Corrections to Massive Neutrino Masses, Caused by Vacuum Polarisation in Strong Coulomb Field of Daughter Nuclei in Weak Decays Of Heavy Ions

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Abstract: We calculate corrections to masses of massive neutrino mass–eigenstates, caused by vacuum polarization in the strong Coulomb fields of daughter heavy nuclei in the K–shell electron capture decays (EC) and positron (β^+) decays of highly ionized heavy ions, investigated experimentally at GSI in Darmstadt. Some applications of the obtained results are discussed.

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1. Introduction

Nowadays the existence of massive neutrinos, neutrino–flavour mixing and neutrino oscillations is well established experimentally and elaborated theoretically [1].

The K–shell electron capture (EC) decays and positron (β^+) decays of the H–like heavy ions [2]–[5] is a nice tool for the investigation of the hyperfine structure of highly

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ionized heavy ions [6]–[8]. In these decays massive neutrinos in the final state appear in a coherent superposition of the electron neutrino $|\nu_e\rangle = \sum_j U^*_{ej}|\nu_j\rangle$, where U^*_{ej} are elements of the unitary mixing matrix U and $|\nu_j\rangle$ are neutrino mass–eigenstates with masses m_j [1]. The reactions of the EC–decays and β^+–decays of the H–like heavy ions can be conventionally defined as follows

$$\begin{aligned} {}^A X^{(Z-1)+} &\to {}^A Y^{(Z-1)+} + \nu_e, \\ {}^A X^{(Z-1)+} &\to {}^A Y^{(Z-2)+} + e^+ + \nu_e, \end{aligned} \tag{1}$$

for example, ${}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + \nu_e$ and ${}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{57+} + e^+ + \nu_e$ and so on [2]–[5].

The description of the electron neutrino in terms of a coherent superposition of neutrino mass–eigenstates $|\nu_e\rangle = \sum_j U^*_{ej}|\nu_j\rangle$ implies that in the final state massive neutrinos move in the strong Coulomb field of the daughter ions ${}^A Y^{(Z-1)+}$ and ${}^A Y^{(Z-2)+}$. According to modern theory of weak interactions [1], neutrino mass–eigenstates can be virtually in the dissociated states $\nu_j \to \sum_\ell U_{j\ell}\ell^- W+$, where ℓ^- is the electron e^-, negatively charged muon μ^- or τ^-–lepton and W^+ is the W–boson of the Weinberg–Salam theory of electroweak interactions. The interaction of electrically charged particles with the Coulomb fields of the daughter ions ${}^A Y^{(Z-1)+}$ and ${}^A Y^{(Z-2)+}$ can be expressed in terms corrections to masses of neutrino mass–eigenstates. This effect is similar to vacuum polarization induced in a strong Coulomb field [9]–[11].

In this paper we calculate corrections to masses of massive neutrinos caused by strong Coulomb fields of highly charged daughter ions and discuss one of the possible applications of these results - the calculation of masses of neutrino mass–eigenstates, using the experimental data on the antineutrino oscillations by KamLAND [12] and experimental data on a time modulation of the EC–decay rates by GSI [3]–[5].

2. Corrections to Massive Neutrino Masses From Vacuum Polarisation in Strong Coulomb Field of Highly Charged Nuclei

A virtual dissociation of massive neutrinos into $\ell^- W^+$ pairs leads to self–energy corrections, which can be defined by Feynman diagrams shown in Fig. 1.

Since the W–boson is a very heavy particle and its mass is much greater than exchange energies, one can use the heavy–boson approximation replacing the propagator of the W–boson by the δ–function and shrinking the line of a virtual W–boson into a point. Since we investigate a dissociation of massive neutrinos into $\ell^- W^+$ pairs in strong Coulomb fields of highly charged daughter ions, in a heavy–boson approximation the analysis of an influence of a strong Coulomb field on massive neutrinos reduces to a research of virtual leptons ℓ^-, moving in a strong Coulomb field. This effect is similar to vacuum polarization caused by a strong attractive Coulomb field, leading to an induced electric charge [9, 10, 11]. In the case of massive neutrinos, vacuum polarization in strong Coulomb fields of highly charged heavy daughter ions, produced in weak decays of highly ionized heavy ions, leads to corrections to masses of massive neutrinos. Following [10, 11], we describe virtual

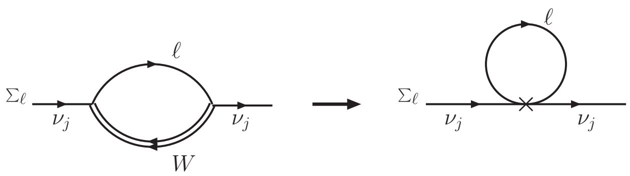

Fig. 1 Feynman diagrams, defining corrections to the mass of a massive neutrino in a strong Coulomb field of a nucleus with charge Ze.

negatively charged leptons ℓ^- by exact Green functions in a strong attractive Coulomb field.

For the calculation of the diagrams in Fig. 1 we use weak leptonic interaction [1]

$$\mathcal{L}_W(x) = -\frac{G_F}{\sqrt{2}}\sum_{j\ell}\sum_{\ell' j'} U_{j\ell}U^*_{\ell' j'}\,[\bar{\psi}_{\nu_j}(x)\gamma^\mu(1-\gamma^5)\psi_\ell(x)]\,[\bar{\psi}_{\ell'}(x)\gamma_\mu(1-\gamma^5)\psi_{\nu_{j'}}(x)], \quad (2)$$

defined by the W–boson exchange, where $x = (t, \vec{r}\,)$, G_F is the Fermi constant, $\psi_{\nu_j}(x)$ and $\psi_\ell(x)$ are operators of the neutrino ν_j and lepton fields $\ell = e^-$, μ^- and τ^-, respectively, and $U_{\ell j}$ are the elements of the unitary neutrino–flavour mixing matrix U [1]. In our analysis neutrinos $\nu_j\,(j = 1,2,3)$ are Dirac particles with masses $m_j\,(j = 1,2,3)$, respectively [1].

The self–energy correction for the neutrino mass–eigenstate ν_j due to weak leptonic interaction Eq.(2) is defined by [13]

$$-\,\delta\Sigma_{\nu_j} = \lim_{T\to\infty}\int\frac{d^4x}{T}\,\langle\nu_j(\vec{k}_j,\sigma_j)|\mathcal{L}_W(x)|\nu_j(\vec{k}_j,\sigma_j)\rangle. \quad (3)$$

Making Fierz transformation and calculating vacuum expectation values of field operators of negatively charged leptons [13] we get the following expression for the matrix element $\langle\nu_j(\vec{k}_j,\sigma_j)|\mathcal{L}_W(x)|\nu_j(\vec{k}_j,\sigma_j)\rangle$ of the $\nu_j \to \nu_j$ transition

$$\begin{aligned}&\langle\nu_j(\vec{k}_j,\sigma_j)|\mathcal{L}_W(x)|\nu_j(\vec{k}_j,\sigma_j)\rangle = \\ &= -\sqrt{2}\,G_F\sum_\ell U_{j\ell}U^*_{\ell j}\psi^\dagger_{\nu_j}(\vec{r},0)\psi_{\nu_j}(\vec{r},0)i\int_{-i\infty}^{+i\infty}\frac{dE}{2\pi}\mathrm{tr}\{\gamma^0 G_\ell(\vec{r},\vec{r};E)\},\end{aligned} \quad (4)$$

where $\psi_{\nu_j}(\vec{r},0)$ is a wave function of a massive neutrino and $G_\ell(\vec{r},\vec{r};E)$ is the energy–dependent Green function of a negatively charged lepton ℓ^- in a strong Coulomb field, produced by a positive electric charge Ze of a daughter ion [10, 11].

Substituting the matrix element Eq.(4) into the r.h.s. of Eq.(3) we arrive at the following expression for the self–energy correction

$$\delta\Sigma_{\nu_j} = \int d^3x\,\bar{\psi}_{\nu_j}(\vec{r},0)\delta m_j(r)\psi_{\nu_j}(\vec{r},0), \quad (5)$$

where $\delta m_j(r)$, the correction to mass of massive neutrino ν_j caused by vacuum polarization in a strong Coulomb field, is given by

$$\delta m_j(r) = \sum_\ell U_{j\ell}U^*_{\ell j}\,\mathcal{M}_\ell(r) \quad (6)$$

with $\mathcal{M}_\ell(r)$ defined by

$$\mathcal{M}_\ell(r) = i\,\sqrt{2}\,G_F \int_{-i\infty}^{+i\infty}\frac{dE}{2\pi}\mathrm{tr}\{\gamma^0 G_\ell(\vec{r},\vec{r};E)\}. \qquad (7)$$

Taking into account the results, obtained in [11], we get

$$\mathcal{M}_\ell(r) = \sqrt{2}\,G_F\,\frac{m_\ell}{\pi^2 r^2}\sum_{n=1}^{\infty} n\int_0^\infty\int_0^\infty dx\,dy\,e^{-2m_\ell r\sqrt{x^2+1}\coth y}\Big\{2Z\alpha\coth y\cos\Big(\frac{2Z\alpha xy}{\sqrt{x^2+1}}\Big)$$
$$\times\,\tilde{I}_{2\nu}\Big(\frac{2m_\ell r\sqrt{x^2+1}}{\sinh y}\Big) - \sin\Big(\frac{2Z\alpha xy}{\sqrt{x^2+1}}\Big)\Big[\frac{2m_\ell r x}{\sinh y}\tilde{I}_{2\nu+1}\Big(\frac{2m_\ell r\sqrt{x^2+1}}{\sinh y}\Big) + \frac{2\nu x}{\sqrt{x^2+1}}$$
$$\times\,\tilde{I}_{2\nu}\Big(\frac{2m_\ell r\sqrt{x^2+1}}{\sinh y}\Big)\Big]\Big\}, \qquad (8)$$

where $\nu = \sqrt{n^2-(Z\alpha)^2}$, $\tilde{I}_{2\nu+1}(z) = I_{2\nu+1}(z) - I_{2n+1}(z)$ and $\tilde{I}_{2\nu}(z) = I_{2\nu}(z) - I_{2n}(z)$ and $I_\mu(z)$ are modified Bessel functions [14], Using the integral representations for the Bessel functions we determine $\tilde{I}_{2\nu}(z)$ as

$$\tilde{I}_{\sqrt{n^2-(Z\alpha)^2}}(z) = I_{\sqrt{n^2-(Z\alpha)^2}}(z) - I_n(z) =$$
$$= \frac{2}{\pi}\int_0^\pi e^{\,z\cos\theta}\sin\Big((n+\sqrt{n^2-(Z\alpha)^2})\theta\Big)\sin\Big(\frac{(Z\alpha)^2}{n+\sqrt{n^2-(Z\alpha)^2}}\theta\Big)\,d\theta$$
$$-\frac{(-1)^n}{\pi}\,\sin\Big(\frac{\pi(Z\alpha)^2}{n+\sqrt{n^2-(Z\alpha)^2}}\Big)\int_0^\infty e^{-z\cosh\xi-\xi\sqrt{n^2-(Z\alpha)^2}}\,d\xi \sim O((Z\alpha)^2). \qquad (9)$$

Following [9, 11] we have renormalised the self–energy correction having subtracted the contribution of order of $O(Z\alpha)$. This is similar to renormalisation of a total induced vacuum–polarization "charge" in a strong attractive Coulomb field [9, 11]. This results in an induced vacuum polarisation leading to mass–corrections to massive neutrino masses of order of $O((Z\alpha)^3)$ [9, 11].

A total renormalised mass of neutrino mass–eigenstate ν_j is equal to $m_j(r) = m_j + \delta m_j(r)$, where m_j is a renormalised proper mass of massive neutrino ν_j and $\delta m_j(r)$ is a neutrino mass–correction, induced by vacuum polarisation in a strong Coulomb field. It vanishes at $Z\alpha \to 0$ as $O((Z\alpha)^3)$. As a result a total energy of massive neutrino ν_j, moving with a momentum $\vec{k}_j$ in a strong Coulomb field of highly charged daughter ions, produced in weak decays of highly ionized heavy ions, is $E_j(r) = \sqrt{\vec{k}_j^{\,2} + m_j^2(r)}$.

3. Numerical Values of Mass–Corrections of Massive Neutrinos, Caused by Coulomb Interaction

Using the definition of matrix elements $U_{\ell j}$ of mixing matrix U in terms of mixing angles [1] we obtain the following expressions for the corrections to massive neutrino masses

$$\delta m_1(r) = \cos^2\theta_{12}\mathcal{M}_{e^-}(r) + \sin^2\theta_{12}(\cos^2\theta_{23}\mathcal{M}_{\mu^-}(r) + \sin^2\theta_{23}\mathcal{M}_{\tau^-}(r)),$$
$$\delta m_2(r) = \sin^2\theta_{12}\mathcal{M}_{e^-}(r) + \cos^2\theta_{12}(\cos^2\theta_{23}\mathcal{M}_{\mu^-}(r) + \sin^2\theta_{23}\mathcal{M}_{\tau^-}(r)), \qquad (10)$$

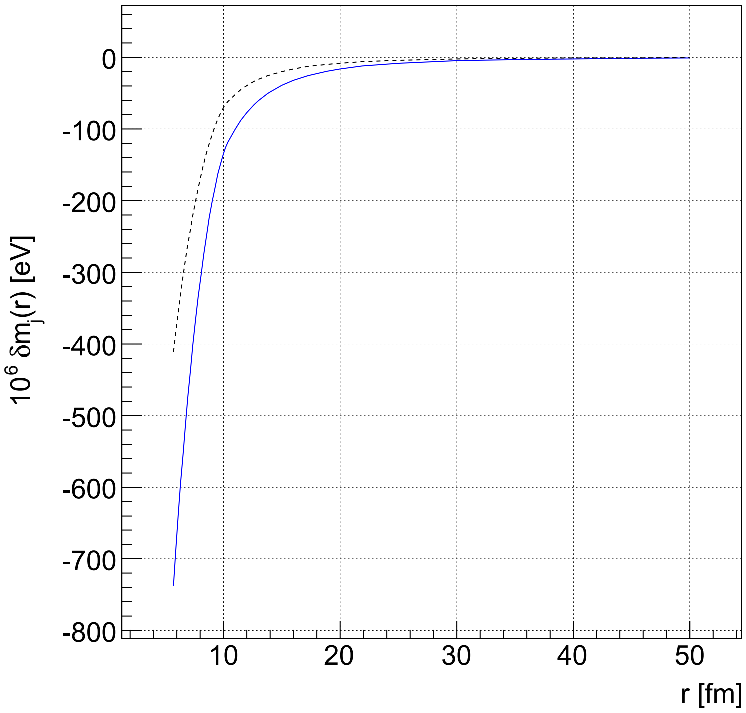

Fig. 2 The corrections to the neutrino masses, caused by a strong nuclear Coulomb field, where $\delta m_1(r)$ and $\delta m_2(r)$ are presented by the solid and dotted line, respectively.

where θ_{12} and θ_{23} are mixing angles [1]. The corrections to the neutrino masses Eq.(8) are defined for the mixing angle $\theta_{13} = 0$, which is very close to the experimental value, defined by upper limit $\sin^2\theta_{13} < 0.032$ [1], and usually used in theoretical analysis of neutrino oscillations [17]. For numerical estimates we set $\theta_{12} = 34^0$ and $\theta_{23} = 45^0$ [1] (see also [18]). The neutrino mass–corrections $\delta m_j(r)$, induced by strong Coulomb fields of highly charged daughter ions in the weak decays of the H–like heavy ions $^{140}\mathrm{Pr}^{58+}$, are shown in Fig. 2.

In weak decays of heavy nuclei neutrinos are emitted from the surface of nuclei at nuclear radius R [19, 15]. Hence, according to [19, 15], corrections to neutrino masses should be calculated at $r = R$. Setting $R = 1.1 \times A^{1/3}$ [20] (see also [7]) we obtain the corrections to neutrino masses $\delta m_1(R)$ and $\delta m_2(R)$ adduced in Table 1.

$^{A}X^{(Z-1)+}$	$\delta m_1(R) \times 10^4\,\mathrm{eV/c^2}$	$\delta m_2(R) \times 10^4\,\mathrm{eV/c^2}$
$^{122}\mathrm{I}^{52+}$	− 12.42	− 6.94
$^{140}\mathrm{Pr}^{58+}$	− 14.74	− 8.22
$^{142}\mathrm{Pm}^{60+}$	− 16.20	− 9.00

Table 1 Numerical values for neutrino mass–corrections, caused by vacuum polarisation in the strong Coulomb field of daughter ions, calculated for $R = 1.1 \times A^{1/3}$ [20] (see also [7]).

The mass–corrections are calculated for massive neutrinos produced in the EC–decays and β^+–decays of the H–like heavy ions

$$\begin{aligned} &{}^{122}\mathrm{I}^{52+} \to {}^{122}\mathrm{Te}^{52+} + \nu_e \quad , \quad {}^{122}\mathrm{I}^{52+} \to {}^{122}\mathrm{Te}^{51+} + e^+ + \nu_e, \\ &{}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + \nu_e \quad , \quad {}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{57+} + e^+ + \nu_e \\ &{}^{142}\mathrm{Pm}^{60+} \to {}^{142}\mathrm{Nd}^{60+} + \nu_e \quad , \quad {}^{142}\mathrm{Pm}^{60+} \to {}^{140}\mathrm{Nd}^{59+} + e^+ + \nu_e, \end{aligned} \qquad (11)$$

measured currently at GSI. The accuracy of the corrections to neutrino masses, given in Table 1, is of about 4 %. It is caused by the experimental uncertainties of the mixing angles $\theta_{12} = 33.9^{+2.4}_{-2.2}$ degrees and $\theta_{23} \le 45$ degrees [1]. The obtained results can be used for the analysis of neutrino energy spectra in weak decays of highly ionised heavy ions.

4. Neutrino Masses From GSI and KamLAND Experiments

One of the interesting applications of the obtained corrections to massive neutrino masses, caused by vacuum polarisation in strong Coulomb fields of highly charged daughter ions, is a calculation of neutrino masses by using the experimental data on a difference of squared values of neutrino masses, measured by KamLAND [12], and the experimental data on a time modulation of the rates of the number of daughter ions, produced in the EC–decays of the H–like heavy ions ${}^{142}\mathrm{Pm}^{60+}$, ${}^{140}\mathrm{Pr}^{58+}$ and ${}^{122}\mathrm{I}^{52+}$, measured at GSI [3]–[5].

Using the reactor antineutrinos $\tilde{\nu}_e$ the experimental group at KamLAND has measured the antineutrino energy spectrum accounting for the $\tilde{\nu}_e \leftrightarrow \tilde{\nu}_e$ oscillations defined by the probability

$$P_{\tilde{\nu}_e\leftrightarrow\tilde{\nu}_e}(E_{\tilde{\nu}_e}) = 1 - \sin^2(2\theta_{12})\,\sin^2\Big(\frac{\Delta m^2_{21}L}{4E_{\tilde{\nu}_e}}\Big), \qquad (12)$$

where $\Delta m^2_{21} = m^2_2 - m^2_1$ is determined by the proper neutrino masses, $L = 180\,\mathrm{km}$ is a distance between a source and a detector of antineutrinos and $E_{\tilde{\nu}_e}$ is an antineutrino energy. Antineutrinos have been detected by the reaction $\tilde{\nu}_e + p \to n + e^+$, having a threshold at $E^{(\mathrm{th})}_{\tilde{\nu}_e} = 1.8\,\mathrm{MeV}$. The experimental value of $\Delta m^2_{21} = m^2_2 - m^2_1$, deduced from the KamLAND experiment data on the antineutrino spectrum, which we call below as $(\Delta m^2_{21})_{\mathrm{KL}}$, is equal to $(\Delta m^2_{21})_{\mathrm{KL}} = 7.59(21)\times 10^{-5}\,\mathrm{eV}^2$ [12].

The experimental investigation of the K–shell electron capture (EC) decays of the H–like heavy ions ${}^{142}\mathrm{Pm}^{60+}$, ${}^{140}\mathrm{Pr}^{58+}$, and ${}^{122}\mathrm{I}^{52+}$

$${}^{142}\mathrm{Pm}^{60+} \to {}^{142}\mathrm{Nd}^{60+} + \nu_e\ ,\ {}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + \nu_e\ ,\ {}^{122}\mathrm{I}^{52+} \to {}^{122}\mathrm{Te}^{52+} + \nu_e \qquad (13)$$

has been recently carried out in the Experimental Storage Ring (ESR) at GSI in Darmstadt [3]–[5]. The measurements of the rates $dN^{EC}_d(t)/dt$ of the number N^{EC}_d of daughter ions ${}^{142}\mathrm{Nd}^{60+}$, ${}^{140}\mathrm{Ce}^{58+}$ and ${}^{122}\mathrm{Te}^{52+}$ showed time modulation of the exponential decay with periods and amplitudes

$$T_{EC} = \begin{cases} 7.10(22)\mathrm{s} \\ 7.06(8)\mathrm{s} \\ 6.11(3)\mathrm{s} \end{cases} \quad a_{EC} = \begin{cases} 0.23(4) & {}^{142}\mathrm{Pm}^{58+} \to {}^{142}\mathrm{Nd}^{60+} + \nu_e \\ 0.18(3) & {}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + \nu_e \\ 0.22(2) & {}^{122}\mathrm{I}^{52+} \to {}^{122}\mathrm{Te}^{52+} + \nu_e. \end{cases} \tag{14}$$

Since the rates of the number of daughter ions are defined by

$$\frac{dN^{EC}_d(t)}{dt} = \lambda_{EC}(t)\, N_m(t), \tag{15}$$

where $\lambda_{EC}(t)$ is the EC–decay rate and $N_m(t)$ is the number of mother ions, time modulation of $dN^{EC}_d(t)/dt$ implies a periodic time–dependence of the EC–decay rate $\lambda_{EC}(t)$ [3]–[5], which can be represented by the form

$$\lambda_{EC}(t) = \lambda_{EC}\,(1 + a_{EC}\cos(\omega_{EC}t + \phi_{EC})), \tag{16}$$

where a_{EC}, $T_{EC} = 2\pi/\omega_{EC}$ and ϕ_{EC} are an amplitude, a period and a phase of a time–dependent term [3]–[5]

As has been proposed in [21] (see also [8]), such a periodic time–dependence of the EC–decay rates can be explained by the differences of squared masses of the neutrino mass–eigenstates. A period of a time modulation T_{EC} has been obtained as

$$T_{EC} = \frac{4\pi\gamma M_m}{\Delta\tilde{m}^2_{21}}, \tag{17}$$

where M_m is a mass of a mother ion, $\gamma = 1.43$ is the Lorentz factor of H–like heavy ions moving in the ESR [3]. Then, $\Delta\tilde{m}^2_{21} = \tilde{m}^2_2 - \tilde{m}^2_1$ is a difference of squared neutrino masses $\tilde{m}_2$ and $\tilde{m}_1$, which can, in principle, include mass–corrections induced by strong Coulomb fields of highly charged daughter ions, i.e. $\tilde{m}_2 = m_2 + \delta m_2(R)$ and $\tilde{m}_1 = m_1 + \delta m_1(R)$.

Using experimental values of mother ion masses and experimental values of periods of time modulation Eq.(14) one can calculate $\Delta\tilde{m}^2_{21}$, which we call $(\Delta m^2_{21})_{\rm GSI}$. They are

$$(\Delta m^2_{21})_{\rm GSI} = \begin{cases} 2.20(7)\times 10^{-4}\,\mathrm{eV}^2 & {}^{142}\mathrm{Pm}^{60+} \to {}^{142}\mathrm{Nd}^{60+} + \nu_e \\ 2.18(3)\times 10^{-4}\,\mathrm{eV}^2 & {}^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + \nu_e \\ 2.19(1)\times 10^{-4}\,\mathrm{eV}^2 & {}^{122}\mathrm{I}^{52+} \to {}^{122}\mathrm{Te}^{52+} + \nu_e, \end{cases} \tag{18}$$

which can be approximated well by an averaged value $(\Delta m^2_{21})_{\rm GSI} = 2.19\times 10^{-4}\,\mathrm{eV}$. Since $M_m \simeq 931.494\,A$, where A is a mass number of a mother nucleus, periods of time modulation of the EC–decay rates in Eq.(17) can be defined by $T_{EC} = A/20\,\mathrm{s}$, where the coefficient 1/20 is calculated as $\kappa = 4\pi\gamma M_m/A(\Delta m^2_{21})_{\rm GSI} \simeq 1/20$. It is seen that the formula $T_{EC} = A/20\,\mathrm{s}$ fits well the experimental values of periods of time modulation.

The observed discrepancy between $(\Delta m^2_{21})_{\rm KL} = 7.59(21)\times 10^{-5}\,\mathrm{eV}^2$ and $(\Delta m^2_{21})_{\rm GSI} = 2.19\times 10^{-4}\,\mathrm{eV}$ can be really reconciled by taking into account corrections to neutrino masses, induced by vacuum polarisation in strong Coulomb fields of highly charged daughter ions.

Since in GSI experiments on the EC–decays ${}^{A}X^{(Z-1)+} \to {}^{A}Y^{(Z-1)+} + \nu_e$ massive neutrinos move in strong Coulomb fields of daughter ions ${}^{A}Y^{(Z-1)+}$, their masses should be corrected by vacuum polarisation induced by strong Coulomb fields. This means that $(\Delta m^2_{21})_{\rm GSI}$ should be taken in the form $(\Delta m^2_{21})_{\rm GSI} = (m_2 + \delta m_2(R))^2 - (m_1 + \delta m_1(R))^2$ as we have assumed above. In turn, in the KamLAND experiment a difference $(\Delta m^2_{21})_{\rm KL}$ is defined by the proper neutrino masses, i.e. $(\Delta m^2_{21})_{\rm KL} = m_2^2 - m_1^2$.

Combining these results together we arrive at a system of algebraical equations for m_1 and m_2

$$\left\{\begin{array}{rcl}(m_2 + \delta m_2(R))^2 - (m_1 + \delta m_1(R))^2 &=& (\Delta m^2_{21})_{\rm GSI},\\ m_2^2 - m_1^2 &=& (\Delta m^2_{21})_{\rm KL},\end{array}\right. \tag{19}$$

the solution of which for the experimental values of $(\Delta m^2_{21})_{\rm GSI}$ and $(\Delta m^2_{21})_{\rm KL}$ gives the following values of neutrino masses, averaged over GSI experimental data: $m_1 = 0.11429\,{\rm eV}$, $m_2 = 0.11463\,{\rm eV}$ and $m_3 = 0.12476\,{\rm eV}$.

The mass m_3 one can calculate using the experimental value $\Delta m^2_{32} = m_3^2 - m_2^2 = 2.40 \times 10^{-3}\,{\rm eV}^2$, deduced from experimental data on atmospheric neutrino oscillations [18] (see also [1]).

The neutrino masses $m_1 = 0.11429\,{\rm eV}$ and $m_2 = 0.11463\,{\rm eV}$ reproduce the experimental data by GSI with an accuracy of about 6 %. This agrees well with the accuracy of the calculation of neutrino mass–corrections in Table 1, which is of about 4 %.

The sum of the calculated neutrino masses $\sum_j m_j \simeq 0.35\,{\rm eV}$ satisfies well the cosmological constraint $\sum_j m_j < 2\,{\rm eV}$ [1].

Conclusion

We have shown that an interaction of virtually produced $\ell^- W^+$ pairs $\nu_j \to \sum_\ell U_{j\ell}\ell^- W^+$ of massive neutrinos ν_j with strong Coulomb fields of highly charged daughter ions, produced in weak decays of highly ionized heavy ions, can induce certain corrections to neutrino masses due to vacuum polarization. For the calculation of corrections to neutrino masses we have taken into account the contribution of the W^+–boson exchange only. The contribution of the Z–boson exchange is proportional to the constant $g_V = -0.040 \pm 0.015$ [1]. This means that the corrections to neutrino masses, caused by the Z–boson exchanges, are smaller compared with corrections, which can be caused by experimental uncertainties of the mixing angles $\theta_{12} = 33.9^{+2.4}_{-2.2}$ degrees and $\theta_{23} \le 45$ degrees [1]. As a result a total energy of massive neutrino ν_j, moving with a momentum $\vec{k}_j$ in a strong Coulomb field of highly charged daughter ions, produced in weak decays of highly ionized heavy ions, takes the form $E_j(r) = \sqrt{\vec{k}^{\,2}_j + m^2_j(r)}$, where $m_j(r) = m_j + \delta m_j(r)$ with a renormalised proper neutrino mass m_j and a correction to neutrino mass $\delta m_j(r)$, caused by vacuum polarization in a strong Coulomb field of daughter ions vanishing as $O((Z\alpha)^3)$

for $Z\alpha \to 0$.

Using corrections to neutrino masses, produced by vacuum polarization in strong Coulomb fields, the experimental data by KamLAND on the antineutrino oscillations and the experimental data by GSI on the periods of time modulation of the K–shell electron capture decay rates and following the hypothesis, that such a time modulation is a consequence of the interference of massive neutrinos, we have calculated the masses of massive neutrino mass–eigenstates: $m_1 = 0.11429\,\mathrm{eV}$, $m_2 = 0.11463\,\mathrm{eV}$ and $m_3 = 0.12476\,\mathrm{eV}$. For the calculation of $m_3 = 0.12476\,\mathrm{eV}$ we have used the experimental value $\Delta m^2_{32} = 2.40 \times 10^{-3}\,\mathrm{eV}^2$, deduced for experimental data on atmospheric neutrino oscillations [1, 18].

The obtained values of neutrino masses agree well with a cosmological constraint on the sum of neutrino masses $\sum_j m_j < 2\,\mathrm{eV}$ [1] and reproduce the experimental data by GSI and KamLAND with an accuracy of about 6 %.

The neutrino masses, which we have calculated above, taking into account corrections to neutrino masses, can be used for the analysis of weak decays of light ions and estimates of upper bounds on neutrinoless double–β^- decay rates [1].

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