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Neutrino Oscillation Probability from Tri-Bimaximality due to Planck Scale Effects

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Abstract: Current neutrino experimental data on neutrino mixing are well describes by Tri-bi-maximal mixing, which is predicts $sin^2\theta_{12} = 1/3$, zero U_{e3} and $\theta_{23} = 45^o$. We consider the planck scale operator on neutrino mixing. We assume that the neutrino masses and mixing arise through physics at a scale intermediate between planck scale and the electroweak braking scale. We also assume, that just above the electroweak breaking scale neutrino mass are nearly degenerate and the mixing is tri-bi-maximal. Quantum gravity (Planck scale) effects lead to an effective $SU(2)_L \times U(1)$ invariant dimension-5 Lagrangian symmetry involving Standard Model. On electroweak symmetry breaking, this operator gives rise to correction to the neutrino masses and mixings these additional terms can be considered as perturbation to the tri-bimaximal neutrino mass matrix. We compute the deviation of the three mixing angles and oscillation probability. We find that the only large change in solar mixing angle and % change in maximum $P_{\mu e}$ is about 10%.

Keywords: Quantum Gravity; Neutrino Oscillation Probability; Planck Scale Effects;Tri-bi-maximal Mixing
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1. Introduction

Recent advance in neutrino physics observation mainly of astrophysical observation suggested the existence of tiny neutrino mass. The experiments and observation has shown evidences for neutrino oscillation. The solar neutrino deficit has been observed [1,2,3,4], the atmospheric neutrino anomaly has been found [5,6,7], and currently almost confirmed by KamLAND [8], and hence indicate that neutrino massive and there is mixing in lepton sector, this indicate to imagine that there occurs CP violation in lepton sector. Several

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physicist has considered whether we can see CP violation effect in lepton sector through long baseline neutrino oscillation experiments. The neutrino oscillation probabilities in general depend on six parameters two independent mass squared difference Δ_{21} and Δ_{31}, there mixing angle, θ_{12}, θ_{13}., θ_{23}.and one CP violating phase δ. There are two large mixing angle θ_{12}, θ_{23} and one small (θ_{13}), and two mass square difference $\Delta_{ij} = m_j^2 - m_i^2$, with m_{ij} the neutrino masses.

Where

$$\Delta_{21} = \Delta_{solar}, \tag{1}$$

$$\Delta_{31} = \Delta_{atm}. \tag{2}$$

The angle θ_{12} and θ_{23} represent the neutrino mixing angles corresponding to solar and atmospheric neutrino oscillation. Much progress has been made towards determining the values of the three mixing angle. In this paper we discuss the effect of Planck's scale on neutrino mixing and neutrino oscillation probability.

2. Neutrino Mixing Angle and Mass Squared Differences due to Planck Scale Effects

To calculate the effects of perturbation on neutrino observables. The calculation developed in an earlier paper [12]. A natural assumption is that unperturbed (0^{th} order mass matrix) M is given by

$$\mathbf{M} = U^* diag(M_i) U^\dagger, \tag{3}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i , the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U_{e3}. We adopt the usual parametrization.

$$\frac{|U_{e2}|}{|U_{e1}|} = tan\theta_{12}, \tag{4}$$

$$\frac{|U_{\mu 3}|}{|U_{\tau 3}|} = tan\theta_{23}, \tag{5}$$

$$|U_{e3}| = sin\theta_{13}. \tag{6}$$

In term of the above mixing angles, the mixing matrix is

$$U = diag(e^{if1}, e^{if2}, e^{if3}) R(\theta_{23}) \Delta R(\theta_{13}) \Delta^* R(\theta_{12}) diag(e^{ia1}, e^{ia2}, 1). \tag{7}$$

The matrix $\Delta = diag(e^{\frac{1\delta}{2}}, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation $a1$ and $a2$ are the so called Majoring phase, which effects the neutrino less double beta decay. $f1$, $f2$ and $f3$ are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [12]

$$U^{'} = U(1 + i\delta\theta),$$

$$= \begin{pmatrix} U_{e1} \ U_{e2} \ U_{e3} \\ U_{\mu 1} \ U_{\mu 2} \ U_{\mu 3} \\ U_{\tau 1} \ U_{\tau 2} \ U_{\tau 3} \end{pmatrix} + i \begin{pmatrix} U_{e2}\delta\theta^*_{12} + U_{e3}\delta\theta^*_{23,} \ U_{e1}\delta\theta_{12} + U_{e3}\delta\theta^*_{23}, \ U_{e1}\delta\theta_{13} + U_{e3}\delta\theta^*_{23} \\ U_{\mu 2}\delta\theta^*_{12} + U_{\mu 3}\delta\theta^*_{23,} \ U_{\mu 1}\delta\theta_{12} + U_{\mu 3}\delta\theta^*_{23}, \ U_{\mu 1}\delta\theta_{13} + U_{\mu 3}\delta\theta^*_{23} \\ U_{\tau 2}\delta\theta^*_{12} + U_{\tau 3}\delta\theta^*_{23}, \ U_{\tau 1}\delta\theta_{12} + U_{\tau 3}\delta\theta^*_{23}, \ U_{\tau 1}\delta\theta_{13} + U_{\tau 3}\delta\theta^*_{23} \end{pmatrix}. \tag{8}$$

Where $\delta\theta$ is a hermition matrix that is first order in μ[12,13]. The first order mass square difference $\Delta M^2_{ij} = M^2_i - M^2_j$,get modified [12,13] as

$$\Delta M^{'2}_{ij} = \Delta M^2_{ij} + 2(M_i Re(m_{ii}) - M_j Re(m_{jj}), \tag{9}$$

where

$$m = \mu U^t \lambda U,$$

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} eV.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$[12], is given by

$$\delta\theta_{ij} = \frac{iRe(m_{jj})(M_i + M_j) - Im(m_{jj})(M_i - M_j)}{\Delta M^{'2}_{ij}}. \tag{10}$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance. Using Eq(8), we can calculate neutrino mixing angle due to Planck scale effects,

$$\frac{|U^{'}_{e2}|}{|U^{'}_{e1}|} = tan\theta^{'}_{12}, \tag{11}$$

$$\frac{|U^{'}_{\mu 3}|}{|U^{'}_{\tau 3}|} = tan\theta^{'}_{23}, \tag{12}$$

$$|U^{'}_{e3}| = sin\theta.^{'}_{13} \tag{13}$$

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that $U^{'}_{e1}$ and $U^{'}_{e2}$ are much larger than $U^{'}_{e3}$, $U^{'}_{\mu 3}$ and $U^{'}_{\tau 3}$. Hence we can expect much larger change in θ_{12} compared to θ_{13} and θ_{23}. As one can see from the above expression of mixing angle due to Planck scale effects, depends on new contribution of mixing $U^{'} = U(1 + i\delta\theta)$. We assume that, just above the electroweak breaking scale, the neutrino masses are nearly degenerate and the mixing are Tri-bimaximal, with the value of the mixing angle as $\theta_{12} = 35^o$, $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. Taking the common degenerate neutrino mass to be 2 eV, which is the upper limit coming from tritium beta decay [9]. We compute the modified mixing angles using Eqs (11)-(13). We have taken $\Delta_{31} = 0.002eV^2$[10] and $\Delta_{21} = 0.00008eV^2$[11]. For simplicity we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. Since we have set the $\theta_{13} = 0$, the Dirac phase δ drops out of the zeroth order mixing angle. Next section , we discuss the neutrino oscillation probability under Planck scale effects

3. Neutrino Oscillation Probability Under Planck Scale Effects

The flux of solar neutrino observed by the Homestake detector was on third of that predicted by Standard Solar Model (SSM). The phenomenon of neutrino oscillation can be used to explain neutrino deficit. suppose an electron neutrino is produced at $t = 0$. A set of neutrino mass eigen state at $t = 0$ as

$$|\nu(t=0) > |\nu_e >= cos\theta_{12}|\nu_1(0) > +sin\theta_{12}|\nu_2(0) > . \tag{14}$$

After time t it becomes

$$|\nu(t=t) > |\nu_\mu >= cos\theta_{12}e^{-iE_1t}|\nu_1(0) > +sin\theta_{12}e^{-iE_2t}|\nu_2(0) > . \tag{15}$$

Then the oscillation probability becomes

$$P(\nu_e \rightarrow \nu_\mu) = sin^2 2\theta_{12} sin^2\left(\frac{1.27\Delta_{21}L}{E}\right), \tag{16}$$

and the survival probability

$$P(\nu_e \rightarrow \nu_e) = 1 - sin^2 2\theta_{12} sin^2\left(\frac{1.27\Delta_{21}L}{E}\right). \tag{17}$$

In the above two equation units of $\Delta_{21} = m_2^2 - m_1^2$ is ev^2,L (baseline length) is in meter and E is neutrino energy in MeV. For a maximum oscillation case the phase term in eq(16), $\left(\frac{1.27\Delta_{21}L}{E}\right)$ equal to $\frac{\pi}{2}$, then oscillation probability only depend on θ_{12}

$$P(\nu_e \rightarrow \nu_\mu) = sin^2 2\theta_{12}. \tag{18}$$

The oscillation probability due to Planck scale effects is

$$P(\nu_e \rightarrow \nu_\mu) = sin^2 2\theta_{12}^{'}, \tag{19}$$

In the above Eq(19), $\theta_{12}^{'}$ is the mixing angle due to Planck scale effects.

4. Numerical Results

We assume that, just above the electroweak breaking scale, the neutrino masses are nearly degenerate and the mixing are Teri-bi maximal, with the value of the mixing angle as $\theta_{12} = 35^o$, $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. Taking the common degenerate neutrino mass to be 2 eV, which is the upper limit coming from tritium beta decay [9]. We compute the modified mixing angles using Eqs (11)-(13). We have taken $\Delta_{31} = 0.002eV^2$[10] and $\Delta_{21} = 0.00008eV^2$[11]. For simplicity we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. Since we have set the $\theta_{13} = 0$, the Dirac phase δ drops out of the zeroth order mixing angle. We compute the modified mixing angles as function of a_1 and a_2 using Eq(11). In table 1, we list the modified neutrino mixing angle $\theta_{12}^{'}$ and maximum $P(\nu_e \rightarrow \nu_\mu)$ oscillation probability for some sample of a_1 and a_2. From Table 1, we see that planck scale effects change the θ_{12} from the Tri-bimaximal value of $\theta_{12} = 35^o$to a value close the the best fit value of the data [15,16]. The Planck scale effects give rise the correction to neutrino mass matrix on electroweak symmetry breaking. It is imperative to cheack that these correction do not spoil the good agreement between the experiments fits and the predection of the tri-bimaximal mixing scenorio. It is expected that dynamics at a higher scale generates the neutrino mass matrix, which will eventually provides the presently observed neutrino mass and mixing. In an attractive scenario, the neutrino mixing pattern generated by high scale dynamics is predicted to be tri-bimaximal. However the solar neutrino data show that the mixing angle θ_{12} is substantially less than 35^o. It is argued in the literature that renormalization group evolution effects from the higher scale to electroweak scale, can bring down the value of θ_{12} from 35^o to a value which is within the experimentally acceptable range. However, for a large range of neutrino parameters, the renormalization group evolution leads to negligible change in the neutrino mass matrix. Then it become imperative to search for such alternate mechanism for which the necessary reduction in θ_{12} can be achieved.

5. Conclusions

In this paper, we studied, how Planck scale effects the mixing and oscillation probability. The effective dimension-5 operator from Planck scale [12], leads to correction in neutrino mass matrix at the electroweak symmetry breaking scale. We compute the change in the mixing angle due to additional mass terms for the case of Tri-bimaximal. The change in θ_{12} is more than $3^o$from the Tri-bimaximal value. Therefore corresponding maximum change in oscillation probability is about 10%. The change of θ_{12}occurs of course, for degenerate neutrino mass with a common mass of about 2 eV. Cosmology constraints, from WMAP measurement [14] impose an upper limit of 0.7eV on neutrino mass.Then the change in the value of $\theta_{12}$is smaller. One summarizing statement of this work might be the following, due to Planck scale effects only θ_{12} deviated by 3.5^o and other mixing angle have very small deviation and maximum change of $P(\nu_e \rightarrow \nu_\mu)$ oscillation probability is about 10%, this can be achieved by our calculation of “Tri-Bimaximal” neutrino mixing.

a_1	a_2	θ'_{12}	$P(\nu_e \to \nu_\mu) = sin^2 2\theta'_{12}$
0^o	0^o	36.63^o	0.94
0^o	45^o	36.82^o	0.92
0^o	90^o	34.99^o	0.88
0^o	135^o	36.88^o	0.87
0^o	180^o	38.51^o	0.94
45^o	0^o	36.63^o	0.91
45^o	45^o	34.97^o	0.96
45^o	90^o	33.26^o	0.84
45^o	135^o	35.09^o	0.88
45^o	180^o	36.63^o	0.91
90^o	0^o	35^o	0.88
90^o	45^o	33.43^o	0.84
90^o	90^o	31.77^o	0.80
90^o	135^o	33.49^o	0.84
90^o	180^o	35^o	0.88
135^o	0^o	36.63^o	0.91
135^o	45^o	35.04^o	0.89
135^o	90^o	33.26^o	0.84
135^o	135^o	35.02^o	0.88
135^o	180^o	36.63^o	0.91
180^o	0^o	38.51^o	0.94
180^o	45^o	36.82^o	0.92
180^o	90^o	34.99^o	0.92
180^o	135^o	36.88^o	0.87
180^o	180^o	38.51^o	0.94

Table 1 Modified mixing angles and maximum $P(\nu_e \to \nu_\mu)$ oscillation probabilities for some sample of a_1 and a_2. Input value are $\Delta_{31} = 0.002eV^2$, $\Delta_{21} = 0.00008eV^2, \theta_{12} = 35^o, \theta_{23} = 45^o, \theta_{13} = 0^o$.

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