frac{\bar{P}B}{\bar{Q}A}$.

If $F>0$, that is $A>0$ and $\Phi>1$, we have a stable equilibrium at the point $Z=\Phi\left(X=\frac{\bar{P}}{\bar{Q}}\right)$.

If $F>0$and $\Phi<1$, then the equilibrium at $Z=\Phi\left(X=\frac{\bar{P}}{\bar{Q}}\right)$is unstable. Under a small increase of the price it steeply up to the value corresponding to $Z=1\left(X=\frac{A}{B}\right)$and under small diminishing falls to zero.

The case $\Phi = 1$is no equilibrium at all since $\dot{Z} = -\frac{1}{F}\left(\dot{X} = \frac{Q}{B}\right)$.

For $B < 0$, the price continuously grows whereas $B > 0$it falls to zero.

In this situation every thing depends on the nature of the sellers' forecast, i.e., do they believe that the raising the price will stimulate the production or one should hide the goals and wait till its price grows up.

Clearly, if the supplies are restricted or if there is a possibility to shrink the production, the case $B > 0$is realized. In other words, in the case of a monopoly of the seller and objective restrictions on supplies, the price grows unboundedly.

Apparently, this model adequately describes the phenomenon of sudden plummeting of prices during social unrests and wars and also the inflation in the case of restricted production.

Let us now consider another type of prognosis, which takes into account the second derivative of price. Resolving equation (36) for the second derivative we get

$$\ddot{X} = \frac{\bar{P} + \alpha T \dot{X} - X\left(Q + \beta\theta\dot{X}\right)}{\beta\frac{\theta^2}{2}X - \alpha\frac{T^2}{2}} \tag{40}$$

We may study this equation another by standard means, see, e.g., [9]. Introduce a new variable $\dot{X} = Y$and in the system of equation obtained eliminate time by dividing $\dot{X}$by $\dot{Y}$.

We get $\frac{dY}{dX} = \frac{\bar{P}+\alpha TY - X\left(\bar{Q}+\beta\theta Y\right)}{Y\left(\beta\frac{\theta^2}{2}X - \alpha\frac{T^2}{2}\right)}$ (40)

We can simplify this equation by setting

$$k_1 = \alpha T, \quad k_2 = \alpha\frac{T^2}{2}, \quad k_3 = \beta\theta, \quad k_4 = \beta\frac{\theta^2}{2}. \tag{41}$$

We get

$$\frac{dY}{dX} = \frac{\bar{P} + \bar{Q}X}{Y\left(k_4 X - k_2\right)} + \frac{k_1 - k_3}{k_4 x - k_2} \tag{42}$$

Here we see that if $k_1 = k_3$then the system is equilibrium at the point $Y = 0$ that is $X = \frac{\bar{P}}{\bar{Q}}$ is the expected equilibrium point.

However, if $k_1 \neq k_3$then $X = \frac{\bar{P}}{\bar{Q}}$ is not an equilibrium point.

This is an astonishing result that shows that under asymmetric conditions taking into account the second derivative we eliminate equilibrium. Asymmetric markets behave totally unexpectedly.

Conclusion

From the above discussed the scope of the phenomenological analysis for Market Fluctuations through Thermodynamics approach. Here we described how market parameters vary with time by using of simplest example, and this extended to the market fluctuations appears with the enforced changes of macro parameters of the market and land speculations with non existence. And also we derived the equation for how market fluctuates with respect to time in an equilibrium state. Finally we analyzed the how the fluctuations affects the perceptions of the market agents on the future.

From the above discussion our considerations resulted in rather unexpected corollaries: The "shadow of future" for the linear forecast restricts the domain of a stable equilibrium but even for the asymmetric forecasts of sellers and buyers it does not totally eliminate the equilibrium. Adding the second derivative into the forecast (which amounts, actually, to professionalization of the forecast) completely eliminates equilibrium for the asymmetric markets. In other words, the market becomes globally unstable. This deduction is extremely important for our further analysis. It means that there appears a possibility to manipulate the market using restricted resources

References

[1] G. Guillaume, E. Guillaume, Sur le fundaments de l'economique rationelle, Gautier-Villars, Paris, 1932.

[2] P. Samuelson, Collected Economics Papers, Vol. 5, MIT Pr., Cambridge, 1986.

[3] P. Mirowski, Machine Dreams, Cambridge, Cambridge, 2002.

[4] E. Smith, D.K. Foley, Is utility theory so different from thermodynamics? SFI preprint, 2002.

[5] D. Ruelle, Statistical Mechanics, Thermodynamic Formalism, Addison-Wesley, Reading, Mass., 1978.

[6] P. Cvitanoviç, G.H. Gunaratne, I. Procaccia, Phys. Rev. A38 (1988) 1503.

[7] J.L. McCauley, Classical Mechanics: flows, transformations, integrability and chaos, Cambridge, Cambridge, 1997.

[8] Landau L. D., Lifshitz E. M. Course of theoretical physics. Vol. 5: Statistical physics. Translated from the Russian by J.B. Sykes and M. J. Kearsley. Second revised and enlarged edition Pergamon Press, Oxford-Edinburgh-New York, 1968, xii+484 pp.

[9] Tricomi F. G., Differential equations. Translated by Elizabeth A. McHarg Hafner Publishing Co., New York 1961 x+273 pp.

[10] Mills T. C., The Econometric Modeling of Financial Time Series. Cambridge Univ. Press, Cambridge, 1993.

[11] Wiener, N., The Fourier integral and certain of its applications. Reprint of the 1933 edition. With a foreword by Jean-Pierre Kahane. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 1988. xviii+201 pp.

[12] Wiener, N., Nonlinear problems in random theory. Technology Press Research Monographs The Technology Press of The Massachusetts Institute of Technology and John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London 1958. ix+131 pp.

[13] Hayek F. A. The Road to Serfdom. Chicago Univ. Press, Chicago, 1977

[14] Hayek F. A. New Studies in Philosophy, Politics, Economics and the History of Ideas,Chicago Univ. Press, Chicago, 1978

[15] Hayek F. A. The Fatal Conceit. In: Collected Works of F.A. Hayek. University of Chicago Press, Chicago, 1988

[16] Axelrod R. The Evolution of Cooperation. New York, Basic, 1984

[17] Sergeev V., The Wild East (Crime and lawlessness in post-Soviet Russia), M. E. Sharp Armonk, NY, 1998

[18] Arthur W.B. Self-reinforcing mechanisms in Economics. In: P. W. Anderson, K. J. Arrow, D. Pines (eds.) The Economy as an Evolving Complex System. Proceedings of the Workshop on the Evolutionary Paths of the Global Economy Held in Santa Fe, New Mexico, September, 1987. Addison-Wesley, Redwood City, CA, 1988, 9 – 31.

[19] Akerloff G.A. The Market for Lemons: Qualitative Uncertainty and the Market Mechanism. Quaterly J. Econom., 1970, 84, 488–500.