Electroweak Standard Model at Finite Temperature in Presence of A Bosonic Chemical Potential

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Abstract: We study the electroweak standard model at finite temperature in presence of a bosonic chemical potential associated with the conserved electromagnetic current. To preserve the thermodynamic equilibrium of the system, the thermal medium is neutralized by the introduction of four background charges related to the four gauge bosons of this model. Using the mean-field approximation, in the high temperature limit, we find that there exists a difference between the effective mass of the spatial and temporal components of the W boson. A W boson condensation induced via the background charges allows to vanish this difference.

Keywords: Electroweak Standard Model, Finite Temperature; Bosonic Chemical Potential; Background Charges; W-Gauge Boson Condensation
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1. Introduction

Since long time ago it is well known that the spontaneous symmetry breaking for gauge theories at finite temperature in presence of chemical potentials can be seen as a Bose-Einstein condensation phenomena [1, 2]. At this respect, the vacuum expectation value of the Higgs field ν at finite temperature can be seen as a variational parameter of Bose-Einstein condensation of the Higgs field. The Electroweak Standard Model (ESM) allows to simultaneously both electroweak phase transition and Bose-Einstein condensation for the Higgs field if a bosonic chemical potential μ related with the conserved electromagnetic current of the model is considered in the system. It was found in [1] that the Critical Temperature (T_c) of the electroweak phase transition increases with μ. This fact suggest

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that μ can be considered as an effective parameter of spontaneous symmetry breaking. To make the system neutral, it was necessary in [1] to add an external charge background to offset the charge density of the scalar field.

On the other hand, a zero temperature condensation of SU(2) vectorial bosons generated by an external lepton density was proposed in [3]-[4], for the case in which the system is electrically neutralized by the inclusion of a external charge. The generalization at finite temperature of a W boson condensation was investigated in [5]-[8] for the case of ESM. This W boson condensation for the ESM can be induced by the presence of a fermion density in the system. Specifically, a W boson condensate was investigated by consideration of the chemical potentials associated with the conserved electromagnetic and leptonic currents [5]. This condensate can also be investigated by the additional consideration of the chemical potential associated with the conserved weak neutral current [7]-[8]. The existence of the W boson condensation was studied in references [5]-[8] through the calculation of the thermodynamical potential, which leads to the phase diagram of this condensation. It is possible to see in these references that if the chemical potentials associated with the conserved leptonic and neutral weak currents vanish, the chemical potential associated with conserved electromagnetic current μ has a zero value in $T = T_c$, being this fact consistent with the gauge symmetry restoration at high temperatures. This behavior is not strange because the condensate considered in these references is neutral and the T_c has not a dependence over μ. In such references the neutralization of the thermodynamic system was performed by the inclusion of the leptonic chemical potential associated with the conserved leptonic current of the ESM.

The W boson condensation phenomenon has been extensively studied in the context of neutral many-particle electroweak theory with the purpose of possible cosmological applications [3]-[8]. In this paper we extend these studies introducing the case of a charged electroweak plasma. The W boson condensation for the ESM at high temperatures is induced by the inclusion of background external charges in the thermodynamical system. The results of this work will have interesting in the study of charged electroweak plasmas that will be obtained in future experiments. Specifically the W boson condensation worked in this paper differs from the one previously studied in the literature [5]-[8] in the sense that it is induced but with an external charge instead of a fermion density. By this reason we do not include in the thermodynamic system the chemical potentials associated with the conserved leptonic and neutral weak currents. We only include a bosonic chemical potential μ associated with the conserved electromagnetic current of the ESM. We neutralize the thermal medium by the introduction of an external charge background which offsets the charge density of the scalar field, as was performed by Kapusta in [1]. Particularly we preserve the thermodynamic equilibrium of the system by the introduction of four background charges j_ν, j_ν^1, j_ν^2 and j_ν^3, which are associated with the four gauge bosons of the ESM [1, 9, 10]. The background charges j_ν^1 and j_ν^2 are associated with the $SU(2)_L$ gauge fields A_ν^1 and A_ν^2 respectively. It is possible to see that if the background charges j_ν^1 and j_ν^2 vanish, then the vacuum average values of the A_ν^1 and A_ν^2 fields vanish too, i. e. $\langle A_\nu^1\rangle_0 = \langle A_\nu^2\rangle_0 = 0$. We note that the bosonic chemical potential that we consider here does not vanish for $T = T_c$, because our interest is focused in the coexistence of the electroweak broken phase and the Higgs condensate.

Considering the ESM at finite temperature in presence of μ, in section II, we calculate the vacuum expectation value of the Higgs field as a function of T and μ, using the mean-field approximation in the high temperature limit [1]. Next we calculate, in section III, the effective masses of the scalar and gauge bosons using a generalized gauge. We find that the effective mass of the spatial component of the W boson has a difference of $-\mu^2$ respect to its temporal component. In section IV, by consideration of non-vanishing background charges j^1_ν and j^2_ν, we assume that the vacuum average values of the A^1_ν and A^2_ν fields do not vanish. i. e. $\langle A^1_\nu\rangle_0 = \xi_{A^1_\nu}$ and $\langle A^2_\nu\rangle_0 = \xi_{A^2_\nu}$. Because the W field is a combination of the A^1_ν and A^2_ν fields, we obtain a W boson condensate associated to the spatial component of W boson. As a consequence of this condensate, the mentioned difference among the effective masses of the spatial and temporal components of the W boson vanishes. In section V, by means of the calculation of the thermodynamic potential, we obtain the electromagnetic charge density of the system and we calculate the critical temperature of the W boson condensation. We demonstrate that the W boson condensate is consistent with the usual condition of condensation $m^2_W = \mu^2$, where m_W is the effective mass of the W boson. Finally the conclusions are presented in section VI.

2. Vacuum expectation value of the Higgs field

The ESM has four conserved currents associated with the four independent generators of the $SU(2)_L \times U(1)_Y$ gauge group. The electromagnetic current J_ν is the only one conventionally conserved [1]. We introduce the chemical potential μ associated with J_ν in the formalism of the quantum field theory at finite temperature [9] through the partition function. We initially restrict our interest to the gauge boson and scalar sectors of the ESM. Performing the functional integrations over the canonical momentums we obtain that the effective Lagrangian density $\mathcal{L}_{eff}$ of the ESM is [1]

$$\begin{aligned}\mathcal{L}_{eff} &= [(D^\nu + i\mu Q\delta^{\nu 0})\Phi]^+[(D_\nu + i\mu Q\delta_{\nu 0})\Phi] + c^2\Phi^+\Phi - \lambda(\Phi^+\Phi)^2\\ &-\frac{1}{4}G^{\mu\nu}G_{\mu\nu} - \frac{1}{4}\widetilde{F}^{\mu\nu}_a\widetilde{F}^a_{\mu\nu} + g'B^\nu j_\nu + gA^\nu_a j^a_\nu,\end{aligned} \tag{1}$$

where $D_\nu = \partial_\nu + igA^a_\nu\tau^a/2 + ig'B_\nu/2$ is the $SU(2)_L \times U(1)_Y$ covariant derivative, $Q = (I+\tau)/2$ is the electromagnetic charge, $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the $U(1)_Y$ effective strength field tensor,

$$\widetilde{F}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc}(A^b_\mu + \frac{\mu}{g}\delta_{\mu 0}\delta^{b3})(A^c_\nu + \frac{\mu}{g}\delta_{\nu 0}\delta^{c3}) \tag{2}$$

is the $SU(2)_L$ effective strength field tensor [1]. We observe that the form of $\widetilde{F}^a_{\mu\nu}$ implies to shifting the non-abelian fields by $A^a_\mu \to A^a_\mu + \frac{\mu}{g}\delta_{\mu 0}\delta^{a3}$. As was explained in [1], the $\delta_{\mu 0}$ arises because there is a preferred reference frame for the medium, and the δ^{a3} arises because it is the A_ν^3 field which mixes with B_ν to form the photon. Because the non-abelian fields have a shift proportional to μ, then for the case $\mu = 0$ the shift vanishes. Therefore, for this case, (2) becomes in the normal non-abelian strength field tensor and the Lagrangian density given by (1) becomes in the usual for the scalar and gauge boson sectors of the ESM. The abelian j_ν and non-abelian j_ν^a background charges, introduced in (1), are given by

$$j_\nu = \ j_0\delta_{\nu 0}, \tag{3}$$
$$j_\nu^a = \ j_0^3\delta_{\nu 0}\delta^{a3}. \tag{4}$$

These background charges were introduced with the purpose of having a vanishing electromagnetic charge density, preserving the thermodynamical equilibrium of the system [1, 7, 11]. Since after electroweak symmetry breaking the electromagnetism is a $U(1)_1$ symmetry, it is possible to introduced the electromagnetic background charge j_ν^{em} in terms of j_ν and j_ν^3.

Starting of the effective Lagrangian density (1) it is possible to show that the equation of motion for the scalar field doublet Φ is given by

$$\begin{aligned}&\Big(\partial^\nu\partial_\nu + i\frac{g}{2}(\partial_\nu A_a^\nu)\tau^a + igA_a^\nu\tau^a\partial_\nu + i\frac{g'}{2}(\partial_\nu B^\nu) + ig'B^\nu\partial_\nu + i\mu(I+\tau^3)\partial_0 \\ &-\frac{1}{4}[gA_a^\nu\tau^a + g'B^\nu + \mu\delta^{\nu 0}(I+\tau^3)]^2 - c^2\Big)\Phi = -2\lambda\Phi\Phi^+\Phi.\end{aligned} \tag{5}$$

In the ESM, the electroweak symmetry is broken spontaneously through the Higgs mechanism. As it is shown in [1], since we are considering a system which has an external charge density, the Higgs mechanism is associated with the existence of a Bose-Einstein condensate. The implementation of this mechanism is performed by mean of the following translation of the scalar field doublet

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_1 + i\psi_2 \\ \psi_3 + i\psi_4\end{pmatrix} + \begin{pmatrix}\xi \\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\nu + \psi_1 + i\psi_2 \\ \psi_3 + i\psi_4\end{pmatrix}, \tag{6}$$

where the vacuum expectation value of the Higgs field ν has been defined in terms of the ξ parameter as $\nu = \sqrt{2}\xi$. The ξ parameter represents the infrared stated of the field Φ, and it can be understood as a variational parameter of the Bose-Einstein condensation of this scalar field. The choosing of a vacuum state of the system through (6) is consistent with the fact that this state is energetically preferred for the case $\mu \neq 0$ [1].

Substituting (6) into (5), using the mean-field approximation in which both the field and mixed field fluctuations vanish [1], we obtain that the non-trivial solution for ξ is

$$\begin{aligned}\xi^2 &= \frac{1}{2\lambda}\Big[c^2 + \mu^2 - \lambda(3\langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle) + e^2\langle A^\nu A_\nu\rangle \\ &\quad + \frac{1}{4}(g'^2 - g^2)^2(\frac{e}{gg'})^2\langle Z^\nu Z_\nu\rangle + \frac{1}{2}g^2\langle W^{+\nu}W_\nu^-\rangle\Big] \\ &= \frac{1}{2\lambda}\Big[c^2 + \mu^2 - \frac{T^2}{4}\Big(\frac{g'^2}{4} + \frac{3g^2}{4} + 2\lambda\Big)\Big],\end{aligned} \tag{7}$$

where the electromagnetic (A_ν), neutral electroweak boson (Z_ν) and charged electroweak boson ($W^\pm_\nu$) fields have been defined as usual in terms of the abelian (B_ν) and non-abelian (A^a_ν) gauge boson fields. We have used in (7) the definition $e = gg'/(g'^2 + g^2)^{\frac{1}{2}}$, and the Gibbs averages have been evaluated in the high temperature limit [10], i. e. $\langle W^{+\nu} W^-_\nu \rangle = \langle Z^\nu Z_\nu \rangle = \langle A^\nu A_\nu \rangle = \frac{-3T^2}{12}$, $\langle \psi_1^2 \rangle = \langle \psi_2^2 \rangle = \langle \psi_3^2 \rangle = \langle \psi_4^2 \rangle = \frac{T^2}{12}$.

The T_c of the electroweak phase transition corresponds to the temperature in which the electroweak symmetry is restored. The value of T_c is obtained for the temperature in which ξ vanishes, i.e. $\xi(T_c) = 0$. From (7), we obtain that the square of T_c is given by [1]

$$T_c^2 = \frac{4(c^2 + \mu^2)}{\frac{g'^2}{4} + \frac{3g^2}{4} + 2\lambda}. \tag{8}$$

Due ξ is a parameter of Bose-Einstein condensation of the field Φ, the temperature given by (8) also corresponds to the critical temperature of the Bose-Einstein condensation of the same field. The T_c obtained does not contain the fermion contributions and its value is unknown due λ and μ are free parameters. We can observe that μ has the effect to increase the T_c value. For $\mu = 0$, it is known that $T_c \approx 200$ GeV.

3. Effective masses of the bosonic fields

The equation of motion for the scalar field doublet Φ, given by (5), leads to the equations of motion for the scalar fields ψ_1, ψ_2, ψ_3 and ψ_4. If we use the mean-field approximation in these equations of motion, the following mixing terms vanish $\left\langle [igA^{a\nu}\tau_a \partial^\nu + ig' B^\nu \partial_\nu + i\frac{g}{2}\partial_\nu (A^{a\nu}\tau_a) + i\frac{g'}{2}(\partial^\nu B_\nu)]\phi \right\rangle$, $\langle A^{a\nu}\phi \rangle$, $\langle B^\nu \phi \rangle$, $\langle A^\nu B_\nu \phi \rangle$. Using the following result that is obtained from the first-order perturbation theory [1]

$$\langle \psi_1^3 \rangle + \langle \psi_1 \psi_2^2 \rangle + \langle \psi_1 \psi_3^2 \rangle + \langle \psi_1 \psi_4^2 \rangle = \Big[3\langle \psi_1^2 \rangle + \langle \psi_2^2 \rangle + \langle \psi_3^2 \rangle + \langle \psi_4^2 \rangle \Big], \tag{9}$$

we find that the equations of motion of the scalars decouple and they can be written as $(\partial^2 + M^2_{(1,2,3,4)eff})\langle \psi_{1,2,3,4} \rangle = 0$, being $M_{(1,2,3,4)eff}$ the effective masses of the scalar bosons.

We obtain that the square of these effective masses are

$$
\begin{aligned}
M^2_{1eff} &= -c^2 - \mu^2 + \lambda(3\langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle) - e^2\langle A^\nu A_\nu\rangle - \\
&\quad \frac{1}{4}(g'^2 - g^2)^2(\frac{e}{gg'})^2\langle Z^\nu Z_\nu\rangle - \frac{1}{2}g^2\langle W^{+\nu}W^-_\nu\rangle + 6\lambda\xi^2 \\
&= 4\lambda\xi^2, \qquad (10)\\
M^2_{2eff} &= -c^2 - \mu^2 + \lambda(3\langle\psi_2^2\rangle + \langle\psi_1^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle) - e^2\langle A^\nu A_\nu\rangle - \\
&\quad \frac{1}{4}(g'^2 - g^2)^2(\frac{e}{gg'})^2\langle Z^\nu Z_\nu\rangle - \frac{1}{2}g^2\langle W^{+\nu}W^-_\nu\rangle + 2\lambda\xi^2, \\
&= 2\lambda(\langle\psi_2^2\rangle - \langle\psi_1^2\rangle), \qquad (11)\\
M^2_{3eff} &= -c^2 + \lambda(3\langle\psi_3^2\rangle + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_4^2\rangle) - \frac{1}{2}g^2\langle W^{+\nu}W^-_\nu\rangle - \\
&\quad \frac{1}{4}(\frac{gg'}{e})^2\langle Z^\nu Z_\nu\rangle + 2\lambda\xi^2, \\
&= 2\lambda(\langle\psi_3^2\rangle - \langle\psi_1^2\rangle), \qquad (12)\\
M^2_{4eff} &= -c^2 + \lambda(3\langle\psi_4^2\rangle + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle) - \frac{1}{2}g^2\langle W^{+\nu}W^-_\nu\rangle - \\
&\quad \frac{1}{4}(\frac{gg'}{e})^2\langle Z^\nu Z_\nu\rangle + 2\lambda\xi^2, \\
&= 2\lambda(\langle\psi_4^2\rangle - \langle\psi_1^2\rangle). \qquad (13)
\end{aligned}
$$

We observe that in the high temperature limit $M^2_{1eff} = 2\Big[c^2 + \mu^2 - \frac{T^2}{4}\Big(\frac{g'^2}{4} + \frac{3g^2}{4} + 2\lambda\Big)\Big]$ and $M^2_{2eff} = M^2_{3eff} = M^2_{4eff} = 0$. The effective masses obtained for the scalar particles do not include the effects of performing the gauge fixing. In the next we will use the renormalizable R_ρ gauge [8].

The effective Lagrangian density (1) leads to the effective equations of motion for the non-physical gauge bosons B^ν and $A^{a\nu}$. It is possible to prove that the effective equation for the abelian gauge boson B^ν is

$$
\partial^\mu G_{\mu\nu} = -i\frac{g'}{2}\Big[[(D_\nu + i\mu Q\delta_{\nu0})\phi]^+\phi - \phi^+(D_\nu + i\mu Q\delta_{\nu0})\phi\Big] - g' j_\nu, \qquad (14)
$$

while the ones for the non-abelian gauge bosons $A^{a\nu}$ are

$$
\begin{aligned}
\partial^\mu \widetilde{F}^a_{\mu\nu} &= -i\frac{g}{2}\Big\{\big[(D_\nu + i\mu Q\delta_{\nu0})\phi\big]^+\tau^a\phi - \phi^+\tau^a[(D_\nu + i\mu Q\delta_{\nu0})\phi]\Big\} \\
&\quad -\epsilon^{abc}\widetilde{F}^b_{\mu\nu}(gA^{c\mu} + \mu\delta^{\mu0}\delta^{c3}) - gj^a_\nu, \qquad (15)
\end{aligned}
$$

being $a = 1, 2, 3$. After substituting (6) into (14), taking the statistical average, without considering fluctuations of the abelian gauge field, i. e. $< B_\nu >= 0$, we obtain that the background abelian charge j_ν is given by

$$
j_\nu = \frac{-i}{2}\Big\langle(\widetilde{D}_\nu\phi)^+\phi - \phi^+(\widetilde{D}_\nu\phi)\Big\rangle, \qquad (16)
$$

where $\widetilde{D}^\nu = D^\nu + i\mu Q\delta^{\nu0}$. In the mean-field approximation, the mixed field terms of (16) can be forgotten and we obtain

$$
j_\nu = -\mu\delta_{\nu0}\frac{1}{2}\Big(2\xi^2 + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle\Big), \qquad (17)
$$

being j_ν proportional to μ. It is clear that for a vanishing chemical potential this background abelian charge vanishes. In a similar way, starting from (15), without considering fluctuations as $\langle \partial^\nu A^a_\nu \rangle_0 = 0$, we obtain that the background non-abelian charges j^a_ν have the form

$$j^a_\nu = \frac{-i}{2}\Big[\big\langle (\widetilde{D}_\nu \phi)^+ \tau^a \phi \big\rangle - \langle \phi^+ \tau^a (\widetilde{D}_\nu \phi)\big\rangle\Big] - \frac{1}{g}\epsilon^{abc}\big\langle \widetilde{F}^b_{\mu\nu}(gA^{c\mu} + \mu\delta^{\mu 0}\delta^{c3})\big\rangle. \tag{18}$$

After substituting (6) into (18), we obtain the three following background charges:

$$j^1_\nu = -\frac{\mu^2}{g}\big[\langle A^1_0\rangle_0 \delta_{\nu 0} - \langle A^1_\nu \rangle_0\big], \tag{19}$$

$$j^2_\nu = -\frac{\mu^2}{g}\big[\langle A^2_0\rangle_0 \delta_{\nu 0} - \langle A^2_\nu \rangle_0\big], \tag{20}$$

$$\begin{aligned} j^3_\nu &= -\frac{1}{2}\mu\delta_{\nu 0}\big[2\xi^2 + \langle \psi_1^2\rangle + \langle \psi_2^2 \rangle\big] \\ &\quad + \mu\Big[\Big(\big\langle (A^1_\nu)^2\big\rangle + \big\langle (A^2_\nu)^2 \big\rangle\Big)\delta_{\nu 0} - \Big(\big\langle A^1_\nu A^{10}\big\rangle + \big\langle A^2_\nu A^{20}\big\rangle\Big)\Big], \end{aligned} \tag{21}$$

where we note that these three background charges are also proportional to μ. As we have defined $j^a_\nu = j^3_0 \delta^{a3}\delta_{\nu 0}$ in (4), this means that $j^1_\nu = j^2_\nu = 0$. According with (19) and (20), it is necessary by consistency that $\langle A^1_i \rangle_0 = \langle A^2_i \rangle_0 = 0$. It is clear that the background charges j_ν y j^3_ν are not conserved because they have a dependence over the temperature. Additionally, because there are not vacuum fluctuations for the non-abelian gauge fields A^1_ν y A^2_ν, then there not exists a W boson condensation due $\langle W^\pm_\nu \rangle_0 = \frac{1}{\sqrt{2}}(\langle A^1_\nu\rangle_0 \mp i\langle A^2_\nu \rangle_0) = 0$.

Using the mean-field approximation, in which the mixing terms that appears in the equation of motion (14) and (15) vanish, we obtain that the effective masses of the non-physical gauge bosons are given by

$$M^2_{effB_\nu} = \frac{g'^2}{4}\big(2\xi^2 + \langle \psi_1^2\rangle + \langle \psi_2^2\rangle + \langle \psi_3^2\rangle + \langle \psi_4^2\rangle\big), \tag{22}$$

$$M^2_{effA^1_\nu} = M^2_{effA^2_\nu} = \frac{g^2}{4}(2\xi^2 + \langle \psi_1^2\rangle + \langle \psi_2^2\rangle + \langle \psi_3^2\rangle + \langle \psi_4^2\rangle) + q_\nu, \tag{23}$$

$$M^2_{effA^3_\nu} = \frac{g^2}{4}(2\xi^2 + \langle \psi_1^2\rangle + \langle \psi_2^2\rangle + \langle \psi_3^2\rangle + \langle \psi_4^2\rangle), \tag{24}$$

being

$$q_\nu = \begin{cases} 0, & \text{if } \nu = 0. \\ -\mu^2, & \text{if } \nu = i. \end{cases} \tag{25}$$

Because the W boson is defined as $W^\pm_\nu = \frac{1}{\sqrt{2}}(A^1_\nu \mp iA^2_\nu)$, we observe that the effective mass of the spatial component of the W boson has a difference of $-\mu^2$ respect to its temporal component.

The quantity $\langle \psi_1^2\rangle + \langle \psi_2^2\rangle + \langle \psi_3^2\rangle + \langle \psi_4^2\rangle$, which appears in (22), (23) and (24), has its origin in the term $\Big(\psi_1^2 + \psi_2^2 + \psi_3^2 + \psi_4^2\Big)\Big[(A^1_\nu)^2 + (A^2_\nu)^2 + (A^3_\nu)^2 + (B_\nu)^2\Big]$ of the Lagrangian density (1) and contributes also to the effective masses of the scalar bosons. With the purpose to include only the terms that contribute to the effective masses of the bosons, we obtain that the effective action S_E, in the imaginary time formalism, is given by

$$
\begin{aligned}
S_E = \int d^4x \Big[& -\frac{1}{2}\psi_1(\partial^2 + M^2_{1eff})\psi_1 - \frac{1}{2}\psi_2(\partial^2 + M^2_{2eff})\psi_2 - \frac{1}{2}\psi_3(\partial^2 + M^2_{3eff})\psi_3 \\
& - \frac{1}{2}\psi_4(\partial^2 + M^2_{4eff})\psi_4 + \xi^2(\mu^2 + c^2 - \lambda\xi^2) + \frac{1}{2}A^{3\nu}\Big[(\partial^2 + \frac{g^2}{2}\xi^2)g_{\mu\nu} - \partial_\nu\partial_\mu\Big]A^{3\mu} \\
& + \frac{1}{2}A^{1\nu}\Big[(\partial^2 + \frac{g^2}{2}\xi^2 + q^\nu)g_{\mu\nu} - \partial_\nu\partial_\mu\Big]A^{1\mu} \\
& + \frac{1}{2}A^{2\nu}\Big[(\partial^2 + \frac{g^2}{2}\xi^2 + q^\nu)g_{\mu\nu} - \partial_\nu\partial_\mu\Big]A^{2\mu} \\
& + \frac{1}{2}B^\nu[(\partial^2 + \frac{g'^2}{2}\xi^2) - \partial_\nu\partial_\mu]B^\nu + \frac{1}{2}gA^{3\nu}\sqrt{2}\xi\partial_\nu\psi_2 - \frac{1}{2}gA^{2\nu}\sqrt{2}\xi\partial_\nu\psi_3 \\
& + \frac{1}{2}gA^{1\nu}\sqrt{2}\xi\partial_\nu\psi_4 + \frac{1}{2}g'B^\nu\sqrt{2}\xi\partial_\nu\psi_2\Big],
\end{aligned} \tag{26}
$$

where we have depressed the terms of $\mathcal{L}_{eff}$ which do not contribute to the equations of motion and we have only preserved the terms which can be canceled with the gauge fixing.

In this paper we work with the renormalizable R_ρ gauge [8]. For the $U(1)_Y$ abelian part, the gauge fixing function is given by

$$F = \partial^\nu B_\nu - \frac{g'}{2}\sqrt{2}\xi\rho\psi_2 - f(x,\tau), \tag{27}$$

$$\frac{\partial F}{\partial\alpha} = -\partial^2 - 2(\frac{g'}{2})^2\rho\xi^2, \tag{28}$$

being α the abelian gauge transformation phase given by $B_\nu \to B'_\nu = B_\nu - \partial_\nu\alpha(x,\tau)$, while for the $SU(2)_L$ non-abelian gauge part, the gauge fixing function is given by:

$$F^1 = \partial^\nu A^1_\nu - \frac{g}{2}\sqrt{2}\xi\rho\psi_4 - f^1(x,\tau), \tag{29}$$

$$F^2 = \partial^\nu A^2_\nu + \frac{g}{2}\sqrt{2}\xi\rho\psi_3 - f^2(x,\tau), \tag{30}$$

$$F^3 = \partial^\nu A^3_\nu - \frac{g}{2}\sqrt{2}\xi\rho\psi_2 - f^3(x,\tau). \tag{31}$$

Because $f^a(x,\tau)$ labels the three gauge arbitrary functions and $\alpha^a(x,\tau)$ labels the non-abelian gauge transformation phases

$$A^c_\nu \to A'^c_\nu = A^c_\nu + g\epsilon^{abc}A^a_\nu\alpha^b(x,\tau) - \partial_\nu\alpha^c(x,\tau), \tag{32}$$

then

$$\frac{\partial F^i}{\partial\alpha^i} = -\partial^2 - (\frac{g^2}{2})\rho\xi^2, \ \text{ for } i = 1,2,3. \tag{33}$$

We observe that our gauge fixing functions F and F^a do not have a dependence over μ and this fact will allow to obtain a gauge invariant thermodynamical potential in the high temperature limit which does not depend over the ρ parameter.

The effective partition function Z_E that we obtain working in the generalized gauge fixing and in the imaginary time formalism is

$$\begin{aligned} Z_E = & \left(det\Big[\frac{\partial^2}{\partial\tau^2}+\nabla^2-m^2_{C^a}\Big]\right)^3\left(det\Big[\frac{\partial^2}{\partial\tau^2}+\nabla^2-m^2_{C}\Big]\right) \\ & \times \int_{periodic} D[\psi_1]D[\psi_2]D[\psi_3]D[\psi_4]D[A^1_\nu]D[A^2_\nu]D[A^3_\nu]D[B_\nu] \\ & \times \int D[f^1]D[f^2]D[f^3]D[F]\delta(F^1)\delta(F^2)\delta(F^3)\delta(F) \\ & \times \exp\Big\{S_E-\int_0^\beta d\tau\int_V d^3x\frac{1}{2\rho}(f^a)^2+\frac{1}{2\rho}(f)^2\Big\}, \end{aligned} \tag{34}$$

where the two determinants which appear in this partition function are the Fadev-Popov's determinants and they contain the contribution of the ghost fields C^a and C, whose masses are $m^2_{C^a}=2\big(\frac{g}{2}\big)^2\rho\xi^2$ and $m^2_C=2\big(\frac{g'}{2}\big)^2\rho\xi^2$, respectively. Now it is possible to identify in the argument of the exponential the bosonic effective masses. The squares of the effective masses of the scalar bosons are

$$m_1^2(\xi) = M^2_{1eff}, \tag{35}$$

$$m_2^2(\xi) = M^2_{2eff}+\frac{g^2}{2}\rho\xi^2+\frac{g'^2}{2}\rho\xi^2, \tag{36}$$

$$m_3^2(\xi) = M^2_{3eff}+\frac{g^2}{2}\rho\xi^2, \tag{37}$$

$$m_4^2(\xi) = M^2_{4eff}+\frac{g^2}{2}\rho\xi^2, \tag{38}$$

where the effective masses M^2_{1eff}, M^2_{2eff}, M^2_{3eff} and M^2_{4eff} are given by (10), (11), (12) and (13), respectively. In the high temperature limit, the effective masses of the Higgs and Goldstone bosons are:

$$m_H^2(\xi) = 2\lambda(2\xi^2), \tag{39}$$

$$m^2_{\phi^0}(\xi) = \frac{g^2+g'^2}{4}\rho(2\xi^2), \tag{40}$$

$$m^2_{\phi^\pm}(\xi) = \frac{g^2}{4}\rho(2\xi^2), \tag{41}$$

being $2\xi^2=\frac{1}{\lambda}\Big[c^2+\mu^2-\frac{T^2}{4}\Big(\frac{g'^2}{4}+\frac{3g^2}{4}+2\lambda\Big)\Big]$. We note that the effective masses of the Goldstone bosons depend on the gauge parameter ρ and therefore the Goldstone bosons are not part of the physical particle spectrum. On the other hand, the squares of the effective masses of the non-physical gauge bosons are given by

$$m^2_{B_\nu}(\xi) = \frac{g'^2}{4}(2\xi^2), \tag{42}$$

$$m^2_{A^1_\nu}(\xi,\mu) = m^2_{A^2_\nu}(\xi,\mu) = \frac{g^2}{4}(2\xi^2)+q_\nu, \tag{43}$$

$$m^2_{A^3_\nu}(\xi) = \frac{g^2}{4}(2\xi^2). \tag{44}$$

Starting from (42), (43) and (44), we can write the squares of the effective masses of the physical gauge bosons as

$$\begin{aligned} m^2_{A_\nu}(\xi) &= \Big(\frac{g'^2}{g^4-g'^4}\Big)\Big(\frac{g}{e}\Big)^2[g^2 m^2_{B_\nu} - g'^2 m^2_{A^3_\nu}] \\ &= 0, \end{aligned} \tag{45}$$

$$\begin{aligned} m^2_{Z_\nu}(\xi) &= \Big(\frac{g^2}{g'^4-g^4}\Big)\Big(\frac{g'}{e}\Big)^2[g'^2 m^2_{B_\nu} - g^2 m^2_{A^3_\nu}] \\ &= \frac{g^2+g'^2}{4\lambda}\big[c^2+\mu^2-\frac{T^2}{4}\big(\frac{g'^2}{4}+\frac{3g^2}{4}+2\lambda\big)\big], \end{aligned} \tag{46}$$

$$\begin{aligned} m^2_{W^\pm_\nu}(\xi,\mu) &= m^2_{A^1_\nu}(\xi,\nu) = m^2_{A^2_\nu}(\xi,\mu) \\ &= \frac{g^2}{4\lambda}\big[c^2+\mu^2-\frac{T^2}{4}\big(\frac{g'^2}{4}+\frac{3g^2}{4}+2\lambda\big)\big]+q_\nu. \end{aligned} \tag{47}$$

The effective masses given by (46) and (47) allow to the physical masses of the electroweak gauge bosons taking $T=0$ and $\mu=0$. For the case $T\neq 0$ and $\mu=0$, these masses are in agreement with the effective masses given in [12]. We note that the effective mass of the W boson is a function of the amount q_ν, this implies that the effective mass of the spatial component of the W boson has a difference of $-\mu^2$ respect to its temporal component. This behavior is in agreement with the observation realized by Kapusta in [8] when he mentioned that the transverse W's have associated a chemical potential μ whereas the longitudinal W's does not have one. This strange behavior is contrary to what happens in the non-gauge theories where all three spin states of a massive spin-1 bosons have associated a common chemical potential [8].

For high temperatures above the T_c of the electroweak phase transition, i. e. $T\geq T_C$, we observe from (46) that the effective mass of the Z-gauge boson vanishes because $\xi^2=0$, while from (47) the effective mass for the W boson non-vanishes, and it takes the value $m_{W^\pm_\nu}(\xi,\mu)=q_\nu$.

4. Induction of a W boson condensate

In the effective Lagrangian density of the ESM, given by (1), we have introduced the abelian j_μ and non-abelian j^3_μ background charges with the purpose to have a vanishing electromagnetic charge density, and so, to preserve the thermodynamic equilibrium of the system. It is important to note that a non-abelian gauge theory with external charges requires a special quantization scheme as was studied in [13]. However we can introduce the non-abelian background charges j^1_μ and j^2_μ, in such a way that the thermodynamic equilibrium of the system is not affected. These non-abelian charges should be only dynamical, and satisfying the neutrality condition, with the purpose to preserve the gauge symmetry of the theory [13]. The fact of considering $j^1_\mu\neq 0$ and $j^2_\mu\neq 0$ implies from (19) and (20) that $\langle A^1_i\rangle_0\neq 0$ and $\langle A^2_i\rangle_0\neq 0$. We parameterize the non-vanishing vacuum average values of the spatial components of the A^1_ν and A^2_ν fields as

$$\langle A^1_i\rangle_0 = \xi_{A^1_i}(x), \tag{48}$$

$$\langle A^2_i\rangle_0 = \xi_{A^2_i}(x), \tag{49}$$

where they acquire non-homogeneous classical components. The non-homogeneity is shown by the dependence of $\xi_{A_i^1}(x)$ and $\xi_{A_i^2}(x)$ on the space-time, consequently the later means that the vacuum average value of the spatial components of the W boson fields are non-homogeneous, and it is given by

$$\langle W_i^{\pm}\rangle_0(x) = \xi_{W_i^{\pm}}(x) = \frac{1}{\sqrt{2}}\big(\xi_{A_i^1}(x) \mp i\xi_{A_i^2}(x)\big). \tag{50}$$

This fact implies that we have obtained a W boson condensation associated with the spatial component of the W boson for a system in which was not included the chemical potentials associated with the conserved leptonic and neutral weak currents. The background charges (19) and (20) are written as

$$gj_\nu^1 = -\mu^2\big[\langle A_0^1\rangle_0\delta_{\nu 0} - \langle A_\nu^1\rangle_0\big] = -\mu^2\big[\xi_{A_0^1}(x)\delta_{\nu 0} - \xi_{A_\nu^1}(x)\big], \tag{51}$$

$$gj_\nu^2 = -\mu^2\big[\langle A_0^2\rangle_0\delta_{\nu 0} - \langle A_\nu^2\rangle_0\big] = -\mu^2\big[\xi_{A_0^2}(x)\delta_{\nu 0} - \xi_{A_\nu^2}(x)\big]. \tag{52}$$

We observe that the time components of j_ν^1 and j_ν^2 vanish, while the spatial components of these charges do not. This fact means that the non-abelian background charges given by (51) and (52) are

$$gj_\nu^1 = \mu^2\xi_{A_\nu^1}(x), \tag{53}$$

$$gj_\nu^2 = \mu^2\xi_{A_\nu^2}(x), \tag{54}$$

where is clear that $\xi_{A_0^1}(x) = \xi_{A_0^2}(x) = 0$.

If we use the mean-field approximation in the equation of motion (15), modified by the introduction of the two news background charges, we obtain

$$\Big((\partial^2 g_\nu^\mu - \partial^\mu\partial_\nu) + \frac{g^2}{4}\big(2\xi_1^2 + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle\big) + q\Big)\langle A_\mu^1\rangle = -gj_\nu^1, \tag{55}$$

$$\Big((\partial^2 g_\nu^\mu - \partial^\mu\partial_\nu) + \frac{g^2}{4}\big(2\xi_1^2 + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle\big) + q\Big)\langle A_\mu^2\rangle = -gj_\nu^2, \tag{56}$$

where the square of the vacuum expectation value of the Higgs field ξ_1, for this case, is

$$\xi_1^2 = \xi^2 + \frac{1}{2\lambda}\Big(\frac{g^2}{4}(\xi_{W_\nu^+})^2 + \frac{g^2}{4}(\xi_{W_\nu^-})^2\Big), \tag{57}$$

being $(\xi_{W_\nu^+})$ and $(\xi_{W_\nu^-})$ the parameters of W boson condensation.

The effective masses of the A_ν^1 and A_ν^2 fields can be obtained from the equations of motion of these fields in the vacuum state. If we substitute (53) and (54) into (55) and (56), respectively, and remembering that $\langle A_\nu^{1,2}\rangle_0 = \xi_{A_\nu^{1,2}}(x)$, we obtain.

$$\Big((\partial^2 g_\nu^\mu - \partial^\mu\partial_\nu) + \frac{g^2}{4}(2\xi_1^2 + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle)\Big)\xi_{A_\mu^{1,2}}(x) = 0, \tag{58}$$

where is clear the cancelation of the term q. For an excited state it is also possible to find the effective masses of the non-abelian gauge fields by the consideration of a small variation of the vacuum state as $\langle A_\mu^{1,2}\rangle \to \xi_{A_\mu^{1,2}}(X) + \delta\xi_{A_\mu^{1,2}}(X)$. Applying this result into (55) and (56), we obtain

$$\Big((\partial^2 g_\nu^\mu - \partial^\mu\partial_\nu) + \frac{g^2}{4}(2\xi_1^2 + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle + q)\Big)\delta\xi_{A_\mu^{1,2}}(x) = 0, \tag{59}$$

due there not exist a W boson condensate in an excited state, i. e. $\xi_{A_\mu^{1,2}}(x) = 0$. So, the effective masses of the non-abelian gauge bosons are

$$M^2_{effA_\nu^{1,2}} = \begin{cases} \frac{g^2}{4}(2\xi_1^2 + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle + q), & \text{(Excited state)}, \\ \frac{g^2}{4}(2\xi_1^2 + \langle\psi_1^2\rangle + \langle\psi_2^2\rangle + \langle\psi_3^2\rangle + \langle\psi_4^2\rangle), & \text{(Vacuum state)}, \end{cases} \tag{60}$$

being clear that the term q vanish in the vacuum state of the A_ν^1 and A_ν^2 fields. This vacuum state corresponds to the phase of W boson condensation. In the condensation phase the effective mass of the spatial components of the W boson is equal to its temporal component. The difference among the effective masses of the spatial and temporal components of the W boson has vanished as a consequence of the introduction of the non-abelian background charges j_ν^1 and j_ν^2 given by (53) and (54), respectively.

5. Critical temperature of the W boson condensation

The critical temperature of the W boson condensation T_w can be obtained from (7) and (57), for the case in which $(\xi_{W_\nu^\pm})^2 = 0$. This critical temperature is given by

$$T_w^2 = \frac{4\Big(c^2 + \mu^2 - 2\lambda\xi_1^2(T_w)\Big)}{\frac{1}{4}g'^2 + \frac{3}{4}g^2 + 2\lambda}, \tag{61}$$

where ξ_1^2 is evaluated in $T = T_w$. The expression (61) is not useful to calculate T_w because $\xi_1^2(T_w)$ is unknown. To know T_w it is first necessary to calculate the thermodynamic potential Ω_f, using the mean-field approximation, in the high temperature limit.

To calculate Ω_f, by convenience, we initially calculate the thermodynamic potential Ω_0 that we obtain for the case in which the background charges j_μ and j_μ^3 are not included. The effective partition function Z_E^0 for this case is given by

$$\begin{aligned} Z_E^0 = &\left(det\Big[\frac{\partial^2}{\partial\tau^2} + \nabla^2 - m^2_{C^a}\Big]\right)^3\left(det\Big[\frac{\partial^2}{\partial\tau^2} + \nabla^2 - m^2_{C}\Big]\right) \\ &\times \int_{periodic} D[\psi_1]D[\psi_2]D[\psi_3]D[\psi_4]D[\delta\xi_{A_\nu^1}]D[\delta\xi_{A_\nu^2}]D[A_\nu^3]D[B^\nu] \\ &\times \int D[f^1]D[f^2]D[f^3]D[F]\delta(F^1)\delta(F^2)\delta(F^3)\delta(F)\, \exp[A], \end{aligned} \tag{62}$$

where the effective action A is

$$A = \int_0^\beta d\tau \int_V d^3x \left\{ \frac{1}{2}\psi_1\Big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - m_1^2(\xi_1)\Big)\psi_1 + \frac{1}{2}\psi_2\Big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - m_2^2(\xi_1)\Big)\psi_2 \right.$$
$$+\frac{1}{2}\psi_3\Big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - m_3^2(\xi_1)\Big)\psi_3 + \frac{1}{2}\psi_4\Big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - m_4^2(\xi_1)\Big)\psi_4$$
$$+\frac{1}{2}\xi_{A^{1\mu}}(x,\tau)\Big(\big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - \frac{g^2}{2}\xi_1^2\big)\delta_{\mu\nu} + \frac{1-\rho}{\rho}\partial_\mu\partial_\nu\Big)\xi_{A^{1\nu}}(x,\tau)$$
$$+\frac{1}{2}\xi_{A^{2\mu}}(x,\tau)\Big(\big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - \frac{g^2}{2}\xi_1^2\big)\delta_{\mu\nu} + \frac{1-\rho}{\rho}\partial_\mu\partial_\nu\Big)\xi_{A^{2\nu}}(x,\tau)$$
$$+\frac{1}{2}A^{3\mu}\Big(\big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - \frac{g^2}{2}\xi_1^2\big)\delta_{\mu\nu} + \frac{1-\rho}{\rho}\partial_\mu\partial_\nu\Big)A_3^\nu$$
$$+\frac{1}{2}B^{\mu}\Big(\big(\frac{\partial^2}{\partial\tau^2} + \nabla^2 - \frac{g'^2}{2}\xi_1^2\big)\delta_{\mu\nu} + \frac{1-\rho}{\rho}\partial_\mu\partial_\nu\Big)B^\nu$$
$$+\xi_1^2\big(\mu^2 + c^2 - \lambda\xi_1^2\big) + \frac{g^2}{4}\xi_1^2\Big[(\xi_{W_\nu^+})^2 + (\xi_{W_\nu^-})^2\Big]$$
$$\left. +\frac{1}{2}\mu^2\Big[(\xi_{W_\nu^+})^2 + (\xi_{W_\nu^-})^2\Big]\right\}. \tag{63}$$

After performing the functional integrals over $\ln Z_E^0$, using the high temperature limit, we obtain that the thermodynamic potential Ω_0 is

$$\Omega_0 = -\frac{\ln Z_E^0}{\beta V} = -\xi_1^2\Big[\mu^2 + c^2 - \Big(\frac{3g^2}{4} + \frac{g'^2}{4} + 2\lambda\Big)\frac{T^2}{4}\Big] - (2\mu^2 + 4c^2)\frac{T^2}{24}$$
$$+\frac{T^4}{24}\Big(\frac{3g^2}{4} + \frac{g'^2}{4} + 2\lambda\Big) - \frac{2\pi^2}{15}T^4$$
$$+\lambda\xi_1^4 - \frac{g^2}{4}\xi_1^2\Big((\xi_{W_\nu^+})^2 + (\xi_{W_\nu^-})^2\Big). \tag{64}$$

We observe that Ω_0 does not dependent from the gauge parameter ρ. For the case $\mu = 0$, we obtain that (64) is in agreement with the thermodynamic potential known in the literature [9].

Starting from (64) we find that the electromagnetic charge density σ_0 for the non-neutralized system is

$$\sigma_0 = -\frac{\partial\Omega_0}{\partial\mu} = \frac{\mu T^2}{6} + 2\mu\xi_1^2. \tag{65}$$

If we include the background charges j_μ and j_μ^3, the system has now a vanishing electromagnetic charge density. The electromagnetic charge density σ for the neutralized system is given by

$$\sigma = \frac{1}{\beta V}\frac{\partial \ln Z}{\partial\mu} = \frac{1}{\beta V}\frac{1}{Z}\frac{\partial Z}{\partial\mu}, \tag{66}$$

where Z is the partition function given by (34) modified by the inclusion of the non-abelian background charges j_μ^1 and j_μ^2. Using the mean-field approximation, in the high temperature limit, we obtain that

$$\sigma = \sigma_0 + \sigma_1, \tag{67}$$

where σ_0 is given by (65) and σ_1 is

$$\begin{aligned}\sigma_1 &= -\frac{1}{4}\frac{\partial}{\partial\mu}\langle \widetilde{F}^a_{\mu\nu}\widetilde{F}^{\mu\nu}_a\rangle + \frac{\partial}{\partial\mu}\langle g'B^0J_0 + gA^0_3J^3_0\rangle \\ &= \frac{3\mu T^2}{6} + \frac{\partial}{\partial\mu}\langle g'B^0J_0 + gA^0_3J^3_0\rangle.\end{aligned} \tag{68}$$

Because the system has been neutralized, then the electromagnetic charge density vanishes, i. e. $\sigma = \sigma_0 + \sigma_1 = 0$, and then

$$\langle g'B^0J_0 + gA^0_3J^3_0\rangle = -\frac{1}{3}\mu^2T^2 - \int 2\mu\xi_1^2\, d\mu + f(T), \tag{69}$$

where $f(T)$ is an arbitrary function over temperature, that we choose as zero. For $T = T_c$, we obtain that the background charges are

$$j_0\Big|_{T=T_c} = -\frac{\mu T_c^2}{12}, \tag{70}$$

$$j_0^3\Big|_{T=T_c} = -\frac{7\mu T_c^2}{12}. \tag{71}$$

Substituting (70) and (71) into (69), and assuming a non-vanishing chemical potential we obtain that

$$(g'\langle B^0\rangle + 7g\langle A^0_3\rangle) = 4\mu. \tag{72}$$

We observe, for $T = T_c$, that a possible solution of the last equation is $\langle B^0\rangle = \frac{\mu}{2g'}$ and $\langle A^0_3\rangle = \frac{\mu}{2g}$. This means that any value of μ satisfies $\sigma = 0$, and then μ can describes a phase transition.

The thermodynamic potential of the neutralized system can be written as

$$\Omega_f = \Omega_0 - \int \sigma_1\, d\mu - g\langle A^\nu_1 J^1_\nu + A^\nu_2 J^2_\nu\rangle. \tag{73}$$

With the goal to find $(\xi_{W^\pm_\nu})^2$, we calculate Ω_f near to T_w, and we obtain

$$\Omega_f = \Omega_0 - \frac{6\mu^2T^2}{24} - \langle g'B^0J_0 + gA^0_3J^3_0\rangle - \frac{\mu^2}{2}\Big[(\xi_{W^+_\nu})^2 + (\xi_{W^-_\nu})^2\Big]. \tag{74}$$

If we minimize (74) respect to $(\xi_W)^2$, we find that

$$\frac{\partial\Omega_f}{\partial(\xi_W)^2} = \frac{\partial\Omega_0}{\partial(\xi_W)^2} - \mu^2 = 0. \tag{75}$$

where we have taken in consideration that the quantity $\left\langle g'B^0 J_0 + gA_3^0 J_0^3 \right\rangle$, for $T \approx T_w$, does not depend over $(\xi_W)^2$. We have also considered that $(\xi_W)^2 = (\xi_{W_\nu^+})^2 = (\xi_{W_\nu^-})^2$ and we have used $\frac{\partial \xi_1^2}{\partial (\xi_W)^2} = \frac{g^2}{4\lambda}$. It is easy to probe that the equation (75) can be written as

$$\frac{\partial \Omega_f}{\partial (\xi_W)^2} = \frac{g^2}{4\lambda}[c^2 + \mu^2]\Big(1 - \frac{T^2}{T_c^2}\Big) + \frac{3g^4}{8\lambda}(\xi_W)^2 - \mu^2 = 0. \tag{76}$$

From this last equation, we can obtain

$$\begin{aligned}(\xi_W)^2 &= -\frac{2}{3g^2}\left[[c^2 + \mu^2]\Big(1 - \frac{T^2}{T_c^2}\Big) - \frac{4\lambda\mu^2}{g^2}\right] \\ &= -\frac{2}{3g^2}\left[c^2 + \mu^2 - \Big(\frac{g'^2}{4} + \frac{3g^2}{4} + 2\lambda\Big)\frac{T^2}{4} - \frac{4\lambda\mu^2}{g^2}\right],\end{aligned} \tag{77}$$

where we have substituted T_c by the value given by (8). The critical temperature of the W boson condensation T_w is obtained from (77) doing $(\xi_W)^2 = 0$ and is given by

$$T_w^2 = \frac{4(c^2 + \mu^2 - \frac{4\lambda\mu^2}{g^2})}{\frac{g'^2}{4} + \frac{3g^2}{4} + 2\lambda}. \tag{78}$$

Comparing (78) with (61), we observe that

$$2\lambda\xi_1^2(T_w) = 4\frac{\lambda\mu^2}{g^2}, \tag{79}$$

and it is possible to obtain from this last expression the known W condensation condition given by [5, 7, 8]

$$m_W^2(\xi_1) = \frac{g^2}{2}\xi_1^2(T_w) = \mu^2, \tag{80}$$

being this fact a good indicator of the theoretical consistency of the W boson condensation induced by the introduction of the background charges j_μ^1 and j_μ^2.

If the fermions are included in the ESM, considering the Yukawa terms in the Lagrangian, the square of the critical temperature of the electroweak phase transition T_c is given by

$$T_c^2 = \frac{4(c^2 + \mu^2)}{\frac{g'^2}{4} + \frac{3g^2}{4} + 2\lambda + \sum_{m=1}^{3}[Y_{dm}^2 + Y_{um}^2 + \frac{Y_{em}^2}{3}]}, \tag{81}$$

being the Y's the Yukawa coupling constants of the quarks and charged leptons. On the other hand, the square of the critical temperature of the W boson condensation, for this case, is

$$T_w^2 = \frac{4(c^2 + \mu^2 - \frac{4\lambda\mu^2}{g^2})}{\frac{g'^2}{4} + \frac{3g^2}{4} + 2\lambda + \sum_{m=1}^{3}[Y_{dm}^2 + Y_{um}^2 + \frac{Y_{em}^2}{3}]}. \tag{82}$$

We observe that the inclusion of the fermions in the system allows to low the value of the critical temperature of the W boson condensation. This fact is also valid for the case of the critical temperature of the electroweak phase transition.

6. Conclusions

In this paper we have shown a W boson condensation in the ESM induced by the inclusion of background charges in the thermodynamical system. The W boson condensation that we have presented here has an origin different from the one studied in references [5]-[8]. Specifically we have not included in the system the chemical potentials associated with the conserved leptonic and neutral weak currents. We have considered the ESM at finite temperature in presence of only a bosonic chemical potential μ associated with the conserved electromagnetic current. We have neutralized the thermal medium by the introduction of a background external charge which offsets the charge density of the scalar field. Particularly we have preserved the thermodynamic equilibrium of the system by the introduction of two charges j_ν and j_ν^3, which are associated with the $U(1)_Y$ gauge field and the third $SU(2)_L$ gauge field, respectively.

We have calculated the effective masses of the scalar and gauge bosons using the mean-field approximation in the high temperature limit. We have found that the effective mass of the spatial component of the W boson has a difference of $-\mu^2$ respect to its temporal component. By the inclusion in the system of the background charges j_ν^1 and j_ν^2 which are associated with the first and second gauge fields of the $SU(2)_L$ gauge group, we have obtained a W boson condensation associated to the spatial component of W boson. As a consequence of this condensate, the mentioned difference among the effective masses of the spatial and temporal components of the W boson has vanished. We have obtained the critical temperature of the W boson condensation as a function of μ. We have demonstrated that the W boson condensate is consistent with the usual condition of condensation $m_W^2 = \mu^2$, where m_W is the effective mass of the W boson.

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