# Cylindrically Symmetric Inhomogeneous String Cosmological Models of Perfect Fluid Distribution with Electromagnetic Field

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**Abstract:** Two new cylindrically symmetric inhomogeneous string cosmological models are investigated in presence of magnetic field. We have assumed that $F_{12}$ is the only non-vanishing component of electromagnetic field tensor $F_{ij}$. The Maxwell's equations show that $F_{12}$ is the function of $x$ alone whereas the magnetic permeability $\bar{\mu}$ is the function of $x$ and $t$ both. To get the deterministic solution, it has been assumed that the metric coefficients are separable in the form as $A = f(x)\ell(t)$, $B = g(x)k(t)$, $C = g(x)\nu(t)$. Also, the Einstein field equations have been solved with string source in which magnetic field is absent. Some physical and geometric aspects of the models in presence and absence of magnetic field are discussed.


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## 1. Introduction

Cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1]– [5]. It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies [6]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings.

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The general treatment of strings was initiated by Letelier [7, 8] and Stachel [9]. The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zel'dovich [10]. Also Harrison [11] has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [12]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors (Misner, Thorne and Wheeler [13]; Asseo and Sol [14]; Pudritz and Silk [15]; Kim, Tribble, and Kronberg [16]; Perley and Taylor [17]; Kronberg, Perry and Zukowski [18]; Wolfe, Lanzetta and Oren [19]; Kulsrud, Cen, Ostriker and Ryu [20]; Barrow [21]). Melvin [22], in his cosmological solution for dust and electromagnetic field suggested that during the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Hence the presence of magnetic field in string dust universe is not unrealistic.

Benerjee et al. [23] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field. The string cosmological models with a magnetic field are also discussed by Chakraborty [24], Tikekar and Patel [25, 26]. Patel and Maharaj [27] investigated stationary rotating world model with magnetic field. Ram and Singh [28] obtained some new exact solution of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [29]. Singh and Singh [30] investigated string cosmological models with magnetic field in the context of space-time with $G_3$ symmetry. Singh [31] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey, Wands and Copeland [32] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali et al. [33, 34, 35] have investigated Bianchi type I magnetized string cosmological models.

Cylindrically symmetric space-time play an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Inhomogeneous cylindrically symmetric cosmological models have significant contribution in understanding some essential features of the universe such as the formation of galaxies during the early stages of their evolution. Bali and Tyagi [36], Pradhan et al. [37]$-$[40], Kilinc [41] have investigated cylindrically symmetric inhomogeneous cosmological models in presence of electromagnetic field. Barrow and Kunze [42, 43] found a wide class of exact cylindrically symmetric flat and open inhomogeneous string universes. In their solutions all physical quantities depend on at most one space coordinate and the time. The case of cylindrical symmetry is natural because of the mathematical simplicity of the field equations whenever there exists a direction in which the pressure equal to energy density. In recent time, cylindrically symmetric inhomogeneous string cosmological models in presence of magnetic field have been studied by several authors [44]−[49] in various contexts.

Recently Kilinc and Yavuz [50] have investigated some string cosmological models with magnetic field in cylindrically symmetric space-time. Motivated by the situation discussed above, in this paper, we have revisited these solutions by assuming metric coefficient as separable in new form. We have taken string and electromagnetic field together as the source gravitational field as magnetic field are anisotropic stress source and low strings are one of anisotropic stress source as well. This paper is organized as follows. The metric and the field equations are presented in section 2. In section 3 we deal with the solution of the field equations by revisiting solutions obtained by Kilinc and Yavuz [50]. Section 4 describes solutions in absence of magnetic field. Finally in section 5 concluding remarks have been given.

## 2. The Metric and Field Equations

We consider the metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2, \tag{1}$$

where $A$, $B$ and $C$ are functions of $x$ and $t$. The energy momentum tensor for the string with electromagnetic field has the form

$$T_i^j = (\rho + p)u_i u^j + p g_i^j - \lambda x_i x^j + E_i^j, \tag{2}$$

where $u_i$ and $x_i$ satisfy conditions

$$u^i u_i = -x^i x_i = -1, \tag{3}$$

and

$$u^i x_i = 0. \tag{4}$$

Here $\rho$ being the rest energy density of the system of strings, $p$ is the isotropic pressure, $\lambda$ the tension density of the strings, $x^i$ is a unit space-like vector representing the direction of strings so that $x^1 = 0 = x^2 = x^4$ and $x^3 \neq 0$, and $u^i$ is the four velocity vector satisfying the following conditions

$$g_{ij} u^i u^j = -1. \tag{5}$$

In Eq. (2), $E_i^j$ is the electromagnetic field given by Lichnerowicz [51]

$$E_i^j = \bar{\mu} \left[ h_l h^l \left( u_i u^j + \frac{1}{2} g_i^j \right) - h_i h^j \right], \tag{6}$$

where $\bar{\mu}$ is the magnetic permeability and $h_i$ the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} {}^*F_{ji} u^j, \tag{7}$$

where the dual electromagnetic field tensor $^*F_{ij}$ is defined by Synge [52]

$$^*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ijkl}F^{kl}. \tag{8}$$

Here $F_{ij}$ is the electromagnetic field tensor and $\epsilon_{ijkl}$ is the Levi-Civita tensor density.

In the present scenario, the comoving coordinates are taken as

$$u^i = \left(0, 0, 0, \frac{1}{A}\right). \tag{9}$$

We choose the direction of string parallel to x-axis so that

$$x^i = \left(\frac{1}{A}, 0, 0, 0\right). \tag{10}$$

We consider the magnetic field as flowing along the $z$-axis so that $F_{12}$ is the only non-vanishing component of $F_{ij}$. Maxwell's equations

$$F_[ij; k] = 0, \tag{11}$$

$$\left[\frac{1}{\bar{\mu}}F^{ij}\right]_{;j} = J^i, \tag{12}$$

require that $F_{12}$ is the function of x-alone and the magnetic permeability is the functions of $x$ and $t$ both. The semicolon represents a covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$)

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j, \tag{13}$$

for the line-element (1) lead to the following system of equations:

$$-\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC} + \frac{C_{44}}{C} + \frac{B_{44}}{B}$$
$$= -pA^2 + \lambda A^2 - \frac{F_{12}^2}{2\bar{\mu}B^2}, \tag{14}$$

$$\frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_{14}}{B} - \frac{C_{14}}{C} = 0, \tag{15}$$

$$\frac{A_{11}}{A} - \frac{A_1^2}{A^2} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{C_{11}}{C} - \frac{C_{44}}{C} = pA^2 + \frac{F_{12}^2}{2\bar{\mu}B^2}, \tag{16}$$

$$\frac{A_{11}}{A} - \frac{A_1^2}{A^2} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{B_{11}}{B} - \frac{B_{44}}{B} = pA^2 - \frac{F_{12}^2}{2\bar{\mu}B^2}, \tag{17}$$

$$\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC} - \frac{C_{11}}{C} - \frac{B_{11}}{B}$$
$$= \rho A^2 + \frac{F_{12}^2}{2\bar{\mu}B^2}, \tag{18}$$

where the sub indices 1 and 4 in A, B, C and elsewhere denote ordinary differentiation with respect to $x$ and $t$ respectively.

The velocity field $u^i$ is irrotational. The scalar expansion $\theta$, shear scalar $\sigma^2$, acceleration vector $\dot{u}_i$ and proper volume $V^3$ are respectively found to have the following expressions:

$$\theta = u^i_{;i} = \frac{1}{A}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right), \tag{19}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\theta^2 - \frac{1}{A^2}\left(\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA}\right), \tag{20}$$

$$\dot{u}_i = u_{i;j}u^j = \left(\frac{A_1}{A}, 0, 0, 0\right) \tag{21}$$

$$V^3 = \sqrt{-g} = A^2BC, \tag{22}$$

where $g$ is the determinant of the metric (1). Using the field equations and the relations (19) and (20) one obtains the Raychaudhuri's equation as

$$\dot{\theta} = \dot{u}^i_{;i} - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}\rho_p, \tag{23}$$

where dot denotes differentiation with respect to $t$ and

$$R_{ij}u^iu^j = \frac{1}{2}\rho_p. \tag{24}$$

With the help of Eqs. (1) - (4), (9) and (10), the Bianchi identity $\left(T^{ij}_{;j}\right)$ reduced to two equations:

$$\rho_4 - \frac{A_4}{A}\lambda + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\rho = 0 \tag{25}$$

and

$$\lambda_1 - \frac{A_1}{A}\rho + \left(\frac{A_1}{A} + \frac{B_1}{B} + \frac{C_1}{C}\right)\lambda = 0. \tag{26}$$

Thus, due to all the three (strong, weak and dominant) energy conditions, one finds $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of $\lambda$ is unrestricted, it may take values positive, negative or zero as well.

## 3. Solution of the Field Equations

We have revisited the solutions obtained by Kilinc and Yavuz [41]. Equations (16) and (17) lead to

$$\frac{F_{12}^2}{\bar{\mu}B^2} = \frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{C_{11}}{C} - \frac{B_{11}}{B}. \tag{27}$$

and

$$\frac{2A_{11}}{A} - \frac{2A_1^2}{A^2} - \frac{2A_{44}}{A} + \frac{2A_4^2}{A^2} + \frac{C_{11}}{C} + \frac{B_{11}}{B} - \frac{C_{44}}{C} - \frac{B_{44}}{B} = 0. \tag{28}$$

The field equations (14) - (18) constitute a system of five equations with six unknowns parameters $A$, $B$, $C$, $\lambda$, $\rho$ and $F_{12}$. Therefore some additional constraints relating these parameters are required to obtain explicit solutions of the system of equations. Assuming that the metric coefficients are separable as the following

$$A = f(x)\ell(t), \tag{29}$$

$$B = g(x)k(t), \tag{30}$$

$$C = g(x)\nu(t), \tag{31}$$

and

$$\frac{\ell_4}{\ell} = m(\text{constant}). \tag{32}$$

From Eq. (15) we get

$$\frac{\frac{k_4}{k} + \frac{\nu_4}{\nu}}{\frac{k_4}{k} + \frac{\nu_4}{\nu} - \frac{2\ell_4}{\ell}} = \frac{\frac{g_1}{g}}{\frac{f_1}{f}} = n(\text{constant}). \tag{33}$$

Eq. (33) leads to

$$\frac{g_1}{g} = n\frac{f_1}{f}, \tag{34}$$

which after integration gives

$$g = \alpha f^n, \tag{35}$$

where $\alpha$ ($\neq 0$) is a constant of integration. Eqs. (32) and (33) reduce to

$$\frac{k_4}{k} = -a - \frac{\nu_4}{\nu}, \tag{36}$$

where $a = \frac{2mn}{1-n}$ is constant. From Eq. (32) we get

$$\ell = e^{mt}. \tag{37}$$

Using Eqs. (29) - (31) in (28), we get

$$2\left(\frac{f_{11}}{f} - \frac{f_1^2}{f^2}\right) + 2\frac{g_{11}}{g} = \frac{k_{44}}{k} + \frac{\nu_{44}}{\nu} = s(\text{constant}). \tag{38}$$

From the right hand side of Eq. (38), we have

$$\frac{k_{44}}{k} + \frac{\nu_{44}}{\nu} = s. \tag{39}$$

Equations (36) and (39) lead to

$$\nu_4^2 + a\nu\nu_4 + \left(\frac{a^2 - s}{2}\right)\nu^2 = 0. \tag{40}$$

The differential equation (40) has two solutions

$$\nu = e^{-\frac{1}{2}(a \pm \sqrt{2s - a^2})t}. \tag{41}$$

Now we consider the following two cases.

## 3.1 Model I

Taking negative sign, Eq. (41) leads to

$$\nu = e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}. \tag{42}$$

From Eqs. (36) and (42), we get

$$k = e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}. \tag{43}$$

From the left hand side of Eq. (38), we have

$$2\frac{f_{11}}{f} - 2\frac{f_1^2}{f^2} + 2\frac{g_{11}}{g} = s. \tag{44}$$

From Eqs. (34) and (44), we have

$$\frac{f_{11}}{f} + \frac{2(n^2-n-1)}{(2n+1)}\frac{f_1^2}{f^2} = s, \tag{45}$$

which leads to

$$f = \left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{2n+1}{2n^2-1}}, \tag{46}$$

where $b = \sqrt{\frac{(2n^2-1)s}{(2n+1)}}$ and $c_1$, $c_2$ are constants of integration. Hence Eq. (35) gives

$$g = \alpha\left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}}. \tag{47}$$

Therefore, we have

$$A = \left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}, \tag{48}$$

$$B = \alpha\left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}, \tag{49}$$

$$C = \alpha\left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}. \tag{50}$$

Hence, the metric (1) reduces to

$$\begin{aligned} ds^2 &= \left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}(dx^2 - dt^2) \\ &+ \alpha^2\left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{-(a+\sqrt{2s-a^2})t}dy^2 \\ &+ \alpha^2\left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{(-a+\sqrt{2s-a^2})t}dz^2. \end{aligned} \tag{51}$$

**Some Physical and Geometric Properties of the Model**

The pressure ($p$), the string tension density ($\lambda$), the energy density ($\rho$), the particle density ($\rho_p$), for the model (51) are given by

$$p = \frac{1}{\left(c_1e^{bx} + c_2e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}}\left[-\frac{1}{2}s + \frac{(2n+1)(n+1)b^2}{(2n^2-1)}\right.$$

$$+\frac{(2n+1)(1+n-n^2)b^2}{(2n^2-1)^2}\frac{(c_1e^{bx}-c_2e^{-bx})^2}{(c_1e^{bx}+c_2e^{-bx})^2}\Bigg], \quad (52)$$

$$\lambda = \frac{1}{(c_1e^{bx}+c_2e^{-bx})^{\frac{2(2n+1)}{2n^2-1}}\, e^{2mt}}\Bigg[ma+\frac{1}{2}(s+a^2+a\sqrt{2s-a^2})$$

$$-\frac{n(n+2)(2n+1)^2b^2}{(2n^2-1)^2}\frac{(c_1e^{bx}-c_2e^{-bx})^2}{(c_1e^{bx}+c_2e^{-bx})^2}\Bigg], \quad (53)$$

$$\rho = \frac{1}{(c_1e^{bx}+c_2e^{-bx})^{\frac{2(2n+1)}{2n^2-1}}\, e^{2mt}}\Bigg[-ma-\frac{1}{2}(s-a^2+a\sqrt{2s-a^2})$$

$$-\frac{2n(2n+1)b^2}{(2n^2-1)}-\frac{n^2(4n^2-1)b^2}{(2n^2-1)^2}\frac{(c_1e^{bx}-c_2e^{-bx})^2}{(c_1e^{bx}+c_2e^{-bx})^2}\Bigg], \quad (54)$$

$$\rho_p = \frac{1}{(c_1e^{bx}+c_2e^{-bx})^{\frac{2(2n+1)}{2n^2-1}}\, e^{2mt}}\Bigg[-s-a(2m+\sqrt{2s-a^2})$$

$$-\frac{2n(2n+1)b^2}{(2n^2-1)}+\frac{2n(3n+1)(2n+1)b^2}{(2n^2-1)^2}\frac{(c_1e^{bx}-c_2e^{-bx})^2}{(c_1e^{bx}+c_2e^{-bx})^2}\Bigg]. \quad (55)$$

The non-vanishing component $F_{12}$ of electromagnetic field tensor for model (51) is given by

$$F_{12}^2 = \bar{\mu}\alpha^2 a\sqrt{2s-a^2}\left(c_1e^{bx}+c_2e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{-(a+\sqrt{2s-a^2})t}. \quad (56)$$

The scalar of expansion ($\theta$), shear tensor ($\sigma$), acceleration vector ($\dot{u}_i$) and the proper volume ($V^3$) for the model (51) are given by

$$\theta = \frac{(m-a)}{(c_1e^{bx}+c_2e^{-bx})^{\frac{2n+1}{2n^2-1}}\, e^{mt}}, \quad (57)$$

$$\sigma^2 = \frac{(2m^2+2ma+3s-a^2)}{6\left(c_1e^{bx}+c_2e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}}\, e^{2mt}}, \quad (58)$$

$$\dot{u}^i = \left(\frac{(2n+1)b}{(2n^2-1)}\frac{(c_1e^{bx}-c_2e^{-bx})^2}{(c_1e^{bx}+c_2e^{-bx})^2}, 0, 0, 0\right), \quad (59)$$

$$V^3 = \alpha^2(c_1e^{bx}+c_2e^{-bx})^{\frac{2(2n^2+3n+1)}{2n^2-1}} e^{(2m-a)t}. \quad (60)$$

From Eqs. (57) and (58), we have

$$\frac{\sigma^2}{\theta^2} = \frac{(2m^2+2ma+3s-a^2)}{6(m-a)^2} = \text{constant}. \quad (61)$$

The model (51) is expanding (for $m < 0$ only), shearing, accelerating, non-rotating and singularity free model. For $m > 0$, $\theta =$ constant when $t = 0$ and $\theta \to 0$ when $t \to \infty$. For $m < 0$, $\theta =$ constant when $t = 0$ and $\theta \to \infty$ when $t \to \infty$. From Eq. (59), we observe that for $b = 0$ or $n = -\frac{1}{2}$, $\dot{u}^i$ vanishes. In this case the pressure $p$, the energy density

$\rho$, the string tension density $\lambda$ and particle density $\rho_p$ tend to a constant value as $t \to 0$ and $x \to 0$. At a later stage $p, \rho$, $\lambda$ and $\rho_p$ approach to zero when $t \to \infty$ and $x \to \infty$, as expected. For suitable values of constants, our solution satisfies the energy conditions $\rho > 0$, $\rho_p \geq 0$. In the electromagnetic field tensor (56), $\bar{\mu}$ remains undetermined as function of both $x$ and $t$. The electromagnetic field tensor $F_{12}$ does not vanish if $s \neq \frac{a^2}{2}$. The proper volume increases as time increases. Since $\frac{\sigma}{\theta}$ = constant, hence the model does not approach isotropy.

## 3.2 Model II

Taking positive sign, Eq. (41) leads to

$$\nu = e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}. \tag{62}$$

From Eqs. (36) and (62), we get

$$k = e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}. \tag{63}$$

Therefore, we have

$$A = \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}, \tag{64}$$

$$B = \alpha \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{\frac{1}{2}(-a+\sqrt{2s-a^2})t}, \tag{65}$$

$$C = \alpha \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{-\frac{1}{2}(a+\sqrt{2s-a^2})t}. \tag{66}$$

Hence, the metric (1) reduces to

$$\begin{aligned} ds^2 = {} & \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}(dx^2 - dt^2) \\ & + \alpha^2 \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{(-a+\sqrt{2s-a^2})t} dy^2 \\ & + \alpha^2 \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{-(a+\sqrt{2s-a^2})t} dz^2. \end{aligned} \tag{67}$$

**Some Physical and Geometric Properties of the Model**

The pressure ($p$), the string tension density ($\lambda$), the energy density ($\rho$), the particle density ($\rho_p$), for the model (67) are given by

$$\begin{aligned} p = {} & \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -\frac{1}{2}s + \frac{(2n+1)(n+1)b^2}{(2n^2-1)} \right. \\ & \left. + \frac{(2n+1)(1+n-n^2)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \end{aligned} \tag{68}$$

$$\lambda = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ ma + \frac{1}{2}(s + a^2 - a\sqrt{2s-a^2}) \right.$$

$$- \frac{n(n+2)(2n+1)^2 b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \Bigg], \tag{69}$$

$$\rho = \frac{1}{(c_1 e^{bx} + c_2 e^{-bx})^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \Bigg[ -ma + \frac{1}{2}(a^2 - s + a\sqrt{2s - a^2})$$

$$- \frac{2n(2n+1)b^2}{(2n^2-1)} - \frac{n^2(4n^2-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \Bigg], \tag{70}$$

$$\rho_p = \frac{1}{(c_1 e^{bx} + c_2 e^{-bx})^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \Bigg[ -2ma - s + a\sqrt{2s - a^2})$$

$$- \frac{2n(2n+1)b^2}{(2n^2-1)} + \frac{2n(3n+1)(2n+1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \Bigg]. \tag{71}$$

The non-vanishing component $F_{12}$ of electromagnetic field tensor for model (67) is given by

$$F_{12}^2 = \bar{\mu} R \sqrt{2s - a^2} \alpha^2 \left( c_1 e^{bx} + c_2 e^{-bx} \right)^{\frac{2n(2n+1)}{2n^2-1}} e^{(-a + \sqrt{2s-a^2})t}, \tag{72}$$

where $R = \frac{mn}{n-1}$.

The kinematics parameters $\theta$, $\sigma$, $\dot{u}^i$ for model (67) are same as in the case I. The model (67) possesses the same behaviour as the previous model (51).

## 4. Solution in the Absence of Magnetic Field

In absence of magnetic field, if we use same assumption with presence of magnetic field, we obtain the following field equations

$$-\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} + \frac{C_{44}}{C} + \frac{B_{44}}{B}$$

$$= -pA^2 + \lambda A^2, \tag{73}$$

$$\frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_{14}}{B} - \frac{C_{14}}{C} = 0, \tag{74}$$

$$\frac{A_{11}}{A} - \frac{A_1^2}{A^2} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{C_{11}}{C} - \frac{C_{44}}{C} = pA^2, \tag{75}$$

$$\frac{A_{11}}{A} - \frac{A_1^2}{A^2} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} + \frac{B_{11}}{B} - \frac{B_{44}}{B} = pA^2, \tag{76}$$

$$\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} - \frac{C_{11}}{C} - \frac{B_{11}}{B} = \rho A^2. \tag{77}$$

Following the same techniques with presence of magnetic field, in this case also, we find equations (33) - (40). From Eqs. (73) and (74), we obtain

$$\frac{k_{44}}{k} = \frac{\nu_{44}}{\nu}. \tag{78}$$

Eqs. (39) and (78) leads to

$$\frac{k_{44}}{k} = \frac{s}{2}. \tag{79}$$

Therefore our solutions will be equivalent to (41). But Eq. (78) will be satisfied if $2s = a^2$. In this case our solutions (41) are reduced only to one equation

$$\nu = e^{-\frac{1}{2}at}. \tag{80}$$

Accordingly, Eq. (36) reduces to

$$k = e^{-\frac{1}{2}at}. \tag{81}$$

But in absence of magnetic field the expressions for $f$ and $g$ are same as with presence of magnetic field i.e. given by Eqs. (46) and (47) respectively.

Hence, in this case, we have

$$A = \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}, \tag{82}$$

$$B = C = \alpha \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{n(2n+1)}{2n^2-1}} e^{-\frac{1}{2}at}. \tag{83}$$

Therefore the geometry of the universe (1) reduces to

$$ds^2 = \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}(dx^2 - dt^2) + \alpha^2 \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n(2n+1)}{2n^2-1}} e^{-at}(dy^2 + dz^2). \tag{84}$$

**Some Physical and Geometric Properties of the Model**

The pressure ($p$), the string tension density ($\lambda$), the energy density ($\rho$), the particle density ($\rho_p$) the kinematics parameters $\theta$, $\sigma$, $\dot{u}_i$ and $V^3$ for the model (84) are given by

$$p = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -\frac{a^2}{4} + \frac{(2n+1)(n+1)b^2}{(2n^2-1)} + \frac{(2n+1)(1+n-n^2)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{85}$$

$$\lambda = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ a^2 \left(m + \frac{1}{2}\right) + \frac{(2n+1)(n+1)b^2}{(2n^2-1)} - \frac{(2n+1)(2n^3+6n^2+n-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{86}$$

$$\rho = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -a^2 \left(m - \frac{1}{4}\right) - \frac{2n(2n+1)b^2}{(2n^2-1)} - \frac{n^2(4n^2-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{87}$$

$$\rho_p = \frac{1}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}} \left[ -a^2 \left(2m + \frac{1}{4}\right) - \frac{(2n+1)(3n+1)b^2}{(2n^2-1)} \right.$$
$$\left. - \frac{(2n+1)(4n^3+5n^2+n-1)b^2}{(2n^2-1)^2} \frac{(c_1 e^{bx} - c_2 e^{-bx})^2}{(c_1 e^{bx} + c_2 e^{-bx})^2} \right], \tag{88}$$

$$\theta = \frac{(m-a)}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2n+1}{2n^2-1}} e^{mt}}, \tag{89}$$

$$\sigma^2 = \frac{\frac{1}{3}(m-a)^2 + a\left(m - \frac{a}{4}\right)}{\left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{2(2n+1)}{2n^2-1}} e^{2mt}}, \tag{90}$$

$$V^3 = \alpha^2 \left(c_1 e^{bx} + c_2 e^{-bx}\right)^{\frac{4(2n+1)}{2n^2-1}} e^{4mt}. \tag{91}$$

From Eqs. (89) and (90), we have

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} + \frac{a(4m-a)}{4(m-a)^2} = \text{constant}. \tag{92}$$

## 5. Concluding Remarks

In this paper, we have investigated new exact solutions of Einstein's field equations for cylindrically symmetric space-times with string source in presence and absence of magnetic field. In this solution, we take magnetic field and string together as the source gravitational field. It is known that magnetic field are anisotropic stress source. In general, the expressions for physical and kinematics quantities depend on at most one space coordinate and the time. Our solutions are new and different from the other author's solutions.

If we choose suitable values for constants, we see that our solutions in presence and in absence of magnetic field satisfy energy conditions (strong, weak and dominant). We also observe that our solutions also satisfy the Raychaudhuri's equation. In presence and absence of magnetic field, the pressure $p$ the energy density $\rho$, the string tension density $\lambda$ and the particle density $\rho_p$ tend to constant values as $t \to 0$ and $x \to 0$. At a later stage, $p$, $\rho$, $\lambda$ and $\rho_p$ approach zero when $t \to \infty$ and $x \to \infty$, as expected. In all these cases, the proper volumes also increase as time increases. For $m > 0$, $\theta =$ constant when $t = 0$ and $\theta \to 0$ when $t \to \infty$. For $m < 0$, $\theta =$ constant when $t = 0$ and $\theta \to \infty$ when $t \to \infty$.

In both cases 3.1 and 3.2, the electromagnetic field tensor given in equations (52) and (72), $\bar{\mu}$ remains undetermined as function of both x and t. The electromagnetic field tensor does not vanish if $s \neq \frac{a^2}{2}$. Either in presence of magnetic field or in the absence of it, we observe that all the kinematics quantities $\theta$, $\sigma$, $\dot{u}^i$ and proper volume $V^3$ tend to constant as $t \to 0$ and $x \to 0$. Our solutions are free from singularity. Since $\frac{\sigma}{\theta} =$ constant, the models (51), (67) and (84) do not approach to isotropy at any time. For $b = 0$ or $n = -\frac{1}{2}$, the acceleration in the models vanishes.

Either in presence of a magnetic field or in the absence of it, if we put $m = 0$, our solutions become a function of x only. Thus, these solutions reduce to static solutions. For $n = -\frac{1}{2}$, our solutions become homogeneous either in presence or absence of magnetic field. Therefore inhomogeneity dies out for $n = -\frac{1}{2}$. In general our models represent expanding, shearing and non-rotating universe. We found that the magnetic field is decreasing function of time. For large value of time, the magnetic field will be very small. The magnetic field imposed the restriction on constants such as $a$, $\alpha$ and $s$. It is observed that in presence of magnetic field, the rate of expansion of the universe is faster than the rate of expansion in absence of magnetic field. The idea of primordial magnetism is appealing because it can potentially explain all the large-scale fields seen in the universe today, especially those found in remote protogalaxies. As a result, the literature contains many studies examining the role and the implications of magnetic fields for cosmology.

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