EJTP 6, No. 21 (2009) 125–138 *Electronic Journal of Theoretical Physics*

Time scale synchronization between two different time-delayed systems

Dibakar Ghosh*

Department of Mathematics, Dinabandhu Andrews College, Garia, Calcutta-700 084, India

Received 30 January 2009, Accepted 15 March 2009, Published 5 May 2009

Abstract: In this paper we consider time scale synchronization between two different time-delay systems. Due to existence of intrinsic multiple characteristic time scales in the chaotic time series, the usual definition for the calculation of phase failed. To define the phase, we have used empirical mode decomposition and the results are compared with those from continuous wavelet transform. We investigate the generalized synchronization between these two different chaotic time delay systems and find the existence condition for the generalized synchronization. It has been observed that the generalized synchronization is a weaker than the phase synchronization. Due to the presence of scaling factor in the wavelet transform it has more flexibility for application.

Keywords: Time scale; Synchronization; Krasovskii-Lyapunov Theory; Time Delay System
PACS (2008): 05.45.Vx; 05.45.Xt; 05.45.Pq

1. Introduction

Synchronization of chaotic oscillators is one of the fundamental phenomena in nonlinear dynamics. Various types of synchronization in chaotic systems have been classified[1], such as complete synchronization(CS), generalized synchronization(GS), lag synchronization(LS) and phase synchronization(PS). Among them, PS refers to the condition where the phases between two chaotic oscillators are locked, or the weaker condition where the mean frequencies between two chaotic oscillators are locked[2]. But the case of phase synchronization between two different time-delayed systems have not yet been identified and addressed. A main problem here is to define even the notion of phase in chaotic time delay system due to the intrinsic multiple characteristic time scales in these systems[3].

* E-mail: drghosh_math@yahoo.co.in, Fax No. 091-033-28671480, Telephone No. 091-9883285599.

In that case, its trajectories on the complex plane may show multiple centers of rotation and broadband power spectrum of the signal indicates multiple Fourier modes, which creates a difficulty in the estimation of phase. There is no unique method to determine the phase in complex chaotic oscillators and different definition of phase are found in [4]. All the synchronization types mentioned above are associated with each other, but the relationship between them is not completely clarified yet for time delay system.

In this paper, we propose a universal method to detect all types of synchronization and find a relationship between them. The main idea of this approach is to analyze the behavior of different time scale of the coupled time delay system. To study the time scale synchronization between two different time delay systems, we use empirical mode decomposition(EMD)[5] and continuous wavelet transform(CWT)[6]. Using CWT, we introduce the continuous set of time scales s and the instantaneous phases $\phi_s(t)$ associated with them. In other words, $\phi_s(t)$ is a continuous function of time t and time scale s. For the case of synchronization, the time series generated by these systems involve time scales s that are necessarily correlated and satisfy the phase locking condition

$$\mid \phi_{s_1}(t) - \phi_{s_2}(t) \mid < \text{constant} \tag{1}$$

The structure of this paper is as follows. In section 2, we discuss the method of EMD. In section 3, we consider the phase synchronization of two different chaotic time-delayed systems with unidirectional coupling by EMD. The CWT method are discussed in section 4. In section 5, the synchronization between Logistic and Mackey-Glass time-delayed systems are discussed. The GS versus time scale synchronization are discussed in section 6. We summarize our results in section 7.

2. Empirical Mode Decomposition

The study of synchronization basically requires the analysis of the signal or data which is available in the form of a time series. Many applications that involve signal or data processing require the use of transforms such as the Fast Fourier Transform (FFT) or Discrete Fourier Transform (DFT). These transforms allow a signal or data set that satisfies certain conditions to be converted to the frequency domain. A new signal processing technology called the Hilbert-Huang Transform (HHT) [5] has recently found that accurately analyzes physical signals. It calculate instantaneous frequencies based on the EMD method when intrinsic mode functions(IMFs) are generated for complex data. Then a Hilbert transform converts the local energy and instantaneous frequency derived from the IMFs to a full energy-frequency-time distribution of the data (i.e., a Hilbert spectrum). Finally, the physical signal is filtered by reconstruction from selected IMFs and a curve can be fitted to the filtered signal. Such a curve fitting might not have been possible with the original, unfiltered signal.

To use EMD method to decompose a function $f(t)$ as a linear combination of IMF's ψ_n, the first step is to choose a time scale which is inherent in the function and has physical meaning. We also require to know the time between successive zero-crossings, successive extrema and successive curvature extrema. Here $n = 0$, $f_0 = f$, $h_0 = f_n$ and $k = 0$ are taken. The upper envelope is constructed for h_k. For this all the local extrema are identified and all the local maxima are fitted by a cubic spline interpolation for use as the upper envelope $U(t)$. Similarly by taking all the local minima the lower envelope L(t) is constructed. The functions U(t) and L(t) should envelop the data between them, i.e., $L(t) \leq h_k(t) \leq U(t)$, for all t. Their mean is denoted by $m_k(t)$ and the k-th component is defined as

$$h_{k+1} = h_k - m_k$$

If h_{k+1} is not an IMF, h_k is replaced by h_{k+1} and the envelopes are again constructed. Otherwise, h_{k+1} is defined as IMF ψ_n and the residual is $f_{n+1} = f_n - \psi_n$. If the convergence criterion is not satisfied then n is increased and the whole procedure is repeated. The convergence criterion is tested by considering the residual either smaller than a predetermined value or a monotonic function. Adding all the IMF's together with the residual slow trend reconstructs the original signal without information loss or distortion. Explicit procedure of EMD for a given signal $x(t)$ can be summarized[7] as follows.
(i) All extrema of $x(t)$ should be identified.
(ii) Interpolation should be done between the minima (respectively maxima)
ending up with some envelope $e_{min}(t)$ (respectively $e_{max}(t)$).
(iii) The mean $m(t) = (e_{min}(t) + e_{max}(t))/2$ should be computed.
(iv) The detail $d(t) = x(t) - m(t)$ should be extracted.
(v) The residual $m(t)$ should be iterated.
In practice, a *sifting process* refines the above procedure which amounts to first iterating steps (i) to (iv) upon the detail signal $d(t)$, until the latter can be considered as zero-mean according to some stopping criterion. Once this is achieved, the detail can be referred to as IMF. The corresponding residual is computed and step (v) applies. The number of extrema decreases while going from one residual to the next and the whole decomposition is completed within a finite number of modes.

The phase variable $\phi(t)$ can be easily estimated from a scalar time series $x(t)$. But the problem arises when the signal possesses a multi component or a time varying spectrum. In that case, its trajectory on the complex plane may show multiple centers of rotation and an instantaneous phase cannot be defined easily [4]. The power spectrum of the signal indicates multiple Fourier modes, which creates a difficulty in the estimation of phase. To overcome this difficulty we have used the algorithm of EMD. EMD ensures that the complex plane of $C_j(t)$ rotate around a unique rotation center, the resulting signal can be considered as a proper rotation mode and the phase can be defined.

We have decomposed the original chaotic signals $x(t)$ and $y(t)$ as

$$x(t) = \sum_{j=1}^{M} C_j(t) + R(t) \quad \text{and} \quad y(t) = \sum_{j=1}^{N} C'_j(t) + R'(t)$$

where $R(t)$ and $R^{'}(t)$ are residuals of the signals $x(t)$ and $y(t)$ respectively. The functions $C_j(t)$ (and $C'_j(t)$) are nearly orthogonal to each other. Thus each mode generates a proper rotation on the complex plane with the analytic signal

$$C_j(t) = A_j(t)e^{i\phi_j(t)} \text{ and } C'_j(t) = A'_j(t)e^{i\phi'_j(t)}$$

and the two phases ϕ_j and ϕ'_j of the signals C_j and C'_j respectively are obtained.

Thus the phases ϕ_j and ϕ'_j are obtained by using Hilbert transform of each C_j and C'_j respectively and the frequencies ω_j and ω'_j are obtained by averaging the instantaneous phases $\frac{d\phi_j}{dt}$ and $\frac{d\phi'_j}{dt}$ respectively, separately for each mode. The instantaneous frequencies of each mode can vary with time and the fast oscillations present in the signal are in general extracted into the lower and the slow oscillations into the higher modes so that $\omega_0 > \omega_1 > \omega_2 > ... > \omega_M$ and $\omega'_0 > \omega'_1 > \omega'_2 > ... > \omega'_N$. Moreover, the mode amplitudes usually decay fast with j so that the signal can be decomposed into a small number of empirical modes. In this context one should note that the phase synchronization between the drive and response systems can always be characterized either as a phase locking or a weaker condition of frequency locking. One should note that phase locking and mean frequency locking are two independent conditions to characterize phase synchronization [1]. In the synchronized state the phase difference between the oscillators is bounded and the frequency difference is zero or at least close to zero. By using the EMD method, the transition to phase synchronization is basically analyzed as a process of merging of the frequencies of the corresponding modes and the phase difference is bounded as the coupling strength is increased. Further, when synchronization is set, different types of phase interactions may simultaneously arise at specific time scales. Then the two signals $x(t)$ and $y(t)$ can be written in polar form as

$$x(t) = \sum_{j=1}^{M} A_j(t)exp[i\int_0^t \Omega_j(t)dt] \quad \text{and}$$

$$y(t) = \sum_{j=1}^{N} A'_j(t)exp[i\int_0^t \Omega'_j(t)dt]$$

3. Synchronization between Logistic and Mackey-Glass time-delayed systems via EMD method: From phase to lag synchronization

Let us consider the more complicated example when it is impossible to correctly introduce the instantaneous phase $\phi(t)$ of chaotic signal $\mathbf{x(t)}$. It is clear, that for such cases the traditional methods of the phase synchronization detection fail and it is necessary to use the other techniques, e.g. indirect measurement. As the first attempt to study PS between two different coupled time-delayed chaotic systems, we couple two such chaotic time-delayed systems in our model. The drive system is a delay Logistic system[8] and response system as time-delayed Mackey-Glass system[9].

To illustrate it we consider the following unidirectionally coupled drive $x(t)$ and response $y(t)$ systems as

$$\dot{x}(t) = -ax(t) + rx(t-\tau_1)[1 - x(t-\tau_1)] \quad (2a)$$

$$\dot{y}(t) = -\alpha y(t) + \frac{\beta y(t-\tau_2)}{1 + y^{10}(t-\tau_2)} + k(x(t) - y(t)) \quad (2b)$$

where k is a coupling strength. We choose the values of parameter as $a = 15, r = 54, \tau_1 = 5, \alpha = 1, \beta = 2, \tau_2 = 10$. In the absence of coupling strength the drive system $x(t)$ and response system $y(t)$ exhibit chaotic attractors. It is necessary to note that under these control parameter values none of the direct measurement methods for phase permits us to define the phase of chaotic signal correctly in the whole range of coupling parameter k variation.

The behavior of phase difference $[\Delta(\phi)]$ at different intrinsic time scales for different coupling strength $k = 2.0$ and $k = 6.0$ are shown in fig. (1a) and fig. (1b) respectively. We observed that in fig. (1a) for low coupling strength $k = 2.0$ the phase difference between different intrinsic mode are not bounded. The phase difference $\phi_j(t) - \phi'_j(t)$ are not bounded for almost all intrinsic time scales. At this case the intrinsic time scales of both the systems are not correlated. For further increase of coupling strength (i.e. $k = 6.0$), some of intrinsic time scales of the first chaotic oscillator becomes correlated with the other intrinsic modes of the second oscillator and the phase synchronization[PS] occur [in fig.(1b)]. With further increase of coupling ($i.e.k = 13.53$), all intrinsic time scales of two systems are correlated. Phase difference between any two time scales are correlated with each other[Fig. (1c)]. With increase the value of coupling, PS converted to lag synchronization[LS]. The LS between oscillators means that all intrinsic modes are correlated. From the condition of LS we have $x(t-\tau) \simeq y(t)$ and therefore $\phi_j(t-\tau) \simeq \phi'_j(t)$ where τ is the time lag. This time lag depends on the coupling parameter. At high coupling parameter, the time lag decreases and LS transfered to CS.

4. Continuous wavelet transform(CWT)

An alternative approach for the analysis of phase of a complicated time series is the wavelet transform. Several people have already worked on this approach [6]. To elaborate the basis of wavelet technique, consider a time series $x(t)$. The behavior of such systems can be characterized by a continuous phase set define wavelet transform of the chaotic signal $x(t)$;

$$W(s_0, t_0) = \int_{-\infty}^{\infty} x(t)\psi^*_{s_0,t_0} dt$$

where the asterisk means complex conjugation and

$$\psi_{s_0,t_0}(t) = \frac{1}{\sqrt{s_0}}\psi_0\left(\frac{t-t_0}{s_0}\right)$$

is the wavelet function obtained from the mother wavelet $\psi_{s_0,t_0}(t)$. The time scale s_0 determines the width of $\psi_0(t)$, where t_0 stands for time shift of the wavelet function.

Here it should be noted that the time scale s_0 is replaced by the frequency of Fourier transform and can be considered as the quantity inverse to it. In this paper we have used Morlet wavelet which is given by;

$$\psi_0(\eta) = \frac{1}{\sqrt[4]{\pi}} \, exp\,(i\omega_0\eta)\; exp\left(-\frac{\eta^2}{2}\right)$$

The wavelet parameter $\omega_0 = 2\pi$ ensures the relation $s = \frac{1}{f}$ between the time scale s and frequency f of Fourier transform. One then considers

$$W\,(s_0, t_0) = |W\,(s_0, t_0)|\, e^{i\phi_{s_0}(t_0)}$$

which determines the wavelet surface characterizing the behavior of the system for every time scale s_0 at any time t_0. The magnitude of $W\,(s_0, t_0)$ represents the relative presence and magnitude of the corresponding time scale s_0 at t_0. Usually it is very standard to consider the integral energy distribution over all time as

$$< E(s_0) >= \int |W\,(s_0, t_0)|^2 \; dt_0$$

The phase $\phi_{s_0}(t) = argW\,(s_0, t_0)$ also proves to be naturally defined for time scale s_0. So that the behavior of each time scale s_0 can be identified using the phase $\phi_{s_0}(t)$. We now apply this idea to the two time series obtained from the two chaotic systems. It is observed that the time scales accounting for the greatest fraction of the wavelet spectrum energy $< E(s_0) >$ are obviously synchronized first. For other time scales there is no synchronization. Actually we should have $|\phi_{s_1}(t) - \phi_{s_2}(t)| <$ constant, for some s, leading to phase locking in the situation of phase synchronization.

5. Synchronization between the Logistic and Mackey-Glass time-delayed systems via CWT

We now apply continuous wavelet transform to the two time series obtained from the two chaotic time-delayed systems. It is observed that the time scales accounting for the greatest fraction of the wavelet spectrum energy $< E(s_0) >$ are obviously synchronized first. For the other time scales there is no synchronization. Actually we should have $|\phi_{s_1}(t) - \phi_{s_2}(t)| <$ constant, for some time-scale s, leading to phase locking in the situation of phase synchronization.

We consider the dynamics of different time scale s of two different coupled time-delayed systems (2) when the coupling parameter value increases. If there is no phase synchronization between the oscillators, their dynamics remain uncorrelated for all time scales s. The dynamics of the coupled system (2) when the coupling parameter k is sufficiently small $k = 2.0$ are shown in figure (2a) and (2b). The power spectrums $< E(s) >$ of the Logistic and Mackey-Glass time delay systems are different from each other [fig. 2(a)], but the maximum value of the power spectrum for both the systems are occurred to the same time scale $s_0 = 10.2$. It is clear that the phase difference

$\phi_{s_1}(t) - \phi_{s_2}(t)$ are not bounded for all the time scales $s_0 = 3.0, 10.2, 12.0$ [Fig. 2(b)]. This means that the systems under consideration do not involve synchronized time scales. Therefore the systems are unsynchronized.

As the coupling increases, the systems are brought to PS regime as illustrates in figure (2c, 2d). The power spectrum of the response system is near similar to the power spectrum of driving system [in fig. 2(c)]. It is seen that phase locking occurs at the time scale $s_0 = 10.2$ corresponding to the maximum energy in the wavelet spectrum, $< E(s) >$ [in fig 2(d)]. But the other time scales $s_0 = 3.0, 12.0$, the phase difference are same as before [Fig. 2(b)] i.e. not bounded. As soon as any of the time scales of the driving system becomes correlated with another time scale of the response system (e.g. when the coupling parameter increases), phase synchronization occurs. The time scale s_0 is characterized by the largest value of energy in the wavelet spectrum $< E(s) >$ is more likely to be correlated first. The other time scales remain uncorrelated as before. The phase synchronization between chaotic systems leads to phase locking (1) at the correlated time scales s_0. With a further increase in the coupling parameter, the unsynchronized time scales become synchronized. The number of time scales for which the phase locking occurs increases and one can say that the degree of synchronization grows. For coupling parameter $k = 13.53$, we observe that the normalized energy spectrum totally overlap [Fig. 2(e)]. The time scales $s_0 = 3.0, 12.0$ which are not synchronized in the previous [Fig. 2(b), 2(d)] are synchronized. The phase difference remain bounded for all time scales s_0 where we have shown this variation for low s_0 and also for high s_0 value [in Fig. 2(f)].

The occurrence of lag synchronization between time delay system means that all the time scales are correlated. The lag synchronization condition $x(t - \tau) \approx y(t)$ implies that $\omega_x(s, t - \tau) \approx \omega_y(s, t)$ and therefor $\phi_{s_1}(t - \tau) \approx \phi_{s_2}(t)$. In that case phase locking condition(1) is satisfied for all time scales. For instance, where the coupling parameter k is sufficient large then lag synchronization of coupled system (2) occurs. In this case the power spectrum of two system are coincident with each other and the phase locking condition satisfied for all time scales. It is to be noted that the phase difference $\phi_{s_1}(t) - \phi_{s_2}(t)$ will not be zero in the case of lag synchronization. This difference depends on time lag τ. We can therefore say that the time-scale synchronization is the most general synchronization and phase, lag synchronization are particular cases of time-scale synchronization.

6. Generalized synchronization versus time scale synchronization

Let us consider another type of synchronization behavior, the so-called generalized synchronization(GS). It has been shown in the above section that the PS, LS and CS are the particular type of time scale synchronization. All the above type synchronization are depend on the number of synchronized time scales. But the relation between the PS and GS is not clear. Several work are studied the problem, how the PS and GS are correlated with each other. In this section we will study the GS of the coupled time-delayed system(2). In a paper[10] the CS and GS of one way, linearly coupled Mackey-Glass system is studied. In recent paper[11] the GS between two unidirectionally linearly and nonlinearly coupled chaotic nonidentical Ikeda models are discussed. For this purpose, we use the auxiliary system method to detect GS[10]; that is, given another identical driven auxiliary system $z(t)$, GS between $x(t)$ and $y(t)$ is established with the achievement of CS between $y(t)$ and $z(t)$. In fact, the auxiliary method allows us to find the local stability condition of the GS[10].

To illustrate the above procedure we consider the coupled Mackey-Glass system as

$$\dot{y}(t) = -\alpha y(t) + \frac{\beta y(t-\tau_2)}{1+y^{10}(t-\tau_2)} + k(x(t)-y(t)) \tag{3a}$$

$$\dot{z}(t) = -\alpha z(t) + \frac{\beta z(t-\tau_2)}{1+z^{10}(t-\tau_2)} + k(x(t)-z(t)) \tag{3b}$$

We derive the existence conditions of the CS in coupled time-delayed systems (3) with the help of Krasovskii-Lyapunov functional approach. We denote the error signal $\Delta = y - z$. Then the error dynamics can be written as

$$\dot{\Delta} = -r(t)\Delta + s(t)\Delta_\tau \tag{4}$$

where $r(t) = \alpha + k$, $s(t) = f'(y_{\tau_2})$, $f(y_{\tau_2}) = \frac{\beta y(t-\tau_2)}{1+y^{10}(t-\tau_2)}$. The driving and response subsystem described by (3) are synchronized if the fixed point $\Delta = 0$ of system (4) is stable. By the Krasovskii-Lyapunov theory, a positively defined functional was introduced as,

$$V(t) = \frac{\Delta^2}{2} + \mu \int_{-\tau}^{0} \Delta^2(t+\theta)d\theta \tag{5}$$

where $\mu > 0$ is an arbitrary positive parameter. According to [12-14], the sufficient stability condition for the trivial solution $\Delta = 0$ of the time delayed equation (4) is

$$r(t) > \mid s(t) \mid \qquad \text{with} \qquad \mu = \frac{\mid s(t) \mid}{2} \tag{6}$$

For a particular problem, two cases may arises in equation(6), for the first case s is constant and $r(t)$ is variable and the second case r is constant and $s(t)$ is variable. The second case arise in our case where $s(t)$ is a variable. For the general cases the stability condition $\mu = \frac{|s(t)|}{2}$ is not always true because μ is a parameter and $s(t)$ is a variable. The above problem can be removed if we define μ as a function of time and the derivative of μ can be considered in the expression of $\dot{V}$.

Suppose $\mu = \zeta(t) > 0$, then the Krasovskii-Lyapunov functional can be taken as[12-14]

$$V(t) = \frac{1}{2}\Delta^2(t) + \zeta(t)\int_{-\tau}^{0} \Delta^2(t+\xi)d\xi$$

Then

$$\dot{V}(t) = -r(t)\Delta^2 + s(t)\Delta\Delta_\tau + \zeta(t)(\Delta^2 - \Delta_\tau^2) +$$

$$\dot{\zeta}(t)\int_{-\tau}^{0}\Delta^2(t+\xi)d\xi$$

If $\dot{\zeta}(t) \leq 0$ then we have

$$\dot{V}(t) \leq -[r(t) - s^2(t)/4\zeta(t) - \zeta(t)]\Delta^2$$

We obtain the stability condition as

$$r(t) - s^2(t)/4\zeta(t) - \zeta(t) > 0$$

$$i.e.\;\; r(t) > h(s,\zeta) \quad \text{where} \quad h(s,\zeta) = s^2(t)/4\zeta(t) + \zeta(t)$$

For any function of $s(t)$, $h(s,\zeta)$ is a function of $\zeta(t)$ and has an absolute minimum for $\zeta(t) = \frac{s}{2}$ and $h_{min}(s,\zeta) =\mid s(t) \mid$. Thus $h(s,\zeta) \geq\mid s(t) \mid$ for any s and $\zeta > 0$. The stability condition for synchronization is $r(t) >\mid s(t) \mid$. Then the stability condition for the trivial solution $\Delta = 0$ of linear time delay system is $r(t) >\mid s(t) \mid$ for $\zeta(t) = \frac{|s(t)|}{2}$.

The stability condition for CS is $\alpha + k >\mid f'(y_{\tau_2}) \mid$ that is $k > max\mid f'(y_{\tau_2}) \mid - 1$, where the maximum is defined on the trajectory of the driving system. Therefore, GS between (2a) and (2b) exists if $k > 3.05$. Figure(3a) shows the CS between (3a) and (3b). GS between (2a) and (2b) shown in figure(3b).

Thus, the GS of the unidirectional coupled time-delayed systems (2) appears as the time scale synchronized dynamics, as another types of synchronization does not occurs before. At this coupling parameter $k = 4.0$ the PS does not occurs [Fig. 3c, 3d]. The above results are similar to the results in Ref [15], in which the GS between two unidirectional coupled Rossler systems are occurred while the PS has not been observed. It is also clear why the PS has not been observed in our case. The instantaneous phases $\phi_{s_1}(t) - \phi_{s_2}(t)$ of the chaotic signal $x(t)$ and $y(t)$ determined by CWT for the time scales $s_0 = 3.0, 10.2, 12.0$ but only the time scale $s_0 = 10.2$ are synchronized[Fig. 3d]. The other time scale are not synchronized. The instantaneous phases does not allow to detect PS in that case although the synchronization of time scales occurs.

Conclusion

In conclusion, we have considered the time scale synchronization between two different time-delayed systems by means of EMD and CWT and compared their results. Since the two systems are in chaotic regime, the time series contain multiple Fourier modes. The several definition for detection of phase are failed. We observe that while the EMD method separates the complex signal in various IMF's corresponding to a definite frequency, the CWT approach does the same but with respect to different scales. Actual comparison between EMD and CWT is possible only through statistical methods. One can observe that different synchronization (CS, LS, PS and GS) come from universal position, i. e. time scale synchronization is a common type of synchronization where CS, LS, PS and GS are the particular cases of time scale synchronization. We have investigated the relation between GS and PS. We observe that GS is a weaker than PS. PS could be stranger than GS, and they can also occurs for chaotic time delay systems. The sufficient condition for generalized synchronization are studied analytically and also we have shown numerically the effectiveness of the synchronized system. According to our paper one can see that it will be a unified framework for different types of chaotic synchronization for any dynamical systems. These two methods are also applicable for experimental data because it does not require any information about the dynamical systems.

References

[1] J.Madore, S.Schraml,P.Schupp and J.Wess, Eur.Phys.Jour.C16(2000)161

[2] M.R.Douglas and N.A.Nekrasov,Rev.Mod.Phys.73,(2001)977.

[3] A. Pikovsky, M. Rosenblum and J. Kurths, Synchronization, A Universal concept in Nonlinear Systems, Cambridge University Press, UK 12 2001.

[4] S. Boccaletti, J. Kurths, G. Osipovand, D. L. Valladams and C. S. Zhou, *Phys. Rep* **366**(2002)1.

[5] D. V. Senthilkumar, M. Lakshmanan and J. Kurths, *Phys. Rev. E* **74**(2006) 035205.

[6] M. Chavez, C. Adam, V. Navarro, S. Boccaletti and J. Martinerie, *Chaos* **15**(2005) 023904.

[7] N. E. Huang et al., *Proc. R. Soc. London* **Ser A 454**(1998)903.

[8] A. E. Hramov and A. A. Koronovskii, *Chaos* **14**(2004) 603.

[9] G. Rilling, P. Flandrin and P. Goncalves, *In Proc. IEEE-EURASIP workshop on Nonlinear Signal and Image Processing NSIP-03*(2003).

[10] M. Jiang, Y. Shen, J. Jian and X. Liao, *Phys. Lett. A* **350**(2006) 221.

[11] M. C. Mackey and L. Glass, *Science* **197**(1977) 287.

[12] M. Zhan, X. Wang, X. Gong, G. W. Wei and C.-H. Lai, *Phys. Rev. E* **68**(2003) 036208.

[13] E. M. Shahverdiev and K. A. Shore, *Phys. Rev. E* **68** (2003)036208.

[14] D. Ghosh, S. Banerjee and A. Roy Chowdhury, *Europhys. Letts.* **80**(2007) 30006.

[15] S. Banerjee, D. Ghosh, A. Ray and A. Roy Chowdhury, *Europhys. Letts.* **81**(2008) 20006.

[16] S. Banerjee, D. Ghosh and A. Roy Chowdhury, *Phys. Scr.* **78**(2008) 015010.

[17] Z. Zheng and G. Hu, *Phys. Rev. E* **62**(2000) 7882.

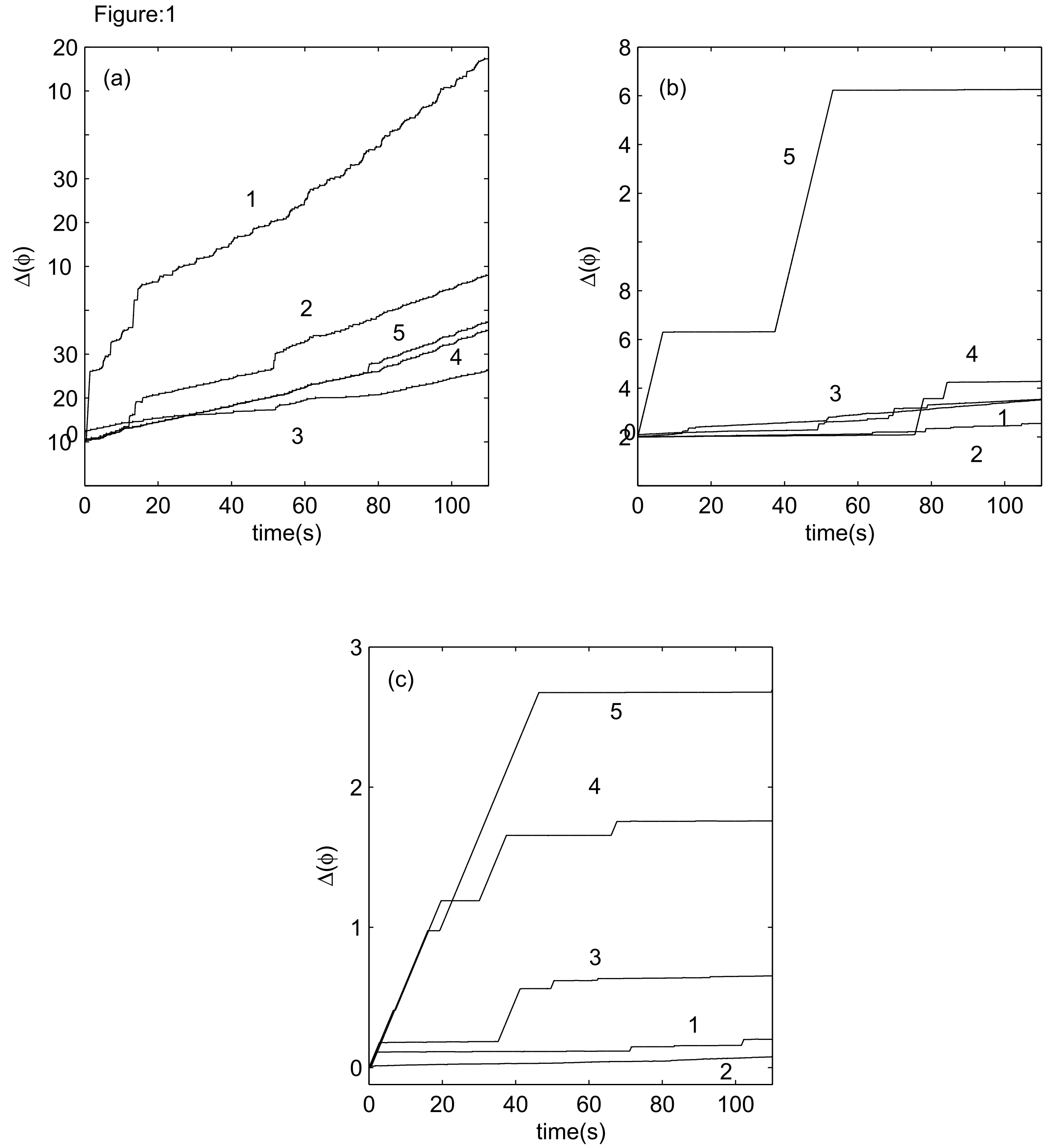

Fig. 1 Phese difference between the IMF's of systems (2a) and (2b) at (a) $k = 2.0$, (b) $k = 6.0$ and (c) $k = 13.53$, where 1 represents the phase difference between the corresponding C_1 and C_1' IMF's, 2 between C_2 and C_3' IMF's, 3 between C_3 and C_4' IMF's, 4 between C_4 and C_4' IMF's, 5 between C_6 and C_4' IMF's.

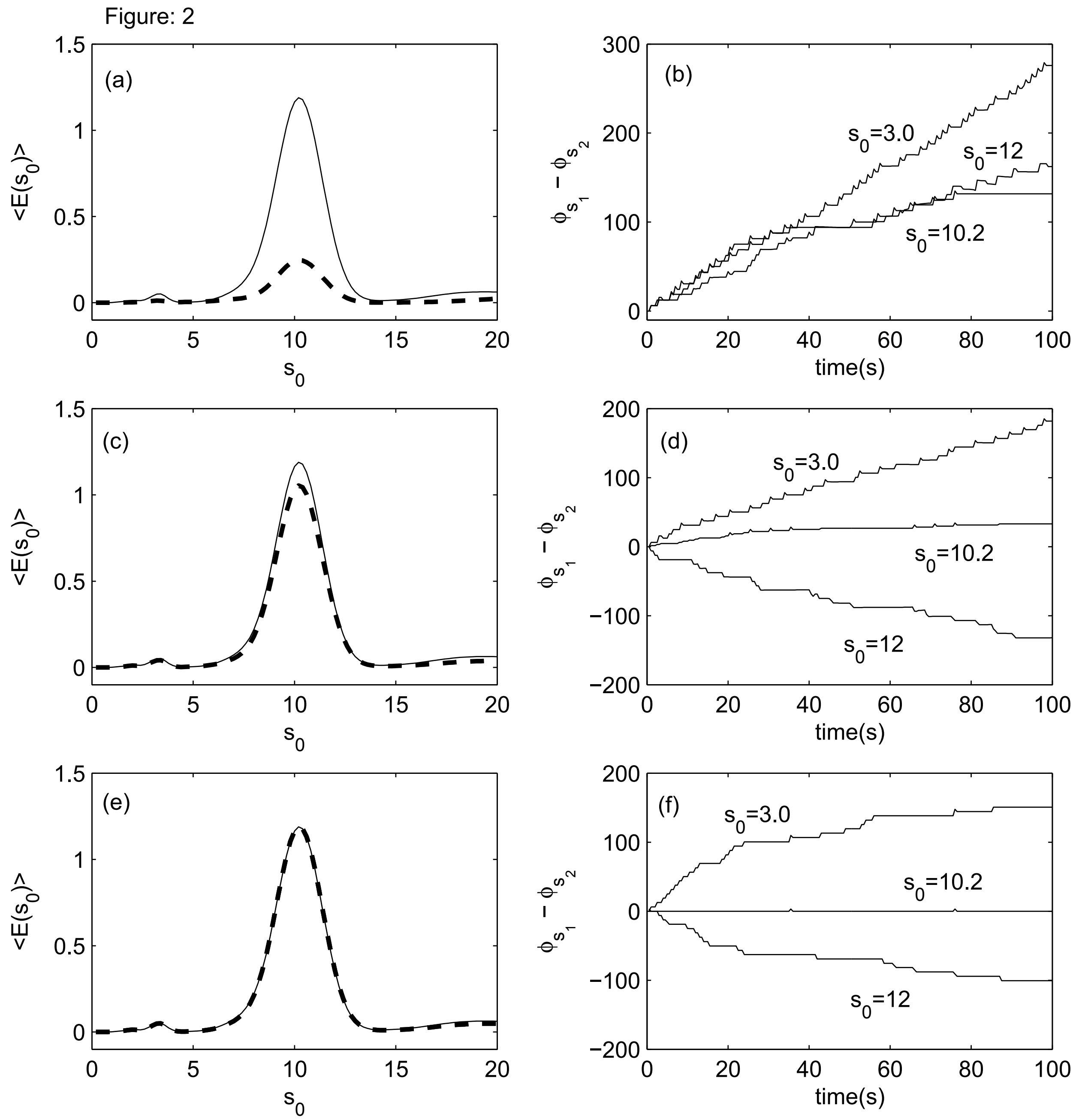

Fig. 2 a) The normalized energy distribution in the wavelet spectrum $< E(s_0) >$ for the drive (solid line) and response (dashed line) of coupled system (2), b) the dependence of the phase difference $\phi_{s_1}(t) - \phi_{s_2}(t)$ on time t for different time scales s. The coupling parameter between the oscillators is $k = 3.0$. There is no phase synchronization between the systems. c) The normalized energy distribution in the wavelet spectrum $< E(s_0) >$ for the drive (solid line) and response system (dashed line), d) phase difference $\phi_{s_1}(t) - \phi_{s_2}(t)$ for coupling system (2). The coupling parameter between the oscillators is $k = 6.0$. The time scale $s_0 = 10.2$ are correlated with each other and synchronization is observed. e) The normalized energy distribution in the wavelet spectrum $< E(s_0) >$ for the drive (solid line) and response system (dashed line), f) phase difference $\phi_{s_1}(t) - \phi_{s_2}(t)$ for coupling system (2). The coupling parameter between the oscillators is $k = 13.53$. All the time scales are correlated and synchronization is observed.

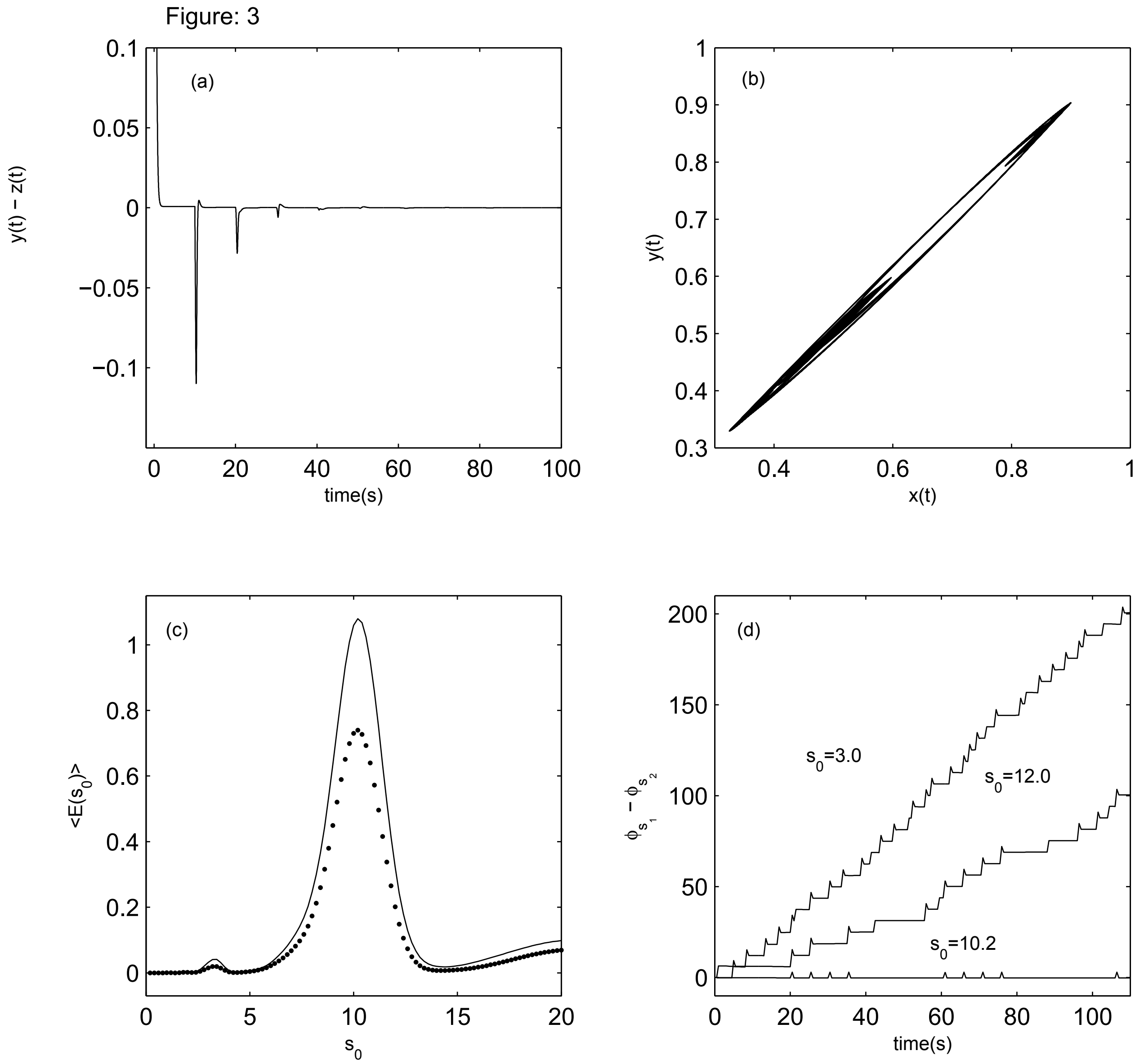

Fig. 3 a) Complete synchronization between $y(t)$ and $z(t)$ in system 3, b) generalized synchronization between $x(t)$ and $y(t)$ in system 2. c) normalized energy distribution and d) phase difference $\phi_{s_1}(t) - \phi_{s_2}(t)$ at $k = 4.0$.