Thermodynamic Fluctuation Theory and Gravitational Clustering of Galaxies

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Abstract: We study the phase transitions occurring in the gravitational clustering of galaxies on the basis of thermodynamic fluctuation theory. This is because the fluctuations in number and energy of the particles are constantly probing the possibility of a phase transition. A calculation of various moments of the fluctuating thermodynamic extensive parameters like the number and energy fluctuations, has been performed. The correlated fluctuations $\langle \triangle N \triangle U \rangle$, have shown some interesting results. For weak correlations, their ensemble average is positive, indicating that a region of density enhancement typically coincides with a region of positive total energy. Its perturbed kinetic energy exceeds its perturbed potential energy. Similarly an underdense region has negative total energy since it has preferentially lost the kinetic energy of the particles that have fled. For larger correlations the overdense regions typically have negative total energy, underdense regions have positive total energy. The critical value at which this switch occurs is the critical temperature $T = T_C$, whose value has been calculated analytically. At this critical value T_C, a positive $\langle \triangle N \rangle$ is just as likely to be associated with a positive or a negative $\triangle U$.

1. Introduction

Galaxies cluster on very large scales under the influence of their mutual gravitation and the characterization of this clustering is a problem of current interest.Observations indicate that while the large- scale distribution of galaxies appears to be essentially uniform, however, small-scale distribution is appreciably influenced by the well known tenancy

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towards clustering. The universe is homogeneous and isotropic on scales $\geq$ 100-200 Mpc where as on smaller scales its fundamental units- galaxies cluster together to form groups, clusters and even super clusters.

Our universe emerged from a singularity at a very high temperature about 2×10^{10} years ago. At such temperatures all matter behaves like photons, hence the initial state was a chaotic gaseous inferno of high energy elementary particles and photons. As the universe expanded, the temperature dropped and the heavier particles annihilated and decayed in to the less massive stable particles (protons, neutrons and neutrinos). At a time of about 10^{-4} seconds, the temperature had dropped to below that needed to make the heavy particles such as protons. At a time of one second this era ended when temperatures fell below 10^{10} K.

After particle formation, most of the energy in the universe was in the form of light which we call radiation era. Near the beginning of this era, cosmic nucleosynthesis took place. Nucleosynthesis began with the production of deuterium. The presence of deutrium then led to other, even faster reactions in which deutrons combined to produce helium. At the end of about five minutes, the nucleosynthesis process was complete. The end product of this phase of cosmic evolution was hydrogen, deuterium and helium. The universe expanded through the nucleosynthesis phase too fast to go for any more fusion than this. After 2000 years when matter (hydrogen and helium) began to dominate the universe and radiation became a secondary constituent, the matter era began. The evolution of this phase is governed by the standard flat model and the relationship R is proportional to $t^{2/3}$. At about 3000K, the matter and radiation decoupled and both evolved independently. Thereafter the radiation cooled to about 2.7K cosmic background radiation observed today and the gaseous matter underwent a critically important transformation- it could form the first galaxies. Prior to decoupling the radiation pressure kept the matter distribution smooth. Afterwards small inhomogeneities could grow and condense into the first gravitationally bound systems.

The clustering of galaxies is amply borne out in three-dimensional red-shift surveys, such as the CFA survey and the southern sky redshift survey (SSRY). Different techniques such as Correlation functions [1, 2, 3], Percolation [4] and distribution functions [5, 6] have been introduced to understand the large scale structure of universe. The theories of the cosmological many body galaxy distribution function have been developed mainly from a thermodynamic point of view. Comparison of gravitational thermodynamics to the cosmological many body problem has been discussed on the basis of N-body computer simulation results [7, 8], along with other theoretical arguments [9] and these support it further. In the present paper we investigate the problem of gravitational clustering of galaxies from the point of view of thermodynamics using the fluctuation theory.

2. Thermodynamic Fluctuation theory and Phase Transitions

The phase transition refers to a change, usually abrupt, from one type of dominant symmetry to another. The fluctuations in a system are constantly probing the possibility of a

phase transition. The fluctuations are an intrinsic part of the thermodynamic equilibrium. Within any sub-region of the system and from region to region, the macroscopic quatities will fluctuate around their average values. Although fluctuations are necessarily small, they can become enormous in gravitational systems. Fluctuations occur between regions, sub-regions or cells of the system- they are not generally isolated. They exchange energy, volume or number of particles with one another or with a much larger reservoir. Away from a phase transition, the fluctuations are usually small and there is no difference between the canonical and microcanonical ensembles. When fluctuations are large we have the case for non-linear galaxy clustering. Since both energy and particles(galaxies) can move across the cell boundaries, the grand canonical ensemble is the relevant one to use. These fluctuations can completely explore new set of states at ease. Often the system changes its character completely and permanently by falling into a state of much lower total energy and altered symmetry i.e phase transition occurs.

A macroscopic system undergoes incessant and rapid transitions among its microstates. Some cells find a microstate of higher entropy or lower free energy and fall into such a microstate. A trapped cell in the new microstate modifies the thermodynamic functions of its surrounding cells. Many such cells may form nuclei and their microstate may propagate through out the entire system i.e; a phase transition occurs. Its consequences are dramatic. Ice turns to water, water boils to steam, crystals change structure and magnetic domains collapse.

In this work we investigate the problem of phase transitions occurring in the gravitational clustering of galaxies from the point of view of thermodynamic fluctuation theory and statistical mechanics. The work done by [10], ensures the application of statistical mechanics to gravitational clustering of galaxies. We consider a large system, which consists of an ensemble of cells of the same volume V(much smaller than the total volume)and average density $\bar{n}$. Both the number of galaxies and their total energy will vary among these cells, that are presented by a grand canonical ensemble, in which galaxies interact pairwise and their distribution is statistically homogeneous over large regions.

3. Gravitational Partition Function

For the cosmological many body system where linear and non-linear clustering of galaxies and their haloes are present over many scales simultaneously, grand canonical ensembles are needed.

The partition function of a system of N particles each of mass m interacting gravitationally with potential energy ϕ, having momenta p_i and average temperature T is

$$Z_N(T,V) \;=\; \frac{1}{\Lambda^{3N} N!} \int \exp\left[-\left\{\sum_{i=1}^{N}\frac{p_i^2}{2m} + \phi_{ij}(r_1, r_2, \cdots, r_N)\right\} T^{-1}\right] dp^{3N} dr^{3N} \quad (1)$$

Here $N!$ takes the distinguishability of classical particles into account and Λ normalizes the phase space volume cell. The integral over phase space in equation (1) has been

evaluated analytically by using virial expansion method and is given by:

$$Z_N(T, V) \ = \ \frac{1}{N!} \left(\frac{2\pi m T}{\Lambda^2} \right)^{\frac{3N}{2}} V^N \left[1 + \beta \bar{n} T^{-3} \alpha(\epsilon/R_1) \right]^{N-1} \tag{2}$$

with $\beta \ = \ \frac{3}{2}(Gm^2)^3$ and $\alpha(\epsilon/R_1)$ as:

$$\alpha(\epsilon/R_1) \ = \ \sqrt{1 + (\epsilon/R_1)^2} + (\epsilon/R_1)^2 \ ln \ \frac{(\epsilon/R_1)}{1 + \sqrt{1 + (\epsilon/R_1)^2}} \tag{3}$$

Here R_1 is the radius of the cell and $\epsilon/R_1 = 0$ corresponds to point mass. The free energy is given by:

$$F \ = \ -TlnZ_N(T, V) \tag{4}$$

Using equation (2) in equation (4), we have:

$$F \ = \ NTln \left(\frac{N}{V} T^{-3/2} \right) - NTln\{ 1 + \beta \bar{n} T^{-3} \alpha(\epsilon/R_1) \} - NT - \frac{3}{2} NTln(\frac{2\pi m}{\Lambda^2}) \tag{5}$$

Again from [6, 10], we have:

$$b \ = \ \frac{\beta \bar{n} T^{-3}}{1 + \beta \bar{n} T^{-3}} \tag{6}$$

$$P \ = \ \frac{b}{1 + \beta T^{-4}} \tag{7}$$

The value of b measures the influence of gravitational correlation potential energy and ranges between 0 and 1.
Using equation(6) in (7) we have:

$$P \ = \ \frac{NT}{V}(1 - b) \tag{8}$$

Where $\bar{n}=N/V$ is the average number density of N point mass galaxies in a volume V. In the thermodynamic fluctuation theory, the mean square deviation is widely used and convenient measure of the magnitude of the fluctuation. The mean square deviation is also called the second moment of the distribution and is expressed as [11]

$$Z_N(T, V) \ = \ \frac{1}{\Lambda^{3N} N!} \int \exp \left[- \left\{ \sum_{i=1}^{N} \frac{p_i^2}{2m} + \phi_{ij}(r_1, r_2, \cdots, r_N) \right\} T^{-1} \right] dp^{3N} dr^{3N} \tag{9}$$

$F_0....F_{j-1}, F_{j+1}, ...F_s, X_{s+1}....X_t$
Where $F_0 = \frac{1}{T}, F_1 = \frac{P}{T}$ and $F_2 = \frac{-\mu}{T}$ etc.
and $X_0 = U, X_1 = V, X_2 = N$

A calculation of various moments of thermodynamic extensive parameters like number and energy fluctuations has been performed for single component point mass galaxies clustering gravitationally in an expanding universe.We derive the moments in terms of

the temperature T, and a critical temperature T_C has been calculated at which a transition occurs from positive to negative total energy. The critical temperature T_C has been equated with the peculiar velocities of galaxies, thereby giving a prediction that our result can be tested in the laboratory by either N-body computer simulations or by observing the velocity catalogues of galaxies or their velocity distribution functions.

4. Thermodynamic Moments

The second order moment is obtained by putting $j = 2$, $k = 2$ in equation(9).

$$\langle \triangle X_2 \triangle X_2 \rangle = -\left[\frac{\partial X_2}{\partial F_2}\right]_{T,V} \tag{10}$$

$$\langle \triangle N^2 \rangle = T \left[\frac{\partial N}{\partial \mu}\right]_{T,V} \tag{11}$$

$$\left[\frac{\partial N}{\partial \mu}\right]_{T,V} = \left[\frac{\partial N}{\partial P}\right]_{T,V} \left[\frac{\partial P}{\partial \mu}\right]_{T,V} \tag{12}$$

$$\left[\frac{\partial N}{\partial \mu}\right]_{T,V} = \frac{N}{V} \left[\frac{\partial N}{\partial P}\right]_{T,V} \tag{13}$$

Where $\frac{\partial P}{\partial \mu} = N/V$

From equation (8), we have

$$\left[\frac{\partial P}{\partial N}\right]_{T,V} = \frac{T}{V}\left[1 - b - N\frac{\partial b}{\partial N}\right] \tag{14}$$

From equation (6),

$$\frac{\partial b}{\partial N} = \frac{b(1-b)}{N} \tag{15}$$

Using equation(15) in equation(14), we have

$$\left[\frac{\partial P}{\partial N}\right]_{T,V} = \frac{T}{V}(1-b)^2 \tag{16}$$

Substuting in equation (13), we have:

$$\left[\frac{\partial N}{\partial \mu}\right]_{T,V} = \frac{N}{T(1-b)^2} \tag{17}$$

Also Substuting equation(17) in equation (11), the result leads to,

$$\langle \triangle N^2 \rangle = \frac{N}{(1-b)^2} \tag{18}$$

Again using equation(6), we get

$$\langle \triangle N^2 \rangle = N \left(1 + \beta \bar{n} T^{-3}\right)^2 \tag{19}$$

Similarly third order moment is given by,

$$\langle \triangle X_i \triangle X_j \triangle X_k \rangle \;=\; \left[\frac{\partial^2 X_i}{\partial F_j \partial F_k} \right] \tag{20}$$

The third order moment is obtained by putting $i=2$, $j=2$ and $k=2$ in equation (20)

$$\langle \triangle N^3 \rangle \;=\; N \left(1 + \beta \bar{n} T^{-3} \right)^4 \left[1 + \frac{2 \beta \bar{n} T^{-3}}{1 + \beta \bar{n} T^{-3}} \right] \tag{21}$$

Finally the fourth order moment is given by,

$$\langle \triangle X_i \triangle X_j \triangle X_k \triangle X_l \rangle \;=\; -\frac{\partial^3 X_i}{\partial F_j \partial F_k \partial F_l} + \frac{\partial X_i}{\partial F_l} \frac{\partial X_j}{\partial F_k} + \frac{\partial X_j}{\partial F_l} \frac{\partial X_i}{\partial F_k} + \frac{\partial X_k}{\partial F_l} \frac{\partial X_i}{\partial F_j} \tag{22}$$

The fourth order moment is obtained by putting $i=j=k=2$ in equation(22):

$$\langle \triangle N^4 \rangle \;=\; N(1 + \beta \bar{n} T^{-3})^6 \left[1 + \frac{8 \beta \bar{n} T^{-3}}{1 + \beta \bar{n} T^{-3}} + 6 \left\{ \frac{\beta \bar{n} T^{-3}}{1 + \beta \bar{n} T^{-3}} \right\}^2 \right] + 3 \langle \triangle N^2 \rangle^2 \tag{23}$$

Since in the grand canonical ensemble of sub-regions or cells, both number of particles and energy can fluctuate between different cells, so it is important to consider the combination of moments, in which both the quantities can fluctuate. In this paper an attempt in this direction is performed.

The combination of moments is obtained by putting $j=2$, $k=0$ in equation (9)

$$\langle \triangle N \triangle U \rangle \;=\; T \left[\frac{\partial U}{\partial \mu} \right]_{1/T, P/T} \tag{24}$$

$$\left[\frac{\partial U}{\partial \mu} \right]_{T,V} \;=\; \left[\frac{\partial U}{\partial b} \right] \left[\frac{\partial b}{\partial \mu} \right] \tag{25}$$

The internal energy is given by

$$U \;=\; \frac{3}{2} NT(1 - 2b) \tag{26}$$

$$\left[\frac{\partial U}{\partial b} \right]_{T,V} \;=\; \frac{3}{2} T \frac{\partial}{\partial b} \left[N(1 - 2b) \right] \tag{27}$$

Using equation (15), we get.

$$\left[\frac{\partial U}{\partial b} \right]_{T,V} \;=\; \frac{3}{2} \frac{NT}{b(1 - b)} \left[1 - 4b + 2b^2 \right] \tag{28}$$

Using equation(6), we have

$$\left[\frac{\partial b}{\partial \mu} \right] = \frac{b}{T(1 - b)} \tag{29}$$

Substituting equations (28) and (29) in equation(25), we get

$$\left[\frac{\partial U}{\partial \mu}\right]_{T,V} = \frac{3N(1 - 4b + 2b^2)}{2(1-b)^2} \tag{30}$$

Substuting equation (30) in equation (24), and using equation(6), we get

$$\langle \triangle N \triangle U \rangle = \frac{3}{2}NT(1 + \beta\bar{n}T^{-3})^2 \left[1 - 4\frac{\beta\bar{n}T^{-3}}{1 + \beta\bar{n}T^{-3}} + 2\left\{\frac{\beta\bar{n}T^{-3}}{1 + \beta\bar{n}T^{-3}}\right\}^2\right] \tag{31}$$

The correlated fluctuations $\langle \triangle N \triangle U \rangle$ are especially interesting. The average energy $\triangle U$ can be either positive or negative. For weak correlations, their ensemble average is positive, indicating that a region of density enhancement typically coincides with a region of positive total energy. Its perturbed kinetic energy exceeds its perturbed potential energy. Similarly an underdense region has negative total energy, since it has preferentially lost the kinetic energy of the particles that have fled. For larger correlations the overdense regions typically have negative total energy; underdense regions usually have positive total energy. The critical value at which this switch occurs is the critical temperature $T = T_c$, whose value has been calculated analytically. At this critical value of T_C, a positive $\triangle N$ is just as likely to be associated with a positive or a negative $\triangle U$.

The critical value is obtained as
$\langle \triangle N \triangle U \rangle = 0$
Using equation(31), we get the result as

$$T_C = (2\beta T^{-3})^{\frac{1}{3}} \tag{32}$$

where $\beta = \frac{3}{2}(Gm^2)^3$ is a positive constant .Equation(32) is especially important because it has direct observational consequences by relating it to observed catalogues and to computer simulations.

5.　Results and Discussion

We have applied the thermodynamic fluctuation theory to study the phase transitions occurring in the gravitational clustering of galaxies in an expanding universe.The calculations of various moments of number and energy fluctuations have been performed in this paper . We have also calculated the correlated fluctuations $\langle \triangle N \triangle U \rangle$ in which both total number of particles(galaxies) as well as their energy can fluctuate between different cells or subsystems of the grand canonical ensemble. The results show that there is a critical temperature T_C at which the transition from positive to negative total energy of the system occurs. We define this as a phase transition. Recently the Lee-Yang theory has been applied to the gravitational clustering of galaxies and a critical temperature T_C [12] has been obtained at which a phase transition occurs in the clustering process. The results of Lee-Yang theory match with the results of the fluctuation theory because the expression for critical temperature is same in both the cases i.e. the phase transition occurs at a unique value of critical temperature T_C, given by equation (32).

The critical temperature T_C at which the phase transition occurs in the cosmological many body problem has been calculated analytically here. We can equate it with the kinetic energy of peculiar motion of galaxies.

$$[T_C]_j \;\; = \;\; \frac{2}{3N}\frac{1}{2}m_J \sum_{k=1}^{N_j} v_k^2, j = 1, 2, .. \tag{33}$$

Where T_j, N_j and m_j are the temperature, the number and the mass respectively of the *jth* component of galaxies.

Peculiar velocities may arise from interactions between individual galaxies as well from collective interaction between a galaxy and a cluster. In the early stages, the more massive galaxies cluster more rapidly and their velocity dispersion increases faster than that of less massive galaxies. Next the less massive galaxies cluster around the massive ones. This implies that in the early stages the massive galaxies speed up the clustering but in the later stages the distinction between different masses diminishes. In other words, although the mass of an individual galaxy is important at early stages, collective effects become more important at late stages and the effects of mass spectrum are substantially reduced.

Equation(33) gives a prediction of observing the phase transition occurring in the gravitational clustering of galaxies in an expanding universe, by either N-body computer simulations or by observing the velocity catalogues or the distribution functions of galaxies. It is therefore very important to see that whether it is a phase transition from the uniform expanding phase to the centrally condensed inhomogeneous state or from a poisson distribution to a correlated distribution, slowly developing on large and large scales as the universe expands. Again it may well be a transition from individual interactions to collective effects between galaxies. From the observations one should be able to calculate T_C in terms of the velocities of galaxies that will lead us to many new aspects of the cosmological N-body gravitational clustering problem.

Acknowledgment

References

[1] H. Totsuji and T. Kihara, *Pub. Astron. Soc. Jpn*; **21**, (1969), 221.

[2] N. Iqbal, F. Ahmad and M. S. Khan; *Journal of Astronomy and Astrophysics*; **27**, (2006), 373.

[3] P. J. E Peebles, *The large scale structure of the universe*, (Princerton: Princeton Univ. Press. 1980).

[4]   S.P. Bhavsar and R.J. Splinter, *MNRAS*; **282**, (1996), 1461.

[5]   W. C. Saslaw and A.J.S. Hamilton, *Ap. J*; **276**, (1984), 13.

[6]   F. Ahmad, W.C.Saslaw and N.I.Bhat, *Ap. J*; **571**,(2002), 576.

[7]   W. C. Saslaw and R.K. Sheth, *Ap. J*; **409**, (1993), 50.

[8]   S. Raychaudhury and W.C.Saslaw , *Ap. J*; **461**, (1996), 514.

[9]   Y. Zhan and C. Dyer, *Ap.J*; **343**, (1989), 107.

[10] F. Ahmad, W. C. Saslaw and M.A. Malik, *Ap. J*; **645**, (2006), 940.

[11] H. B. Callen, *Thermodynamics and an introduction to thermostatics*, (2nd ed. New York, Wiley. 1985).

[12] M. S Khan, N. Iqbal and F.Ahmad,*Chinese Journal of Astronomy and Astrophysics*; submited, (2008).