

Some Bianchi Type I Cosmological Models of the Universe for Viscous Fluid Distribution in Lyra Geometry

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Abstract: Some Bianchi type I cosmological models of the universe with time dependent gauge function β for viscous fluid distribution within the framework of Lyra geometry are investigated in which the expansion is considered only in two dimensions i.e. one of the Hubble parameter ($H_1 = \frac{\dot{A}}{A}$) is zero. To get the deterministic solutions of Einstein's modified field equations, the viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density and the coefficient of shear viscosity is considered as constant in first case whereas in other case it is taken as proportional to scale of expansion in the model. It has been found that the displacement vector $\beta(t)$ behaves like cosmological term Λ in the normal gauge treatment and the solutions are consistent with the observations. Solution in absence of shear viscosity is also obtained. The displacement vector $\beta(t)$ affects entropy. Some physical and geometrical properties of the models are discussed.

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1. Introduction and Motivations

In Einstein's general relativity, the curvature of the space-time is influenced by matter, and it provides the geometrical description of matter. Einstein succeeded in geometrizing gravitation by expressing gravitational potential in terms of metric tensor. In general relativity, spatially homogeneous space-times either belong to Bianchi type or to Kantowski-Sachs models and interpreted as cosmological models [1]. Spatially homoge-

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neous and isotropic universes can be well described by Friedmann-Robertson-Walker [2, 3] (FRW) model. However, the FRW model has the disadvantage of being unstable near the singularity [4] and it fails to describe the early universe. Therefore spatially homogeneous and anisotropic Bianchi type-I models are undertaken to understand the universe at its early stage of evolution. The idea of geometrizing gravitation by Einstein in 1917 inspired Weyl [5] to develop a theory to geometrize gravitation and electromagnetism. Weyl [5] suggested the first so-called unified field theory based on a generalization of Riemannian geometry. With its backdrop, it would seem more appropriate to call Weyl's theory a geometrized theory of gravitation and electromagnetism, instead a unified field theory. It is not clear as to what extent the two fields have been unified, even though they acquire (different) geometrical significance in the same geometry. The theory was never taken seriously in as much as it was based on the concept of non-integrability of length transfer; and, as pointed out by Einstein, this implies that spectral frequencies of atoms depend on their past histories and therefore have no absolute significance. Nevertheless, Weyl's geometry provides an interesting example of non-Riemannian connections, and Folland [6] has given a global formulation of Weyl manifolds clarifying considerably many of Weyl's basic ideas thereby.

In 1951, Lyra [7] proposed a modification of Riemannian geometry by introducing a gauge function into the structure-less manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's geometry. But in Lyra's geometry, unlike that of Weyl, the connection is metric preserving as in Riemannian; in other words, length transfers are integrable. Lyra also introduced the notion of a gauge and in the "normal" gauge the curvature scalar is identical to that of Weyl. In consecutive investigations Sen and co-worker [8, 9] proposed a new scalar-tensor theory of gravitation and constructed an analogue of the Einstein field equations based on Lyra's geometry. It is, thus, possible [8] to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl's "unified" field theory, however, without the inconvenience of non-integrability length transfer. Halford [10] has pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. It is shown by Halford [11] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects within observational limits as the Einstein's general theory of relativity. Several authors Sen and Vanstone [12], Bhamra [13], Karade and Borikar [14], Kalyanshetti and Wagmode [15], Reddy and Innaiah [16], Beesham [17], Reddy and Venkateswarlu [18], Soleng [19], studied cosmological models based on Lyra's manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one for convenience and there is no *a priori* reason for it. Beesham [20] considered Friedmann-Robertson-Walker (FRW) models with time dependent displacement field. Singh and co-workers [21]–[25] studied Bianchi-type I, III, Kantowaski-Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Robertson-Walker models with constant

deceleration parameter in Einstein's theory with cosmological term and in the cosmological theory based on Lyra's geometry. Soleng [19] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology [26]– [28] or contain a special vacuum field, which together with the gauge vector term, may be considered as a cosmological term. In the latter case the solutions are equal to the general relativistic cosmologies with a cosmological term.

Most studies in cosmology involve a perfect fluid. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyze dissipative effects in cosmology. Further, there are several processes which are expected to give rise to viscous effect. These are the decoupling of neutrinos during the radiation era and the recombination era [29], decay of massive super string modes into massless modes [30], gravitational string production [31, 32] and particle creation effect in grand unification era [33]. It is known that the introduction of bulk viscosity can avoid the big bang singularity. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [34] for a review on cosmological models with bulk viscosity). A uniform cosmological model filled with fluid which possesses pressure and second (bulk) viscosity was developed by Murphy [35]. The solutions that he found exhibit an interesting feature that the big bang type singularity appears in the infinite past.

Recently, Pradhan et al. [36], Casama et al. [37], Rahaman et al. [38], Bali and Chandnani [39], Kumar and Singh [40], Singh [41], Rao, Vinutha and Santhi [42] and Pradhan [43] have studied cosmological models based on Lyra's geometry in various contexts. With these motivations, in this paper, we have obtained exact solutions of Einstein's modified field equations for viscous fluid distribution in Bianchi type-I homogeneous space-time within the frame work of Lyra's geometry for time varying displacement vector $\beta(t)$. This paper is organized as follows. In Section 1 the motivation for the present work is discussed. The metric and the field equations are presented in Section 2, in Section 3 are the solutions of field equations. The Subsection 3.1 describes the solution of Case I where $\eta = \text{constant}$ and also deals with some physical and geometrical properties of the model. The Subsection 3.2 deals with the solution of Case II where $\eta = b\theta$, where b is an arbitrary constant and the physical and geometrical aspects of the model are also described. In Section 4 the solution of field equations in absence of shear viscosity is described. Finally, discussion and concluding remarks are given in Section 5.

2. The Metric and Field Equations

We consider the Bianchi type-I metric in the form

$$ds^2 = -dt^2 + dx^2 + B^2 dy^2 + C^2 dz^2, \quad (1)$$

where the metric potentials B and C are functions of t alone. This metric depicts the case when one of the Hubble parameters (here $H_1 = \frac{\dot{A}}{A}$) is zero, i.e. the expansion is only in two directions. The kinematic parameters are then related as $\theta = -3\sigma_1^1$. This condition leads to the metric (1).

The energy-momentum tensor for a viscous fluid distribution is given by Landau and Lifshitz [44]

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \eta(v_{i; \cdot}^j + v_{\cdot; i}^j + v^j v^l v_{i;l} + v_i v^l v_{;l}^j) - \left(\xi - \frac{2}{3}\eta \right) v_{;l}^l (g_i^j + v_i v^j). \quad (2)$$

Here ρ , p , η and ξ are energy density, isotropic pressure, the coefficient of shear viscosity and bulk viscous coefficient respectively and v^i is the flow vector satisfying the relation

$$g_{ij} v^i v^j = -1. \quad (3)$$

The semicolon (;) indicates covariant differentiation. We choose the coordinates to be comoving, so that $v^1 = v^2 = v^3 = 0$ and $v^4 = \frac{1}{A}$.

The field equations, in normal gauge for Lyra's manifold, obtained by Sen [8] as

$$R_i^j - \frac{1}{2} g_i^j R + \frac{3}{2} \phi_i \phi^j - \frac{3}{4} g_i^j \phi_k \phi^k = -8\pi T_i^j, \quad (4)$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = (0, 0, 0, \beta(t)), \quad (5)$$

where other symbols have their usual meaning as in Riemannian geometry.

For the line-element (1), the field Eq. (4) with Eqs. (2) and (5) lead to the following system of equations

$$\left[\frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} \right] + \frac{3}{4}\beta^2 = -8\pi \left[p - \left(\xi - \frac{2}{3}\eta \right) v_{;l}^l \right], \quad (6)$$

$$\frac{\ddot{C}}{C} + \frac{3}{4}\beta^2 = -8\pi \left[p - 2\eta \frac{\dot{B}}{B} - \left(\xi - \frac{2}{3}\eta \right) v_{;l}^l \right], \quad (7)$$

$$\frac{\ddot{B}}{B} + \frac{3}{4}\beta^2 = -8\pi \left[p - 2\eta \frac{\dot{C}}{AC} - \left(\xi - \frac{2}{3}\eta \right) v_{;l}^l \right], \quad (8)$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = 8\pi\rho. \quad (9)$$

Here, and also in the following expressions a dot indicates ordinary differentiation with respect to t .

The energy conservation equation $T^i_{i;j} = 0$ leads to

$$\dot{\rho} + (\rho + \bar{p}) \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0, \quad (10)$$

where \bar{p} is the effective pressure given by

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta \right) v^i_{;i} \quad (11)$$

and

$$(R^j_i - \frac{1}{2}g^j_i R)_{;j} + \frac{3}{2}(\phi_i \phi^j)_{;j} - \frac{3}{4}(g^j_i \phi_k \phi^k)_{;j} = 0. \quad (12)$$

Equation (12) leads to

$$\begin{aligned} \frac{3}{2}\phi_i \left[\frac{\partial \phi^j}{\partial x^j} + \phi^l \Gamma^j_{lj} \right] + \frac{3}{2}\phi^j \left[\frac{\partial \phi_i}{\partial x^j} - \phi_l \Gamma^l_{ij} \right] - \frac{3}{4}g^j_i \phi_k \left[\frac{\partial \phi^k}{\partial x^j} + \phi^l \Gamma^k_{lj} \right] - \\ \frac{3}{4}g^j_i \phi^k \left[\frac{\partial \phi_k}{\partial x^j} - \phi_l \Gamma^l_{kj} \right] = 0. \end{aligned} \quad (13)$$

Equation (13) is identically satisfied for $i = 1, 2, 3$. For $i = 4$, Eq. (13) reduces to

$$\frac{3}{2}\beta \dot{\beta} + \frac{3}{2}\beta^2 \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0. \quad (14)$$

3. Solutions of the Field Equations

Equations (6)-(9) are four independent equations in seven unknowns B , C , p , ρ , η , ξ and β . For complete solutions of equations (6)-(9), we need three extra conditions. The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations.

From Eqs. (6) - (8), we obtain

$$\frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC} = -16\pi\eta \frac{\dot{B}}{B}, \quad (15)$$

and

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} = -16\pi\eta \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right). \quad (16)$$

Eq. (16) on integration leads to

$$C^2 \frac{d}{dt} \left(\frac{B}{C} \right) = L e^{-16\pi\eta t}, \quad (17)$$

where L is an integrating constant. Setting $BC = \mu$ and $\frac{B}{C} = \nu$ in Eqs. (15) - (17) lead to

$$\ddot{\mu} + 16\pi\eta \dot{\mu} = 0, \quad (18)$$

and

$$\frac{\dot{\nu}}{\nu} = \frac{L}{\mu} e^{-16\pi\eta t}. \quad (19)$$

Eq. (18) on integration leads to

$$\dot{\mu} = M e^{-16\pi\eta}, \quad (20)$$

where M is an integrating constant.

Here we consider two cases:

3.1 Case I: Let $\eta = \text{constant} = a$ (say)

In this case Eq. (20) on integration gives

$$\mu = N - \frac{M}{16\pi a} e^{-16\pi a t}, \quad (21)$$

where N is a constant of integration.

Equation (19) on integration leads to

$$\nu = \alpha (16\pi a N - M e^{-16\pi a t})^{L/M}, \quad (22)$$

where α is an integrating constant.

Now we set

$$e^{-16\pi a t} = \cos 2\sqrt{16\pi a \tau}, \quad (23)$$

$$\alpha = \frac{1}{(16\pi a)^{L/M}}, \quad (24)$$

and

$$N = \frac{M}{16\pi a}. \quad (25)$$

Using the above Eqs. (23) - (25) in Eqs. (21) and (22), we obtain

$$\mu = 2M \left(\frac{\sin \sqrt{16\pi a \tau}}{\sqrt{16\pi a}} \right)^2, \quad (26)$$

and

$$\nu = (2M)^{L/M} \left(\frac{\sin \sqrt{16\pi a \tau}}{\sqrt{16\pi a}} \right)^{2L/M}. \quad (27)$$

From Eqs. (26) and (27), we obtain

$$B^2 = \mu\nu = (2M)^{1+(L/M)} \left(\frac{\sin \sqrt{16\pi a \tau}}{\sqrt{16\pi a}} \right)^{2+(2L/M)} \quad (28)$$

$$C^2 = \frac{\mu}{\nu} = (2M)^{1-(L/M)} \left(\frac{\sin \sqrt{16\pi a \tau}}{\sqrt{16\pi a}} \right)^{2-(2L/M)}. \quad (29)$$

Hence the metric (1) reduces to the form

$$\begin{aligned}
 ds^2 = & -4 \left(\frac{\tan 2\sqrt{16\pi a}\tau}{\sqrt{16\pi a}} \right) d\tau^2 + dx^2 \\
 & + (2M)^{1+(L/M)} \left(\frac{\sin \sqrt{16\pi a}\tau}{\sqrt{16\pi a}} \right)^{2+(2L/M)} dy^2 \\
 & + (2M)^{1-(L/M)} \left(\frac{\sin \sqrt{16\pi a}\tau}{\sqrt{16\pi a}} \right)^{2-(2L/M)} dz^2.
 \end{aligned} \tag{30}$$

After suitable transformation of coordinates metric (30) reduces to the form

$$\begin{aligned}
 ds^2 = & -4 \left(\frac{\tan 2kT}{k} \right) dT^2 + dX^2 \\
 & + \left(\frac{\sin kT}{k} \right)^{2+(2L/M)} dY^2 + \left(\frac{\sin kT}{k} \right)^{2-(2L/M)} dZ^2,
 \end{aligned} \tag{31}$$

where $k = \sqrt{16\pi a}$.

The pressure and density for the model (31) are given by

$$\begin{aligned}
 8\pi p = & \frac{1}{4M^2} \left(\frac{k}{\sin kT} \right)^4 \left[M(M-L) \sin^2 kT \cos 2kT \right. \\
 & \left. - \frac{(L+M)}{4} \cos 2kT \{ (L+M) \cos 2kT - 2M \} \right. \\
 & \left. + 16\pi M^2 \left(\xi - \frac{2}{3}a \right) \cos 2kT \left(\frac{\sin kT}{k} \right)^2 \right] - \frac{3}{4}\beta^2,
 \end{aligned} \tag{32}$$

$$8\pi\rho = \frac{1}{4M^2} \left(\frac{k}{\sin kT} \right)^4 \left[\frac{(M^2 - L^2)}{4} \cos^2 2kT \right] + \frac{3}{4}\beta^2. \tag{33}$$

From Eq. (14) we have

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2 \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0,$$

which gives either $\beta = 0$ or $\dot{\beta} + \beta \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0$.

Therefore

$$\frac{\dot{\beta}}{\beta} = - \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right), \tag{34}$$

which reduces to

$$\frac{\dot{\beta}}{\beta} = -2k \cot kT. \tag{35}$$

Integrating above Eq. (35), we obtain

$$\beta = \frac{1}{\sin^2 kT}. \tag{36}$$

For the specification of ξ , we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \quad (37)$$

where $\gamma(0 \leq \gamma \leq 1)$ is a constant. Thus, given $\xi(t)$ we can solve for the cosmological parameters. In most of the investigation involving bulk viscosity it is assumed to be a simple power function of the energy density [45]– [49]

$$\xi(t) = \xi_0\rho^n, \quad (38)$$

where ξ_0 and n are constants. For small density, n may even be equal to unity as used in Murphy's work [35] for simplicity. If $n = 1$, (38) may correspond to a radiative fluid [49]. Near the big bang, $0 \leq n \leq \frac{1}{2}$ is a more appropriate assumption [50] to obtain realistic models.

On using (38) in (32), we obtain

$$\begin{aligned} 8\pi p = & \frac{1}{4M^2} \left(\frac{k}{\sin kT} \right)^4 \left[M(M-L) \sin^2 kT \cos 2kT \right. \\ & \left. - \frac{(L+M)}{4} \cos 2kT \{ (L+M) \cos 2kT - 2M \} \right. \\ & \left. + 16\pi M^2 \left(\xi_0\rho^n - \frac{2}{3}a \right) \cos 2kT \left(\frac{\sin kT}{k} \right)^2 \right] - \frac{3}{4}\beta^2. \end{aligned} \quad (39)$$

For simplicity and realistic models of physical importance, we consider the following two cases ($n = 0, 1$):

3.1.1 Model I: Solution for $n = 0$

When $n = 0$, (38) reduces to $\xi = \xi_0 = \text{constant}$. Using (33) and (37) in Eq. (39) leads to

$$\begin{aligned} 64\pi M^2(1 + \gamma)\rho = & \left(\frac{k}{\sin kT} \right)^4 \cos 2kT \left[M^2 - L^2 + 2(L^2 + M^2) \sin^2 kT \right. \\ & \left. + 32\pi M^2 \left(\xi_0 - \frac{2}{3}a \right) \left(\frac{\sin kT}{k} \right)^2 \right]. \end{aligned} \quad (40)$$

3.1.2 Model II: Solution for $n = 1$

When $n = 1$, (38) reduces to $\xi = \xi_0\rho$. Using (33) and (37) in Eq. (39) leads to

$$\begin{aligned} 16\pi\rho = & \frac{k^4 \cos 2kT}{M^2[2(1 + \gamma) \sin^2 kT - k^2\xi_0 \cos 2kT] \sin^2 kT} \\ & \times \left[\frac{M^2 - L^2}{2} + (L^2 + M^2) \sin^2 kT - \frac{8\pi a}{3} \cos 2kT \left(\frac{k}{\sin kT} \right)^2 \right]. \end{aligned} \quad (41)$$

Some Geometric and Physical Aspects of the Models

With regard to the kinematic properties of the velocity vector v^i in the metric model (31), a straight forward calculation leads to the following expressions for the scalar of the expansion θ , shear scalar σ^2 , deceleration parameter q , and proper volume V^3 of the fluid.

$$\theta = \frac{1}{2} \cos 2kT \left(\frac{k}{\sin kT} \right)^2, \quad (42)$$

$$\sigma^2 = \frac{1}{48} \left(1 + \frac{3L^2}{M^2} \right) \cos^2 2kT \left(\frac{k}{\sin kT} \right)^4, \quad (43)$$

$$q = -\frac{\ddot{V}/V}{\dot{V}^2/V^2} = \frac{1}{2}(3 \tan^2 kT - 1), \quad (44)$$

$$V^3 = \sqrt{-g} = \frac{\sin^2 kT}{k^2} \quad (45)$$

The expressions for $\frac{\sigma}{\theta}$ is found to be

$$\frac{\sigma}{\theta} = \frac{1}{2\sqrt{3}} \left(1 + \frac{3L^2}{M^2} \right)^{\frac{1}{2}} = \text{constant}. \quad (46)$$

The rotation ω is identically zero.

The non-vanishing components of conformal curvature tensor are obtained as

$$C_{12}^{12} = \frac{\cos 2kT}{48M^2} \left(\frac{k}{\sin kT} \right)^4 [M^2 - L^2 - 2L(3M - L) \sin^2 kT], \quad (47)$$

$$C_{13}^{13} = \frac{\cos 2kT}{48M^2} \left(\frac{k}{\sin kT} \right)^4 [M^2 - L^2 + 2L(3M + L) \sin^2 kT], \quad (48)$$

$$C_{14}^{14} = \frac{\cos 2kT}{24M^2} \left(\frac{k}{\sin kT} \right)^4 [L^2 - M^2 - 2L^2 \sin^2 kT]. \quad (49)$$

The rate of expansion H_i (Hubble parameters) in the direction of X, Y, Z are given by

$$H_1 = 0, \quad (50)$$

$$H_2 = \frac{(M + L)}{4MT^2}, \quad (51)$$

$$H_3 = \frac{(M - L)}{4MT^2}. \quad (52)$$

Now since

$$\int_{t_0}^t \frac{dt}{V(t)} = k^{\frac{2}{3}} \int_{t_0}^t \sin^{-\frac{2}{3}kT} dt, \quad (53)$$

which is a convergent integral, so the particle horizon exists.

The models represent shearing, non-rotating and Petrov type I non-degenerate in general, in which the flow is geodetic. However, if $L = 0$ then space-time reduces to Petrov

type D. The model starts expanding at $T \geq 0$ but the initial expansion is slow. When T is closer to $\pi/2k$, it has stiff rise in the expansion and then decreases. This shows the case of $T = 0$ or $T = \pi/k$. The large values of θ near $T = \pi/2k$ is reflection of trigonometric property. But expansion remains finite. As T increases the proper volume also increases. It is observed from Eq. (44) that $q < 0$ when $\tan kT < \frac{1}{\sqrt{3}}$ which implies an accelerating model of the universe. Recent observations of type Ia supernovae [51, 52] reveal that the present universe is in accelerating phase and deceleration parameter lies somewhere in the range $-1 < q \leq 0$. It follows that our models of the universe are consistent with recent observations. From Eq. (46), it can be observed that shear is proportional to scalar of expansion θ in the models and the models do not approach isotropy.

Halford [10] has pointed out that the displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. From Eq. (36), it is observed that the displacement vector $\beta(t)$ is a decreasing function of time which is corroborated with Halford as well as with the recent observations [51, 52] leading to the conclusion that $\Lambda(t)$ is a decreasing function of t .

3.2 Case II: Let $\eta = b\theta$, where b is an arbitrary constant

In this case Eq. (18) reduces to

$$\mu\ddot{\mu} + 16\pi b\dot{\mu}^2 = 0, \quad (54)$$

which on integration leads to

$$\mu = [(1 + 16\pi b)(k_1 t + k_2)]^{1/(1+16\pi b)}, \quad (55)$$

where k_1 and k_2 are constants of integration.

Equation (16) reduces to

$$\mu \frac{d}{dt} \left(\frac{\dot{\nu}}{\nu} \right) = -(1 + 16\pi b) \frac{\dot{\mu}}{\mu}, \quad (56)$$

which on integration leads to

$$\nu = k_3 (k_1 t + k_2)^{k_4 / \{k_1(1+16\pi b)\}}, \quad (57)$$

where k_3 and k_4 are constants of integration.

From Eqs. (55) and (57), we obtain

$$B^2 = \mu\nu = k_3 k_5 (k_1 t + k_2)^{k_6(k_1+k_4)}, \quad (58)$$

$$C^2 = \frac{\mu}{\nu} = \frac{k_5}{k_3} (k_1 t + k_2)^{k_6(k_1-k_4)}, \quad (59)$$

where

$$k_5 = (1 + 16\pi b)^{1/(1+16\pi b)},$$

$$k_6 = \frac{1}{k_1(1 + 16\pi b)}.$$

Hence the metric (1) reduces to

$$ds^2 = -dt^2 + dx^2 + k_3 k_5 (k_1 t + k_2)^{k_6(k_1+k_4)} dy^2 + \frac{k_5}{k_3} (k_1 t + k_2)^{k_6(k_1-k_4)} dz^2. \quad (60)$$

After suitable transformation of coordinates, the metric (60) takes the form

$$ds^2 = -\frac{1}{k_1^2} dT^2 + dX^2 + T^{k_6(k_1+k_4)} dY^2 + T^{k_6(k_1-k_4)} dZ^2. \quad (61)$$

The pressure and density for the model (61) are given by

$$8\pi p = -\frac{k_1^2 k_6}{4T^2} (k_1 + k_4)(k_1 k_6 + k_4 k_6 - 2) + \frac{8\pi k_1^2 k_6}{T} \times \left[\frac{b k_1 k_6}{3T} (k_1 - k_4) + \xi \right] - \frac{k_2}{2k_5^2 T^{2k_1 k_6}} - \frac{3}{4} \beta^2, \quad (62)$$

$$8\pi \rho = \frac{k_1^2 k_6^2 (k_1^2 - k_4^2)}{4T^2} + \frac{3}{4} \beta^2. \quad (63)$$

In this case using the values of B and C in Eq. (34), we obtain

$$\frac{\dot{\beta}}{\beta} = -\frac{k_1 k_6}{T}, \quad (64)$$

which on integration gives

$$\beta = \frac{1}{T^{k_1 k_6}}. \quad (65)$$

On using (38) in (62), we obtain

$$8\pi p = -\frac{k_1^2 k_6}{4T^2} (k_1 + k_4)(k_1 k_6 + k_4 k_6 - 2) + \frac{8\pi k_1^2 k_6}{T} \times \left[\frac{b k_1 k_6}{3T} (k_1 - k_4) + \xi_0 \rho^n \right] - \frac{k_2}{2k_5^2 T^{2k_1 k_6}} - \frac{3}{4} \beta^2, \quad (66)$$

3.2.1 Model I: Solution for $n = 0$

When $n = 0$, (38) reduces to $\xi = \xi_0 = \text{constant}$. Using (63) and (37) in Eq. (66) leads to

$$8\pi(1 + \gamma)\rho = \frac{k_1^2 k_6}{2T^2} (k_1 + k_4)(1 - k_1 k_6) + \frac{8\pi k_1^2 k_6}{T} \times \left[\frac{b k_1 k_6}{3T} (k_1 - k_4) + \xi_0 \right] - \frac{k_2}{2k_5^2 T^{2k_1 k_6}}. \quad (67)$$

3.2.2 Model II: Solution for $n = 1$

When $n = 1$, (38) reduces to $\xi = \xi_0\rho$. Using (63) and (37) in Eq. (66) leads to

$$8\pi \left[1 + \gamma - \frac{k_1^2 k_6 \xi_0}{T} \right] \rho = \frac{k_1^2 k_6^2}{6T^2} \\ \times [3(k_1 + k_4)(1 - k_4 k_6) + 16\pi b k_1(k_1 - 3k_4)] - \frac{k_2}{2k_5^2 T^{2k_1 k_6}}. \quad (68)$$

Some Geometric and Physical Aspects of the Models

The expressions for the scalar of the expansion θ , shear scalar σ^2 , deceleration parameter q , and proper volume V^3 for the model (61) are given by

$$\theta = \frac{k_1^2 k_6}{T}, \quad (69)$$

$$\sigma^2 = \frac{k_1^2 k_6^2 (k_1^2 + k_2^2)}{2T^2}. \quad (70)$$

The expressions for $\frac{\sigma}{\theta}$ and $\frac{\rho}{\theta^2}$ are found to be

$$\frac{\sigma}{\theta} = \frac{(k_1^2 + 3k_4^2)^{1/2}}{2\sqrt{3}k_1} = \text{constant}, \quad (71)$$

$$\frac{\rho}{\theta^2} = \frac{1}{32\pi k_1^4 k_6^2} [(k_1^2 - k_2^2)k_1^2 k_6 + 3T^{2(1-k_1 k_6)}]. \quad (72)$$

$$q = -\frac{(k_1 k_6 - 3)}{k_1 k_6}. \quad (73)$$

$$V^3 = T^{k_1 k_6} \quad (74)$$

The rotation ω is identically zero.

The non-vanishing components of conformal curvature tensor are obtained as

$$C_{12}^{12} = \frac{k_1^2 k_6}{12T^2} [k_1 - 3k_4 + k_4 k_6 (3k_1 - k_4)], \quad (75)$$

$$C_{13}^{13} = \frac{k_1^2 k_6}{12T^2} [k_1 + 3k_4 - k_4 k_6 (3k_1 + k_4)], \quad (76)$$

$$C_{14}^{14} = \frac{k_1^2 k_6}{6T^2} (k_4^2 k_6 - k_1) \quad (77)$$

The models represent an expanding, shearing but non-rotating universe in general. The models explode with a big bang at $T = 0$ and the expansion in the models stops at $T = \infty$. When $k_1 = 0$ then $\theta = 0$, which implies that $\eta = 0$. Therefore, viscosity is due to expansion in the model. We take $k_1 \neq 0$. The space-time is Petrov type I non-degenerate. However, if $k_4 = 0$, the space-time reduces to Petrov type ID. For large values of T , the space-time is conformally flat. Since $\frac{\sigma}{\theta} = \text{constant}$, hence the models do

not approach isotropy at all times.

The rate of expansion H_i (Hubble parameters) in the direction of X, Y, Z are given by

$$H_1 = 0, \quad (78)$$

$$H_2 = \frac{k_1 k_6 (k_1 + k_4)}{2T}, \quad (79)$$

$$H_3 = \frac{k_1 k_6 (k_1 - k_4)}{2T}. \quad (80)$$

As T increases the proper volume also increases. It is observed from Eq. (73) that $q < 0$ when $k_1 k_6 < 3$ which implies an accelerating model of the universe. Recent observations of type Ia supernovae [51, 52] reveal that the present universe is in accelerating phase and deceleration parameter lies somewhere in the range $-1 < q \leq 0$. It follows that our models of the universe are consistent with recent observations. From Eq. (71), it can be observed that shear is proportional to scalar of expansion θ in the models and the models do not approach isotropy.

From Eq. (65), it is observed that the displacement vector $\beta(t)$ is a decreasing function of time which is corroborated with Halford [10] as well as with the recent observations [51, 52] leading to the conclusion that $\Lambda(t)$ is a decreasing function of t .

4. Solution in Absence of Shear Viscosity

When $\eta \rightarrow 0$, then the metric (31) leads to

$$ds^2 = -16T^2 dT^2 + dX^2 + T^{(2+\frac{2L}{M})} dY^2 + T^{(2-\frac{2L}{M})} dZ^2. \quad (81)$$

The pressure and the density for the model (81) are given by

$$8\pi p = \frac{1}{16M^2 T^4} [64\pi M^2 \xi T^2 + M^2 - L^2] - \frac{3}{4}\beta^2, \quad (82)$$

$$8\pi\rho = \frac{(M^2 - L^2)}{16M^2 T^4} + \frac{3}{4}\beta^2. \quad (83)$$

In this case Eq. (34) reduces to

$$\frac{\dot{\beta}}{\beta} = -\frac{1}{2T}, \quad (84)$$

which on integration gives

$$\beta = \frac{1}{T^{\frac{1}{2}}}. \quad (85)$$

4.1 Model I: Solution for $n = 0$

When $n = 0$, (38) reduces to $\xi = \xi_0 = \text{constant}$. Hence in this case Eq. (82), with the use of (37) and (83), leads to

$$8\pi(1 + \gamma)\rho = \frac{1}{8M^2 T^4} [32\pi M^2 \xi_0 T^2 + M^2 - L^2]. \quad (86)$$

4.2 Model II: Solution for $n = 1$

When $n = 1$, (38) reduces to $\xi = \xi_0\rho$. Hence in this case Eq. (82), with the use of (37) and (83), leads to

$$[2(1 + \gamma)T^2 - \xi_0]\rho = \frac{M^2 - L^2}{32\pi M^2 T^2}. \quad (87)$$

Some Geometric and Physical Aspects of the Models

The expressions for the scalar of the expansion θ , shear scalar σ^2 , deceleration parameter q , and proper volume V^3 for the model (81) are given

$$\theta = \frac{1}{2T^2}, \quad (88)$$

$$\sigma^2 = \frac{(M^2 + 3L^2)}{48M^2 T^4}, \quad (89)$$

$$q = \frac{1}{2}, \quad (90)$$

$$V^3 = T^2. \quad (91)$$

The expressions for $\frac{\sigma}{\theta}$ and $\frac{\rho}{\theta^2}$ are found to be

$$\frac{\sigma}{\theta} = \frac{1}{2\sqrt{3}M}(M^2 + 3L^2)^{\frac{1}{2}}, \quad (92)$$

$$\frac{\rho}{\theta^2} = \frac{1}{32\pi M^2}[M^2 - L^2 + 12M^2 T^3]. \quad (93)$$

The rotation ω is identically zero.

The non-vanishing components of conformal curvature tensor are obtained as

$$C_{12}^{12} = C_{13}^{13} = \frac{(M^2 - L^2)}{48M^2 T^4}, \quad (94)$$

$$C_{14}^{14} = \frac{(L^2 - M^2)}{24M^2 T^4}, \quad (95)$$

The models represent an expanding, shearing but non-rotating universe in general. The models explode with a big bang at $T = 0$ and the expansion in the models stops at $T = \infty$. Since $\frac{\sigma}{\theta} = \text{constant}$, hence the models do not approach isotropy at all times. As T increases the proper volume also increases.

5. Discussion and Concluding Remarks

In this paper, we have presented a new class of exact solutions of Einstein's field equations for Bianchi type-I space-time with bulk viscous fluid distribution within the framework of Lyra's geometry for time dependent displacement field. Generally, the models represent shearing, non-rotating and Petrov type D universe in which the flow vector is geodetic.

In all these models, we observe that they do not approach isotropy for large values of time.

It is possible to discuss entropy in our universe. In thermodynamics the expression for entropy is given by

$$TdS = d(\rho V^3) + \bar{p}(dV^3), \quad (96)$$

where $V^3 = BC$ is the proper volume in our case. To solve the entropy problem of the standard model, it is necessary to treat $dS > 0$ for at least a part of evolution of the universe. Hence Eq. (96) reduces to

$$TdS = \dot{\rho} + (\rho + \bar{p}) \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) > 0. \quad (97)$$

The conservation equation $T_{i;j}^j = 0$ for (1) leads to

$$\dot{\rho} + (\rho + \bar{p}) \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2 \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0. \quad (98)$$

Therefore, Eqs. (97) and (98) lead to

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2 \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) < 0. \quad (99)$$

which gives to $\beta < 0$. Thus, the displacement vector $\beta(t)$ affects entropy because for entropy $dS > 0$ leads to $\beta(t) < 0$.

It is observed that the displacement vector $\beta(t)$ coincides with the nature of the cosmological constant Λ which has been supported by the work of several authors as discussed in the physical behaviour of the model in Sections 3 and 4. In recent time Λ -term has attracted theoreticians and observers for many a reason. The nontrivial role of the vacuum in the early universe generates a Λ -term that leads to inflationary phase. Observationally, this term provides an additional parameter to accommodate conflicting data on the values of the Hubble constant, the deceleration parameter, the density parameter and the age of the universe (for example, see Refs. [53] and [54]). In recent past there is an upsurge of interest in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology [55, 56, 57]. Therefore the study of cosmological models in Lyra's geometry may be relevant for inflationary models. There seems a good possibility of Lyra's geometry to provide a theoretical foundation for relativistic gravitation, astrophysics and cosmology. However, the importance of Lyra's geometry for astrophysical bodies is still an open question.

The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. The effect is clearly visible on the p effective (see details in previous sections). We also observe that Murphy's conclusion [35]

about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid, in general, is not true. The results obtained by Myung and Cho [58] also show that, it is, in general, not valid, since for some cases big bang singularity occurs in finite past.

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References

- [1] A. K. Raychudhary, *Theoretical Cosmology*, Oxford Universe Press, London (1979).
- [2] R. B. Partridge and D. T. Wilkinson, *Phys. Rev. Lett.* **18**, 557 (1967).
- [3] J. Ehlers, P. Geren and R. K. Sachs, *J. Math. Phys.* **9**, 1344 (1968).
- [4] E. M. Lifshitz and I. M. Khalatnikov, *Adv. Phys.* **12**, 185 (1963).
- [5] H. Weyl, *Sitzungsber. Preuss. Akad. Wiss. Berlin*, 465 (1918).
- [6] G. Folland, *J. Diff. Geom.* **4**, 145 (1970).
- [7] G. Lyra, *Math. Z.* **54**, 52 (1951).
- [8] D. K. Sen, *Z. Phys.* **149**, 311 (1957).
- [9] D. K. Sen and K. A. Dunn, *J. Math. Phys.* **12**, 578 (1971).
- [10] W. D. Halford, *Austr. J. Phys.* **23**, 863 (1970).
- [11] W. D. Halford, *J. Math. Phys.* **13**, 1399 (1972).
- [12] D. K. Sen and J. R. Vanstone, *J. Math. Phys.* **13**, 990 (1972).
- [13] K. S. Bhamra, *Austr. J. Phys.* **27**, 541 (1974).
- [14] T. M. Karade and S. M. Borikar, *Gen. Rel. Gravit.* **9**, 431 (1978).
- [15] S. B. Kalyanshetti and B. B. Waghmode, *Gen. Rel. Gravit.* **14**, 823 (1982).
- [16] D. R. K. Reddy and P. Innaiah, *Astrophys. Space Sci.* **123**, 49 (1986).
- [17] A. Beesham, *Astrophys. Space Sci.* **127**, 189 (1986).
- [18] D. R. K. Reddy and R. Venkateswarlu, *Astrophys. Space Sci.* **136**, 191 (1987).
- [19] H. H. Soleng, *Gen. Rel. Gravit.* **19**, 1213 (1987).
- [20] A. Beesham, *Austr. J. Phys.* **41**, 833 (1988).
- [21] T. Singh and G. P. Singh, *J. Math. Phys.* **32**, 2456 (1991).

- [22] T. Singh and G. P. Singh, *Il. Nuovo Cimento B* **106**, 617 (1991).
- [23] T. Singh and G. P. Singh, *Int. J. Theor. Phys.* **31**, 1433 (1992).
- [24] T. Singh, and G. P. Singh, *Fortschr. Phys.* **41**, 737 (1993).
- [25] G. P. Singh and K. Desikan, *Pramana-journal of physics*, **49**, 205 (1997).
- [26] F. Hoyle, *Monthly Notices Roy. Astron. Soc.* **108**, 252 (1948).
- [27] F. Hoyle and J. V. Narlikar, *Proc. Roy. Soc. London Ser. A*, **273**, 1 (1963).
- [28] F. Hoyle and J. V. Narlikar, *Proc. Roy. Soc. London Ser.A*, **282**, 1 (1964).
- [29] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison - Wesley, U S A (1990).
- [30] S. Myung and B. M. Cho, *Mod. Phys. Lett. A* **1**, 37 (1986).
- [31] N. Turok, *Phys. Rev. Lett.* **60**, 549 (1988).
- [32] J. D. Barrow, *Nucl. Phys. B* **310**, 243 (1988).
- [33] C. Wolf, *S.-Afr. Tydskr.* **14**, 68 (1991).
- [34] Ø. Grøn, *Astrophys. Space Sci.* **173**, 191 (1990).
- [35] G. L. Murphy, *Phys. Rev. D* **8**, 4231 (1973).
- [36] A. Pradhan, I. Aotemshi and G. P. Singh, *Astrophys. Space Sci.* **288**, 315 (2003).
A. Pradhan and A. K. Vishwakarma, *J. Geom. Phys.* **49**, 332 (2004).
A. Pradhan, L. Yadav and A. K. Yadav, *Astrophys. Space Sci.* **299**, 31 (2005).
A. Pradhan, V. Rai and S. Otarod, *Fizika B*, **15**, 23 (2006).
A. Pradhan, K. K. Rai and A. K. Yadav, *Braz. J. Phys.* **37**, 1084 (2007).
A. Pradhan and Priya Mathur, arXiv:0806.4815 [gr-qc] (2008).
- [37] R. Casama, C. Melo and B. Pimentel, *Astrophys. Space Sci.* **305**, 125 (2006).
- [38] F. Rahaman, B. Bhui and G. Bag, *Astrophys. Space Sci.* **295**, 507 (2005).
F. Rahaman, S. Das, N. Begum, M. Hossain, *Astrophys. Space Sci.* **295**, 507 (2005).
- [39] R. Bali and N. K. Chandnani, *J. Math. Phys.* **49**, 032502 (2008).
- [40] S. Kumar and C. P. Singh, *Int. Mod. Phys. A* **23**, 813 (2008).
- [41] J. K. Singh, *Astrophys. Space Sci.* **314**, 361 (2008).
- [42] V. U. M. Rao, T. Vinutha and M. V. Santhi, *Astrophys. Space Sci.* **314**, 213 (2008).
- [43] A. Pradhan, *Jour. Math. Phys.* **50**, 022501 (2009).
- [44] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Oxford: Pergamon, 1937).
- [45] R. Maartens, *Class. Quant. Grav.* **12**, (1995) 1455.
- [46] D. Pavon, J. Bafaluy and D. Jou, *Class. Quant. Grav.* **8**, (1991) 357.
- [47] N. O. Santos, R. S. Dias and A. Benerjee, *J. Math. Phys.* **26**, 878 (1985).
- [48] W. Zimdahl, *Phys. Rev. D* **53**, 5483 (1996).
- [49] S. Weinberg, *Gravitation and Cosmology*, Wiley, New York (1972), p. 57.
- [50] U. A. Belinskii and I.M. Khalatnikov, *Sov. Phys. JETP* **42**, 205 (1976).

- [51] S. Perlmutter et al., *Astrophys. J.* **483**, 565 (1997).
S. Perlmutter et al., *Nature* **391**, 51 (1998).
S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999).
- [52] A. G. Reiss et al., *Astron. J.* **116**, 1009 (1998).
A. G. Reiss et al., *Astron. J.* **607**, 665 (2004).
- [53] J. Gunn and B. M. Tinsley, *Nature*, **257**, 454 (1975).
- [54] E. J. Wampler and W. L. Burke, in *New Ideas in Astronomy*, Eds. F. Bertola, J. W. Sulentix and B. F. Madora (Cambridge University Press, 1988), p. 317.
- [55] G. F. R. Ellis, *Standard and Inflationary Cosmologies*, Preprint SISSA, Trieste, 176/90/A (1990).
- [56] D. La and P. J. Steinhardt, *Phys. Rev. Lett.* **62**, 376 (1989).
- [57] J. D. Barrow, *Phys. Lett. B* **235**, 40 (1990).
- [58] S. Myung and B. M. Cho, *Mod. Phys. Lett. A* **1** 37 (1986).