Electric Dipole Moment and Neutrino Mixing due to Planck Scale Effects

Bipin Singh Koranga*

Kirori Mal College (University of Delhi,) Delhi-110007, India

Received 23 January 2010, Accepted 5 March 2010, Published 20 March 2010

Abstract: In this paper, we consider the effect of Planck scale operators on electric dipole moment of the electron *de*. The electric dipole moment of the electron, *de* is known to vanish up to three loops in the standard model with massless neutrinos We consider the Planck scale operator on neutrino mixing. We assume that the neutrino masses and mixing arise through physics at a scale intermediate between Planck scale and the electroweak breaking scale. We also assume, that just above the electroweak breaking scale neutrino mass are nearly degenerate and the mixing is bi-maximal. Quantum gravity (Planck scale) effects lead to an effective $SU(2)_L \times U(1)$ invariant dimension-5 Lagrangian symmetry involving Standard Model. On electroweak symmetry breaking, this operator gives rise to correction to the neutrino masses and mixings these additional terms can be considered as perturbation to the bimaximal neutrino mass matrix We assume that the gravitational interaction is flavour blind and we study the neutrino mixing and electric dipole moment due to the Planck scale effects.

Keywords: Neutrino Mixing; Dipole Moment; Planck Scale
PACS (2008): 14.60.Pq; 31.30.jn; 13.15.+g; 12.60.-i

1. Introduction

The electric dipole moment (e.d.m) of fermions can provide a unique window to probe into the nature of the force that are responsible for CP violation [1]. experimental limit on the dipole moment of neutron have reached the level of 6×10^{-26} ecm [2] and have helped to constrain on theoretical model of CP violation. Electric dipole moment of the electron has severely been constrained by atomic measurements in Cs ($de \leqslant 10^{-26}$)and Tl ($de \leqslant 10^{-27} ecm$) [3]. The experimental values [4,5] are given by

* bipiniitb@rediffmail.com

$$dn = (30 \pm 50) \times 10^{-27} e.cm \tag{1}$$

and

$$de = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27} e.cm \tag{2}$$

These values give important constrains on CP violating phase beyond the Standard Model (S.M). CP violation encoded in the complex element of the C.K.M Unitary Matrix, V in the quark sector and observable effects are proportional to the term [5]

$$J = Im\left(V_{ij} V_{kj}^* V_{kl} V_{il}^*\right)$$

Obviously J is non vanishing only if not all the elements of V_{ij} can be made real and this implies the existence of at least three generation of non degenerate massive quarks, standard model de is estimate at the four loop level given by [6]

$$de \sim \frac{eG_F}{\pi^2} \left(\frac{\alpha}{2\pi}\right)^3 meJ \leq 4 \times 10^{-38}, \tag{3}$$

where $J \leq 4 \times 10^{-38}$, for the C.K.M element is used eq(3), indicates the opportunity that de is a clean test of CP violation beyond the S.M. Now the leptonic sector Will exhibit the C.K.M type of mixing as the analogue of the C.K.M matrix in the quark sector. In this paper, we assume the matrix are massive Majorana particle. We consider de as induced by a leptonic C.K.M mechanism which results from such massive Majorana neutrino mixing. The interaction Lagrangian involving the electric dipole moment is given by [7]

$$L = -\frac{1}{2} de F_{\mu\nu} \bar{e} i \sigma^{\mu\nu} \gamma_5 e, \tag{4}$$

and the charge leptonic current interaction is

$$\frac{1}{\sqrt{2}} W^\mu U_{ij} \overline{\nu_i} \gamma_\mu \frac{1-\gamma_5}{2} e_j + hc, \tag{5}$$

where $U_{ij}$is the charged current mixing matrix analogue to the C.K.M matrix in the quark sector. The importance of mixing matrix U is unitary and the elements are in general complex. In the case of Majorona neutrino, after some standard manipulation. de can be write in terms of Feynman integrals is given by [7]

$$de = \pm \frac{e\alpha^2 me}{256\pi^2 s_w^4} m_i m_j J_{ij}^l F, \tag{6}$$

Integral function F and the CP violating factor J is given by

$$F = \frac{x(1-x)^2[(1-s)^2 - (t+u)^2] + xy^2 u(1-u) - x(1-x)y(1+3s+t+u-2tu-2u^2)}{m_i^2 x(10x)(1-s-t-u) + m_j^2(1-x-y)u + m_w^2(x(1-x)(s+t) + yu + m_i^2 xu)^2}, \tag{7}$$

$$J^l_{ij} = Im(U^*_{je}U^*_{jl}U_{il}U_{ie}), \tag{8}$$

where $l = e,\ \mu,\ \tau$ for the S.M

Goldstone boson exchange diagram are less important. We now give a semi quantities estimate of de is given by [8]

$$de \sim \frac{\alpha^2 mem_i m_j(m_i^2 - m_j^2)}{256\pi^2 s_w^2 m_w^6} J_{ij} F(m_l^2/m_w^2, m_i^2/m_w^2, m_j^2/m_w^2)(1.97 \times 10^{-16})e - cm, \tag{9}$$

where all masses are taken in unit of GeV. The G.I.M factor $(m_i^2 - m_j^2)/m_w^2$ for neutrino are explicit the fact that de vanishes, where neutrino are massless. We can see from eq(3), eq(9), expression of electric dipole moment (E.D.M) proportional of factor J. Due to Planck scale effects, E.D.M proportional to the Jarlskog determinat

$$de \propto J^l_{ij}, \tag{10}$$

where $J^{'}_{ij}$is related to mixing angle

Electric dipole moment and Neutrino Mixing Angles due to Planck Scale Effects are given in section 2. In section 3 gives the results on electric dipole moment and neutrino mixing.

2. Electric Dipole Moment and Neutrino Mixing Angles due to Planck Scale Effects

The calculation developed in an earlier paper [9]. A natural assumption is that unperturbed (0^{th} order mass matrix) M is given by

$$\mathbf{M} = U^* diag(M_i) U^\dagger, \tag{11}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i , the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U_{e3}. We adopt the usual parametrization.

$$\frac{|U_{e2}|}{|U_{e1}|} = tan\theta_{12}, \tag{12}$$

$$\frac{|U_{\mu 3}|}{|U_{\tau 3}|} = tan\theta_{23}, \tag{13}$$

$$|U_{e3}| = sin\theta_{13}. \tag{14}$$

In term of the above mixing angles, the mixing matrix is

$$U = diag(e^{if1}, e^{if2}, e^{if3}) R(\theta_{23}) \Delta R(\theta_{13}) \Delta^* R(\theta_{12}) diag(e^{ia1}, e^{ia2}, 1). \tag{15}$$

The matrix $\Delta = diag(e^{\frac{1\delta}{2}}, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation $a1$ and $a2$ are the so called Majoring phase, which effects the neutrino less double beta decay. $f1$, $f2$ and $f3$ are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [9]

$$U^{'} = U(1 + i\delta\theta),$$

$$= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} + i \begin{pmatrix} U_{e2}\delta\theta^{*}_{12} + U_{e3}\delta\theta^{*}_{23,} & U_{e1}\delta\theta_{12} + U_{e3}\delta\theta^{*}_{23}, & U_{e1}\delta\theta_{13} + U_{e3}\delta\theta^{*}_{23} \\ U_{\mu 2}\delta\theta^{*}_{12} + U_{\mu 3}\delta\theta^{*}_{23,} & U_{\mu 1}\delta\theta_{12} + U_{\mu 3}\delta\theta^{*}_{23}, & U_{\mu 1}\delta\theta_{13} + U_{\mu 3}\delta\theta^{*}_{23} \\ U_{\tau 2}\delta\theta^{*}_{12} + U_{\tau 3}\delta\theta^{*}_{23}, & U_{\tau 1}\delta\theta_{12} + U_{\tau 3}\delta\theta^{*}_{23}, & U_{\tau 1}\delta\theta_{13} + U_{\tau 3}\delta\theta^{*}_{23} \end{pmatrix}. \tag{16}$$

Where $\delta\theta$ is a hermition matrix that is first order in μ[9,10]. The first order mass square difference $\Delta M^2_{ij} = M^2_i - M^2_j$,get modified [9,10] as

$$\Delta M^{'2}_{ij} = \Delta M^2_{ij} + 2(M_i Re(m_{ii}) - M_j Re(m_{jj}), \tag{17}$$

where

$$m = \mu U^t \lambda U,$$

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} eV.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$[19], is given by

$$\delta\theta_{ij} = \frac{iRe(m_{jj})(M_i + M_j) - Im(m_{jj})(M_i - M_j)}{\Delta M^{'2}_{ij}}. \tag{18}$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance. Using Eq(14), we can calculate neutrino mixing angle due to Planck scale effects,

$$\frac{|U^{'}_{e2}|}{|U^{'}_{e1}|} = tan\theta^{'}_{12}, \tag{19}$$

$$\frac{|U^{'}_{\mu 3}|}{|U^{'}_{\tau 3}|} = tan\theta^{'}_{23}, \tag{20}$$

$$|U^{'}_{e3}| = sin\theta.^{'}_{13} \tag{21}$$

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that $U^{'}_{e1}$ and $U^{'}_{e2}$ are much larger than

U'_{e3}, $U'_{\mu 3}$ and $U'_{\tau 3}$. Hence we can expect much larger change in θ_{12} compared to θ_{13} and θ_{23}. As one can see from the above expression of mixing angle due to Planck scale effects, depends on new contribution of mixing $U' = U(1 + i\delta\theta)$. Due to Planck Scale Effects, new proportional factor of electric dipole moment (E.D.M) is given by

$$de' \propto J'^{l}_{ij} \propto Im(U'^{*}_{je} U'^{*}_{jl} U'_{il} U'_{le}), \tag{22}$$

where U'_{je},U'_{jl},U'_{il} and U'_{ie} is the mixing angle parameter given by new mixing ($U' = U(1 + i\delta\theta)$)due to Planck scale Effects

3. Results and Discussions

We assume that, just above the electroweak breaking scale, the neutrino masses are nearly degenerate and the mixing are bimaximal, with the value of the mixing angle as $\theta_{12} = 45^o$, $\theta_{23} = 45^o$ and $\theta_{13} = 0$. Taking the common degenerate neutrino mass to be 2 eV, which is the upper limit coming from tritium beta decay [11]. We compute the modified mixing angles using Eqs (11)-(13). We have taken $\Delta_{31} = 0.002eV^2$[12] and $\Delta_{21} = 0.00008eV^2$[13]. For simplicity we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. Since we have set the $\theta_{13} = 0$, the Dirac phase δ drops out of the zeroth order mixing angle. We consider the Planck scale effects on neutrino mixing and we get the given range of mixing parameter of MNS matrix

$$U' = R(\theta_{23} + \epsilon_3) U_{phase}(\delta) R(\theta_{13} + \epsilon_2) R(\theta_{12} + \epsilon_1) \tag{23}$$

In Planck scale, only θ_{12}have reasonable deviation and θ_{13},θ_{23}deviation is very small less than 0.3^o[9,10]. In the new mixing due to Planck scale effects, we get the new multiplicity factor of electric dipole moment , which is proportional to Jarlskog determinant and Planck scale mixing changes the Jarlskog Determinant[14]. If there exist Majorana neutrino with masses $m_i^2 \leq m_w^2$ and $|U_{ei}|^2 \leq 10^{-2}$,obtained from the charged current universality constraints [15]. We get $de \leq 10^{-32} e - cm$ assuming that the integral function F in eq(9) is of the order unity.

4. Conclusions

In conclusions, we find that in general majorana neutrino can induce electric dipole moment at the two loop level. Numerical estimate for $de \leq 10^{-32} e - cm$ even for Majoring neutrino of masses in the 100 GeV range. The electric dipole moment factor in eq(6) can be effected by mixing parameter $\theta_{12,}$ θ_{23} and θ_{13}. We consider the main part of neutrino mixing arise from GUT scale operator. We further assume that GUT scale symmetry the neutrino mixing to be bi-maximal. We compute the first first order correction to neutrino mass eigenvalue and mixing angles. In [10], it was shown that the change in $\theta_{13}\, due$ to perturbation is small. We also show that the change in θ_{23} also is small (less then 3^o) only change in θ_{12}can be substantial (about $\pm 3^o$).

In this paper, we study electric dipole moment due to Planck scale effects. For Majorana neutrino with three flavour, the expression is $de \sim J^l_{ij} F$.In this paper, finally we wish make a important comment due to Planck scale effects mixing angle θ_{12} changes effectively the electric dipole moment (E.D.M) and magnetic moment [16]. Non zero value of electric dipole moment in eq(6) also indicate the existence of E.D.M, which is related to CP violation..

References

[1] W. Bernreuther and M. Suzuki, Rev. Mod. Phys 63,313 (1991).

[2] P.G Harris *et al.*,Phys.Rev.Lett **82**,904 (1999).

[3] E. Commins *et al.*,Phys.Rev. **A** 50,2960 (1994).

[4] L.S Altarev *et al.*,Phys.Lett. B276,242 (1992).

[5] C. Jarlskog, Phys.Rev.Lett **53**,1039 (1985).

[6] S. M Barr and W. J. Marciano, in CP Violation, edited by C. Jarlskog (World Scientific, Singapore, 1989).

[7] K.S Babu *et al.*,Phys.Rev.Lett.**85**:5064-5067 (2000).

[8] Daniel Ng, John N. Ng ,Mod.Phys.Lett.**A11**:211-216 (1996).

[9] F. Vissani *et al.*,Phys.Lett. B571, 209, (2003).

[10] Bipin Singh Koranga, Mohan Narayan and S. Uma Sankar,Phys.Lett.**B665**, 63 (2008).

[11] C Kraus, *et al.*,Eur.Phys. **J** 33 (2004).

[12] Super-Kamiokande Collaboration, J. Hosaka, *et al.*,Phys. Rev**D** 74,32002 (2006).

[13] KamLAND Collaboration, J. Hosaka *et al.*, Phys.Rev.Lett. **94**, 08180 (2005).

[14] Bipin Singh Koranga, S. Uma Sankar , Electron.**J**.Theor.Phys.**5**: 1-6 (2009)

[15] L.N. Chag *et al.*,Phys.Rev D 50,4581 (1999).

[16] Bipin Singh Koranga, Electron.**J**.Theor.Phys.**5**:133-140,(2008).