Neutrino Mixings and Magnetic Moments Due to Planck Scale Effects

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Abstract: In this paper, we consider the effect of Planck scale operators on neutrino magnetic moments. We assume that the main part of neutrino masses and mixings arise through GUT scale operators. We further assume that additional discrete symmetries make the neutrino mixing bi-maximal. Quantum gravitational (Planck scale) effects lead to an effective $SU(2)_L \times U(1)$ invariant dimension-5 Lagrangian involving neutrino and Higgs fields, which gives rise to additional terms in neutrino mass matrix. These additional terms can be considered to be perturbation of the GUT scale bi-maximal neutrino mass matrix. We assume that the gravitational interaction is flavor blind and we study the neutrino mixings and magnetic moments due to the physics above the GUT scale.

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1. Introduction

Neutrino magnetic moments is proportional to the neutrino mass as required by the symmetry principles. At present the solar, atmospheric, reactor and accelerator experiments indicates the existence of non zero neutrino masses. Its indicates that neutrino has a magnetic moment. A minimal extension of solar model yields a neutrino magnetic moment [1]

$$\mu_\nu = \frac{3eG_F m_\nu}{8\pi^2\sqrt{2}} = \frac{3G_F m_e m_\nu}{4\pi^2\sqrt{2}} \mu_B,$$

(1)

where $\mu_B = e/2m$ is the Bohr magnetron, $m_e$ is the electron mass and $m_\nu$ is the neutrino mass.

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The fundamental magnetic moment are associated with the mass eigenstates in the mass eigenstates basis. Dirac neutrino can have diagonal or off diagonal moment, while Majorana neutrino can have transition magnetic moments [2, 3, 4]. The experimental value of neutrino magnetic moment can be determined by only in the recoil electron spectrum from neutrino electron spectrum [5, 6]. In this paper, we study, how Planck scale effects the neutrino magnetic moments. Magnetic moment of neutrinos, in principle, depend on the distance from its source [4]

\[ \mu^2_e = \sum_i \left| \sum_j U_{ej}\mu_{ij} \exp(-iE_jL) \right|^2, \]  

(2)

where \( \mu_{ij} \) is the fundamental constant in term of unit \( \mu_B \) that characterize the coupling of the neutrino mass eigenstate to the electromagnetic field. The expression for \( \mu^2_e \) in the case of Dirac neutrino, with only diagonal magnetic moment (\( \mu_{ij} = \mu_i \delta_{ij} \)); this is used by the Particle Data Group [4]

\[ \mu^2_e = \sum_j |U_{ej}|^2 |\mu_j|^2, \]  

(3)

In this expression, there is no dependence of L and neutrino energy E. In this one can say the neutrino magnetic moments depend on neutrino mixings. In the case of Majorana neutrino, and we assume three mass eigenstates. Then

\[ \mu^2_e = (|\mu_{12}|^2 + |\mu_{13}|^2)(|U_{e1}|^2 + |U_{e2}|^2). \]  

(4)

For the Dirac case this implies that at least nondoagonal magnetic moment is as large as the diagonal ones. In the case of Majorana, it implies that two different nondiagonal magnetic moment are of a similar magnitude [16]. Correction to neutrino mixing and neutrino magnetic moments are given in Section 2. In section 3 give the results on neutrino mixing and magnetic moments.

2. Corrections to Mixing Angles and Neutrino Magnetic Moments

The neutrino mass matrix is assumed to be generated by the see saw mechanism [7, 8, 9]. Here we will assume that the dominant part of neutrino mass matrix arises due to GUT scale operators and they lead to bi-maximal mixing. The effective gravitational interaction of neutrinos with Higgs field can be expressed as \( SU(2)_L \times U(1) \) invariant dimension-5 operator [10],

\[ L_{grav} = \frac{\lambda_{\alpha\beta}}{M_{pl}}(\psi_{A\alpha}^\epsilon AC \psi_C)C_{ab}^{-1}(\psi_{B\beta}\epsilon BD \psi_D) + h.c. \]  

(5)

Here and every where below we use Greek indices \( \alpha, \beta.. \) for the flavor states and Latin indices i, j, k for the mass states. In the above equation \( \psi_{\alpha} = (\nu_\alpha, l_\alpha) \) is the lepton doublet, \( \phi = (\phi^+, \phi^0) \) is the Higgs doublet and \( M_{pl} = 1.2 \times 10^{19} \text{GeV} \) is the Planck mass.
\( \lambda \) is a \( 3 \times 3 \) matrix in flavour space with each element \( O(1) \). In eq(4), all indices are explicitly shown. The Lorentz indices \( a, b = 1, 2, 3, 4 \) are contracted with the charge conjugation matrix \( C \) and the SU(2)L isospin indices \( A, B, C, D = 1, 2 \) are contracted with \( \epsilon \), the Levi-Civita symbol in two dimensions. After spontaneous electroweak symmetry breaking the Lagrangian in eq(4) generates additional terms of neutrino mass matrix

\[
L_{mass} = \frac{v^2}{M_{pl}} \lambda_{\alpha\beta} \nu_\alpha C^{-1} \nu_\beta,
\]

where \( v=174 \text{ GeV} \) is the VEV of electroweak symmetry breaking.

We assume that the gravitational interaction is “flavour blind”, that is \( \lambda_{\alpha\beta} \) is independent of \( \alpha, \beta \) indices. Thus the Planck scale contribution to the neutrino mass matrix is

\[
\mu \lambda = \mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},
\]

where the scale \( \mu \) is

\[
\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} \text{eV}.
\]

In our calculation, we take eq(6) as a perturbation to the main part of the neutrino mass matrix, that is generated by GUT dynamics. We compute the changes in neutrino mass eigenvalues and mixing angles induced by this perturbation. We assume that GUT scale operators give rise to the light neutrino mass matrix, which in mass eigenbasis, takes the form \( M = \text{diag}(M_1, M_2, M_3) \), where \( M_i \) are real and non negative. We take these to be the unperturbed \((0^{th} - \text{order})\) masses. Let \( U \) be the neutrino mixing matrix at \( 0^{th} \) – order. Then the corresponding \( 0^{th} \) – order mass matrix \( M \) in flavour space is given by

\[
M = U^*MU^\dagger.
\]

The \( 0^{th} \) – order MNS matrix \( U \) is given in this form

\[
U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix},
\]

where the nine elements are functions of three mixing angles, one Dirac phase and two Majorana phases. In terms of the above elements, the mixing angles are defined by

\[
\left| \frac{U_{e2}}{U_{e1}} \right| = \tan \theta_{12}, \quad \left| \frac{U_{\mu3}}{U_{\tau3}} \right| = \tan \theta_{23}, \quad |U_{e3}| = \sin \theta_{13}.
\]
In terms of the above mixing angles, the mixing matrix is written as

$$U = \text{diag}(e^{if_1}, e^{if_2}, e^{if_3})R(\theta_{23})\Delta R(\theta_{13})\Delta^* R(\theta_{12})\text{diag}(e^{ia_1}, e^{ia_2}, 1).$$

(12)

The matrix $\Delta = \text{diag}(e^{i\delta}, 1, e^{-i\delta})$ contains the Dirac phase $\delta$. This leads to CP violation in neutrino oscillations. $a_1$ and $a_2$ are the so-called Majorana phases, which affect the neutrinoless double beta decay. $f_1$, $f_2$, and $f_3$ are usually absorbed as a part of the definition of the charge lepton field. It is possible to rotate these phases away, if the mass matrix eq(5) is the complete mass matrix. However, since we are going to add another contribution to this mass matrix, these phases of the zeroth order mass matrix can have an impact on the complete mass matrix and thus must be retained. By the same token, the Majorana phases which are usually redundant for oscillations have a dynamical role to play now. Planck scale effects will add other contributions to the mass matrix. Including the Planck scale mass terms, the mass matrix in flavour space is modified as

$$M \rightarrow M' = M + \mu \lambda,$$

(13)

with $\lambda$ being a matrix whose elements are all 1 as discussed in eq(3). Since $\mu$ is small, we treat the second term (the Planck scale mass terms) in the above equation as a perturbation to the first term (the GUT scale mass terms). The impact of the perturbation on the neutrino masses and mixing angles can be seen by forming the hermitian matrix

$$M'^\dagger M' = (M + \mu \lambda)\dagger(M + \mu \lambda),$$

(14)

which is the matrix relevant for oscillation physics. To the first order in the small parameter $\mu$, the above matrix is

$$M'^\dagger M + \mu \lambda \dagger M + M' \mu \lambda.$$

(15)

This hermitian matrix is diagonalized by a new unitary matrix $U'$. The corresponding diagonal matrix $M'^2$, correct to first order in $\mu$, is related to the above matrix by $U' M'^2 U'^\dagger$. Rewriting $M$ in the above expression in terms of the diagonal matrix $M$ we get

$$U' M'^2 U'^\dagger = U(M'^2 + m^\dagger M + M m)U^\dagger$$

(16)

where

$$m = \mu U^\dagger \lambda U.$$

(17)

Here $M$ and $M'$ are the diagonal matrices with neutrino masses correct to 0th and 1th order in $\mu$. It is clear from eq(15) that the new mixing matrix can be written as:

$$U' = U(1 + i\delta \Theta),$$

where $\delta \Theta$ is the Majorana phase.
\[
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix} + \begin{pmatrix}
U_{e2} \delta \Theta_{12}^* + U_{e3} \delta \Theta_{23}^*, & U_{e1} \delta \Theta_{12} + U_{e3} \delta \Theta_{23}^*, & U_{e1} \delta \Theta_{13} + U_{e3} \delta \Theta_{23}^* \\
U_{\mu 2} \delta \Theta_{12}^* + U_{\mu 3} \delta \Theta_{13}^*, & U_{\mu 1} \delta \Theta_{12} + U_{\mu 3} \delta \Theta_{13}^*, & U_{\mu 1} \delta \Theta_{13} + U_{\mu 3} \delta \Theta_{23}^* \\
U_{\tau 2} \delta \Theta_{12}^* + U_{\tau 3} \delta \Theta_{13}^*, & U_{\tau 1} \delta \Theta_{12} + U_{\tau 3} \delta \Theta_{23}^*, & U_{\tau 1} \delta \Theta_{13} + U_{\tau 3} \delta \Theta_{23}^*
\end{pmatrix},
\]

where \( \delta \theta \) is a hermitian matrix that is first order in \( \mu \). From eq(15) we obtain

\[
M^2 + m^\dagger M + Mm = M''^2 + [i \delta \Theta, M'^2].
\]

Therefore to first order in \( \mu \), the mass squared difference \( \Delta M_{ij}^2 = M_i^2 - M_j^2 \) get modified [11, 13] as:

\[
\Delta M_{ij}'^2 = \Delta M_{ij}^2 + 2(M_i Re[m_{ii}] - M_j Re[m_{jj}]).
\]

The change in the elements of the mixing matrix, which we parameterized by \( \delta \Theta \), is given by

\[
\delta \Theta_{ij} = \frac{i Re(m_{ij})(M_i + M_j)}{\Delta M_{ij}^2} - \frac{Im(m_{ij})(M_i - M_j)}{\Delta M_{ij}^2}.
\]

The above equation determines only the off diagonal elements of matrix \( \delta \Theta_{ij} \). The diagonal elements of \( \delta \Theta \) can be set to zero by phase invariance.

The new Majorana neutrino magnetic moments due to Planck scale is given by

\[
\mu_{x}^2 = \sum_j \sum_k |U'_{xj}|^2 |\mu_{jk}|^2,
\]

where \( (x = e, \mu, \tau) \) is the flavour indices. In the case of three flavour, the magnetic moment of Majorana electron neutrinos is given by

\[
\mu_{e}^2 = (|\mu_{12}|^2 + |\mu_{13}|^2)(|U'_{e2}|^2 + |U'_{e3}|^2).
\]

and there is no dependence on the distance \( L \) or neutrino energy.

3. Results and Discussions

We assume the largest allowed value of 2 eV for degenerate neutrino mass which comes from tritium beta decay [12]. We also assume normal neutrino mass hierarchy. Thus we have \( M_1 = 2 \text{ eV}, M_2 = \sqrt{M_1^2 + \Delta_{21}} \) and \( M_3 = \sqrt{M_1^2 + \Delta_{31}} \). As in the case of 0th order mixing angles, we can compute 1st order mixing angles in terms of 1st order mixing
matrix elements [14]. We expect the mixing angles coming from GUT scale operators to be determined by some symmetries. For simplicity, here we assume a bi-maximal mixing pattern, $\theta_{12} = \theta_{23} = \pi/4$ and $\theta_{13} = 0$. We compute the modified mixing angles for the degenerate neutrino mass of 2 eV. We have taken $\Delta_{31} = 0.0025eV^2$ and $\Delta_{21} = 0.00008eV^2$. For simplicity we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. We have checked that non-zero values for these phases do not change our results. Since we have set $\theta_{13} = 0$, the Dirac phase $\delta$ drops out of the $0^{th}$ order mixing matrix. We consider the Planck scale effects on neutrino mixing and we get the given range of mixing parameter of MNS matrix

$$U' = R(\theta_{23} + \epsilon_3)U_{\text{phase}}(\delta)R(\theta_{13} + \epsilon_2)R(\theta_{12} + \epsilon_1),$$

(24)

In Planck scale, only $\theta_{12}(\epsilon_1 = \pm3^\circ)$ have reasonable deviation and $\theta_{13}$, $\theta_{23}$ deviation is very small less than $0.3^\circ$ [14]. In the new mixing at Planck scale we get the given moments of Majorana neutrinos

$$\mu_e^2 = (|\mu_{12}|^2 + |\mu_{13}|^2)(|U'_{e2}|^2 + |U'_{e3}|^2).$$

(25)

$$\mu_\mu^2 = (|\mu_{21}|^2 + |\mu_{23}|^2)(|U'_{\mu1}|^2 + |U'_{\mu3}|^2).$$

(26)

$$\mu_\tau^2 = (|\mu_{31}|^2 + |\mu_{32}|^2)(|U'_{e\tau2}|^2 + |U'_{e\tau3}|^2).$$

(27)

The best direct limit on the neutrino magnetic moment, $\mu_e \leq 1.8 \times 10^{-10}\mu_B$ at 90% CL [15], coming from neutrino electron scattering with anti-neutrino. However, the limit obtained using the SK data [4], $\mu_e \leq 1.5 \times 10^{-10}\mu_B$. Due to Planck scale effects, mixing angle $\theta_{12}$ and $\theta_{13}$ will contribute the magnetic moments of neutrinos.

**Conclusions**

We assumed that the main part of neutrino masses and mixings arise from GUT scale operators. We considered these to be $0^{th}$ order quantities. We further assumed that GUT scale symmetries constrain the neutrino mixing angles to be either bi-maximal or tri-bi-maximal. The gravitational interaction of lepton fields with SM Higgs field gives rise to an $SU(2)_L \times U(1)$ invariant dimension-5 effective lagrangian, given originally by Weinberg [10]. On electroweak symmetry breaking this operator leads to additional mass terms. We consider these to be a perturbation of GUT scale mass terms. We compute the first order corrections to neutrino mass eigenvalues and mixing angles. In [11], it was shown that the change in $\theta_{13}$, due to this perturbation, is small. Here we show that the change in $\theta_{23}$ also is small (less than $0.3^\circ$) but the change in $\theta_{12}$ can be substantial (about $\pm3^\circ$). The changes in all three mixing angles are proportional to the neutrino mass eigenvalues. To maximize the change we assumed degenerate neutrino masses $\simeq 2.0$ eV. For degenerate neutrino masses, the changes in $\theta_{13}$ and $\theta_{23}$ are inversely proportional to $\Delta_{31}$ and $\Delta_{32}$ respectively, whereas the change in $\theta_{12}$ is inversely proportional to $\Delta_{21}$. Since $\Delta_{31} \simeq \Delta_{32} \gg \Delta_{21}$, the change in $\theta_{12}$ is much larger than the changes in $\theta_{13}$ and
\[ \theta_{23} \]. In this paper, we write the neutrino magnetic moment expression for three flavour neutrino mixing. For majorana neutrino two non diagonal moment, these expression are Eq(4.0) for vacuum mixing. For majorana neutrino with three flavour, the expression is 
\[ \mu_e' = (|\mu_{12}|^2 + |\mu_{13}|^2)(|U_{e2}|^2 + |U_{e3}|^2) \]. In this paper, finally we wish make a important comment. Due to Planck scale effects mixing angle \( \theta_{12} \) and \( \theta_{13} \) contribute the magnetic moments of neutrinos.

References
