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Some LRS Bianchi Type-II String-Dust Cosmological Models in General Relativity

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Abstract: Some LRS Bianchi type-II string- dust cosmological models are investigated in which the expansion (θ) is assumed to be proportional to the shear (σ). To obtain exact solutions, the Einstein's field equations have been solved for two cases (i) Reddy string and (ii) Nambu string. The physical and geometrical behaviour of these models are discussed.

Keywords: LRS Bianchi type II models; Nambu string ; Reddy String
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1. Introduction

Cosmic strings play a vital role in structure formation in cosmology [1]. They arise when the symmetry between the strong and electroweak forces are broken due to the phase transition in the early universe ($t \backsim 10^{-36}s$) [2] as the temperature goes down below some critical temperature ($T_{GUT} = 10^{28}K$) as predicted by grand unified theories (GUT) [2]-[6]. It s believed that the vacuum strings give rise to density fluctuations sufficient enough for formation of galaxies [1]. The cosmic strings have their stress-energy coupled to the gravitational field. Therefore, the study of gravitational effects of such strings will be of interest. The general treatment of strings was initiated by Letelier [8]-[9].
A cloud of strings can be created by massive strings instead of geometrical strings (massless). A massive string is formed when particles are attached to a geometrical string

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(along its extension). A model wherein we have particles and strings together can be consider as the simplest one. Since strings are not observed at the present time of evolution of the universe, one can eliminate the strings and end up with a cloud of particles. String cosmological models have been studied by many authors [10]-[22].

Today, the universe is successfully described by maximally symmetric models given by the Friedmann-Robertson-Walker (FRW) space-time which is homogeneous and isotropic. However, in smaller scales the universe is neither homogeneous nor isotropic. Also we do not expect that the universe have these properties in its early stages. To get a realistic picture of the universe the homogeneous and anisotropic models have been studied in general relativity. Bianchi type II space-time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of universe. The importance of Bianchi type II universe has been emphasized by Asseo and Sol [23].

The purpose of the present work is to obtain Bianchi type-II string cosmological models with the help of relation $A = B^m$ between metric coeffcients. Our paper is organized as follows. In Sect. 2, we derive the field equations with cosmic strings as a source in Bianchi type-II space time. Section 3 deals with some cases and its solutions. The last section contains some conclusions.

2. The Metric and Field Equations

We consider the Bianchi type II metric in the form [24]

$$ds^2 = -dt^2 + B^2(dx + zdy)^2 + A^2(dy^2 + dz^2) \tag{1}$$

where A , B are functions of t only. The energy momentum tensor for a cloud of strings is taken as

$$T_i^j = \rho u_i u^j - \lambda x_i x^j \tag{2}$$

where u_i and x_i satisfy the conditions

$$u_i u^i = -x^i x_i = -1, \quad u^i x_i = 0, \tag{3}$$

ρ is the proper energy density for a cloud string with particles attached to them, λ is the string tension density, u^i the four-velocity of the particles, and x^i is a unit space-like vector representing the direction of string. In a co-moving coordinate system, we have

$$u^i = (0, 0, 0, 1), \quad x^i = (\frac{1}{B}, 0, 0, 0). \tag{4}$$

The particle density of the configuration is given by

$$\rho = \rho_p + \lambda \tag{5}$$

where ρ_p is the rest energy density of the particles attached to the strings. The string tension density, λ, can take positive or negative values. Positive value of λ represents a universe filled with no strings but only an anisotropic fluid whereas its negative value represents strings loaded with particles forming the surface of world sheet[8].

The Einstein's field equations (with $\frac{8\pi G}{C^4} = 1$)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \tag{6}$$

for the metric (1) leads to the following system of equations:

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = -\lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = 0 \tag{8}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} = -\rho \tag{9}$$

where an overdot and double overdot respectively stand for the first and second derivative with respect to t. The scalar expansion θ, the shear scalar σ^2 and the particle density ρ_p are given by

$$\theta = \frac{\dot{B}}{B} + 2\frac{\dot{A}}{A} \tag{10}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{11}$$

$$\rho_p = 2\frac{\ddot{A}}{A} - 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{2}\frac{B^2}{A^2} \tag{12}$$

3. Solution of the Field Equations

The field equations (7)-(9) are a system of three equations with four unknown parameters A, B, ρ, λ. One additional constraint relating these parameters are required to obtain explicit solutions of the system. We assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$A = B^m \tag{13}$$

where m is a constant.
To obtain exact solutions, we solve the field equations for the following two cases.

3.1 Case I: Reddy String

In this case

$$\rho + \lambda = 0 \tag{14}$$

From Eqs. (7), (9), (13) and (14) we obtain

$$2\frac{\ddot{B}}{B} + 4m\frac{\dot{B}^2}{B} = \frac{B^{-4m+3}}{m} \tag{15}$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = \frac{df}{dB}$. Hence (15) can be written as

$$\frac{d}{dB}(f^2) + 4m\frac{f^2}{B} = \frac{B^{-4m+3}}{m} \tag{16}$$

By integrating Eq. (16) we find

$$f^2 = \frac{B^{-4(m-1)}}{m} + NB^{-4m} \tag{17}$$

where M is an integration constant. Therefore, we have

$$\frac{dB}{\sqrt{\frac{B^{-4(m-1)}}{m} + NB^{-4m}}} = dt \tag{18}$$

To get deterministic solution, we assume $m = \frac{1}{2}$. In this case by integrating (18), we obtain

$$B^2 = \sqrt{2N}sinh(\sqrt{2}t) \tag{19}$$

Hence, we have

$$A^2 = (2N)^{\frac{1}{4}}sinh^{\frac{1}{2}}(\sqrt{2}t) \tag{20}$$

where $N > 0$ without any loss of generality.
Thus the metric (1) reduces to

$$ds^2 = -dt^2 + \sqrt{2N}sinh(\sqrt{2}t)(dx + zdy)^2 + (2N)^{\frac{1}{4}}sinh^{\frac{1}{2}}(\sqrt{2}t)(dy^2 + dz^2) \tag{21}$$

3.1.1 The Geometric and Physical Significance of Model

The energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (21) are given by

$$\rho = -\frac{5}{8}coth^2(\sqrt{2}t) + \frac{1}{4} \tag{22}$$

$$\lambda = \frac{5}{8}coth^2(\sqrt{2}t) - \frac{1}{4} \tag{23}$$

$$\rho_p = -\frac{10}{8}coth^2(\sqrt{2}t) + \frac{1}{2} \tag{24}$$

$$\theta = \sqrt{2}coth(\sqrt{2}t) \tag{25}$$

$$\sigma^2 = \frac{1}{24}coth^2(\sqrt{2}t) \tag{26}$$

$$V^3 = \sqrt{2N}sinh(\sqrt{2}t) \tag{27}$$

From Eqs. (25) and (26), we obtain

$$\frac{\sigma^2}{\theta^2} = constant \tag{28}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{S}/S}{\dot{S}^2/S^2} = -\left[\frac{\frac{2}{3} - \frac{4}{9}coth^2(\sqrt{2}t)}{\frac{2}{9}coth^2(\sqrt{2}t)}\right] \tag{29}$$

where $S^3 = A^2B$ is the spatial average scalar factor.
From (29), we observe that

$$q < 0 \qquad if \quad coth^2(\sqrt{2}t) < \frac{3}{2} \tag{30}$$

and

$$q > 0 \qquad if \quad coth^2(\sqrt{2}t) > \frac{3}{2} \tag{31}$$

From (22) and (24), we see that energy conditions, $\rho \geq 0$ and $\rho_p \geq 0$ lead to

$$coth^2(\sqrt{2}t) \leq \frac{2}{5} \tag{32}$$

The model (25) starts with a big bang at $t = 0$. The expansion in the model decreases as time increases, whilst the proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta}$ is constant the model does not approach isotropy. There is a point type singularity in the model at $t = 0$ [25] . For the condition $coth^2(\sqrt{2}t) < \frac{3}{2}$, the solution gives accelerating model of the universe and for the condition $coth^2(\sqrt{2}t) > \frac{3}{2}$, our solution represents decelerating model of the universe. The string tension λ decreases with time. We also observe that $\lambda > 0$ if $coth^2(\sqrt{2}t) > \frac{2}{5}$ and $\lambda < 0$ if $coth^2(\sqrt{2}t) < \frac{2}{5}$.

3.2 Case II: Nambu String

In this case

$$\rho = \lambda \tag{33}$$

From Eqs. (7), (9), (13) and (33) we obtain

$$2\ddot{B} + 2(m-2)\frac{\dot{B}^2}{B} = \frac{1}{2m}B^{-4m+3} \tag{34}$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = \frac{df}{dB}$.

$$\frac{d}{dB}(f^2) + 2(m-2)\frac{f^2}{B} = \frac{1}{2m}B^{-4m+3} \tag{35}$$

By integrating Eq. (35) we obtain

$$f^2 = -\frac{1}{4m^2}B^{-4(m-1)} + MB^{-2(m-2)} \tag{36}$$

where M is an integrating constant.
Therefore, we have

$$\frac{dB}{\sqrt{-\frac{1}{4m^2}B^{-4(m-1)} + MB^{-2(m-2)}}} = dt \tag{37}$$

To get deterministic solution in terms of cosmic time t, we assume $m = 1$. In this case from (37), after integrating, we find

$$B = A = \frac{cosh(\sqrt{M}t)}{\sqrt{4M}} \tag{38}$$

where, $M > 0$ without any loss of generality. Thus the metric (1) reduces to

$$ds^2 = -dt^2 + \frac{cosh^2(\sqrt{M}t)}{4M}(dx + zdy)^2 + \frac{cosh^2(\sqrt{M}t)}{4M}(dy^2 + dz^2) \tag{39}$$

3.2.1 The Geometric and Physical Significance of Model

The energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (40) are given by

$$\rho = \lambda = -2tanh^2(\sqrt{M}t) + M \tag{40}$$

$$\rho_p = 0 \tag{41}$$

$$\theta = 3\sqrt{M}tanh(\sqrt{M}t) \tag{42}$$

$$\sigma = 0 \tag{43}$$

$$V^3 = (\frac{1}{4M})^{\frac{3}{2}}cosh^3(\sqrt{M}t) \tag{44}$$

From (49) and (50) we find

$$\frac{\sigma}{\theta} = 0 \tag{45}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{S}/S}{\dot{S}^2/S^2} = -coth^2(\sqrt{M}t) < 0 \tag{46}$$

From (41) we see that energy conditions, $\rho \geq 0$ leads to

$$coth^2(\sqrt{M}t) \geq \frac{2}{M} \tag{47}$$

The model (39) does not start with a big bang at $t = 0$. Thus our model is free from singularity. The expansion in the model increases as time increases. Since $\frac{\sigma}{\theta}$ is zero, this model presents an isotropic universe. From (47) it is clear that our model represents an accelerating universe. The string tension λ decreases with time. We also observe that $\lambda > 0$ if $coth^2(\sqrt{M}t) > \frac{2}{M}$ and $\lambda < 0$ if $coth^2(\sqrt{M}t) < \frac{2}{M}$.

Concluding Remarks

LRS Bianchi type-II string-dust cosmological models are obtained for two cases, (i) Nambu and (ii) Reddy String. It is found that in the case of Nambu string the model always represent an accelerating universe whereas in the case of Reddy string the solution gives both accelerating and decelerating universes under conditions given by (30) and (31) respectively. To solve the age parameter and density parameter, one requires the deceleration parameter to be negative. From this point of view our results seem to be interesting. It has been shown that in the case of Nambu string the model is free from singularity whereas in the case of Reddy string the model has Point Type singularity at $t = 0$. It is reasonable to say that a cosmological model is required to explain acceleration in the present universe. Therefore, our theoretical models are in agreement with the recent observations [26]-[29]. The cosmic string models studied here will be useful for better understanding of cosmology and structure formation of the universe.

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