

Increasing Effective Gravitational Constant in Fractional ADD Brane Cosmology

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Abstract: Arkani-Hamed-Dimopoulos-Dvali brane model with time-increasing scaling gravitational constant is constructed within the framework of fractional action-like variational approach with one positive parameter ‘ α ’.

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Recently, we formulated a fractional action-like variational approach (**FALVA**) based on the following fractional action

$$S[q] \triangleq \frac{1}{\Gamma(\alpha)} \int_{t_0}^t L(\dot{q}, q, \tau) (t - \tau)^{\alpha-1} d\tau = \int_{t_0=0}^t L(q, \dot{q}, \tau) dg_t(\tau) \quad (1)$$

to model correctly nonconservative and dissipative dynamical systems [1,2,3]. Our formalism is inspired in reality from fractional analysis, in particular the Riemann-Liouville fractional integrals and derivatives, playing an important and significant role in the understanding of complex classical and quantum dynamical systems. $L(\dot{q}, q, \tau)$ is the Lagrangian weighted with $(t - \tau)^{\alpha-1} / \Gamma(\alpha)$ and $\Gamma(1 + \alpha) g_t(\tau) = (t^\alpha - (t - \tau)^\alpha)$ with the scaling properties $g_{kt}(k\tau) = k^\alpha g_t(\tau)$, $k > 0$ and $\alpha \in] - \infty, +\infty[$. For fixed “ t ”, integral (1) becomes a Stieltjes integral. The applications of the fractional calculus to the constrained dynamical systems and the extension of the fractional variational problem of Lagrange were discussed and some interesting cosmological, astrophysical and quantum field features are explored in discussed [4-11]. The extension of Noether’s symmetry theorem to FALVA was also recently discussed [12]. Recently, we have shown that the fractional variational approach when applied to Riemann geometry and Einstein General

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Relativity, will generate perturbed gravity and friction that decay weakly with the time, revealing some important cosmological features by modifying the gravity [9,11]. Encouraged by these results, we would like in this letter to extend our approach and study a simple cosmological application of **FALVA** on brane dissipative cosmology. Recently, the cosmological implications of the brane world scenario are investigated when the gravitational coupling G is not constant but rather vary with time [13,14]. We will show that **FALVA** will lead to a brane world with time-increasing G which may be important to describe a late universe. For this we reconsider the Arkani-Hamed-Dimopoulos-Dvali (**ADD**) brane model of zero tension with space-time topology of type $M = M^4 \otimes \mathbb{R}^n$, where M^4 is the familiar four-dimensional manifold and \mathbb{R}^n is the finite L dimensional extra-dimension. We define the Lagrangian of the theory as $L \rightarrow Lf(\tau)$ where $f(\tau)$ is a function of τ introduced to take into account the Riemann-Liouville fractional integral of order α of f and rewrite the following fractional action-like as [15-18]:

$$\begin{aligned} S_{4+n,\alpha} &= \frac{c^3}{16\pi G_{(3+n,\tau),\alpha}} \int_{M^4 \otimes \mathbb{R}^n} d^{3+n}x \sqrt{-g} R_i \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau) (t-\tau)^{\alpha-1} d\tau, \\ &\sim \frac{c^3}{16\pi G_{(3+n,\tau),\alpha}} \int_M d^3x \sqrt{-\tilde{g}} \tilde{R}_i L^n \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau) (t-\tau)^{\alpha-1} d\tau, \\ &\equiv \frac{c^3 {}_0D_t^{-\alpha} f(t)}{16\pi G_{(3+n,\tau),\alpha}} \int_M d^3x \sqrt{-\tilde{g}} \tilde{R}_i L^n, \end{aligned} \quad (2)$$

where \tilde{R}_i (*the Ricci scalar tensor*), \tilde{g} are the projections of the corresponding quantities on M^4 and

$$D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau) (t-\tau)^{\alpha-1} d\tau, \quad (3)$$

is the Riemann-Liouville fractional integral of order α . The volume of the extra-dimensions is L^n . When compared with the standard **ADD** one corresponding to $\alpha = 1$:

$$S_{4+n,\alpha=1} \sim \frac{c^3}{16\pi G_{(3+n,\tau),\alpha=1}} \int_M d^3x \sqrt{-\tilde{g}} \tilde{R}_i L^n d\tau, \quad (4)$$

we get

$$\begin{aligned} G_{(3+n,\tau),\alpha} &= G_{(3+n,\tau),\alpha=1} D_t^{-\alpha} f(t) \\ &= \frac{G_{(3+n,\tau),\alpha=1}}{\Gamma(\alpha)} \int_0^t f(\tau) (t-\tau)^{\alpha-1} d\tau, \end{aligned} \quad (5)$$

while when compared with the $4D$ action, we easily find:

$$G_{(3+n,\tau),\alpha} \equiv G_{(4+n),\alpha} \propto G_N L^n D_t^{-\alpha} f(t) \equiv \bar{G}_N L^n, \quad (6)$$

where $\bar{G}_N \equiv G_N D_t^{-\alpha} f(t) \rightarrow G_N$ for $\alpha = 1$ (*to be differentiated from ΔG*). As a result, we fall into the **ADD** brane model with time dependent scaling gravitational constant. One can also write:

$$G_{(3+n,\tau),\alpha} \equiv G_{(4+n),\alpha} \propto G_N L^n D_t^{-\alpha} f(t) \equiv G_N \bar{L}^n, \quad (7)$$

where $\bar{L}_N \equiv L_N D_t^{-\alpha} f(t) \rightarrow L_N$ for $\alpha = 1$. In other words, we have an **ADD** brane model with time-dependent scaling volume of the extra-dimensions [19,20,21]. If $f(\tau) = K$, K is a constant, $D_t^{-\alpha} K = K t^\alpha / \Gamma(1 + \alpha)$ [22,23] and as a result the effective \bar{L}_N increases with times for $\alpha > 0$. This mechanism could have important consequences on large extra-compactified dimensions [24]. In order to have $G_{(4+n),\alpha} = G_N L^n$, we need to write a fractional 4D action in the sense of (5), that is:

$$S_{4,\alpha} = \frac{c^3}{16\pi G_{(3,\tau),\alpha}} \int_{M^4} d^3x \sqrt{-\tilde{g}} \tilde{R}_i \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau) (t - \tau)^{\alpha-1} d\tau. \quad (8)$$

If again $f(\tau) = K$, K is a constant, the effective gravitational constant increases with time [25,26,27]. In fact, higher-dimensional theories imply that the gravitational constant is not fundamental constant. Instead it is related to the sizes of the extra-dimensional space, which are moduli fields in the four-dimensional effective theory [28,29]. It will be of interest to explore in details in the future a **FALVA** weak dissipative brane cosmology where the gravitational constant varies as $G = G_0 T^\alpha$, G_0 being a constant.

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