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Klein -Gordon Equation for the Heating of the Fermionic Gases

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Abstract: In this paper the model for the interaction of the ultra-short laser pulses with matter is proposed. The Klein-Gordon equation for heat transport is developed and solved. The condition for the existence of the massless heat carriers is formulated. The condition is $V\tau = \hbar/8$, where V is potential energy, τ is the relaxation time. The new thermal Klein-Gordon equation can be applied to the study of thermal processes for the fermionic gases (electron, nucleon).

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1. Introduction

The development of the new laser projects TESLA, Wake-field accelerators, LASETRON open the new field of investigation: electron and nucleon dynamics study with lasers.
The interaction of the ultrashort laser pulses with matter is the multistep process. In metals and semiconductors the first step is the heating of the electron gas. In nuclei it is the heating the nucleon gas The heating is described by the relaxation time τ_i=h/m$_i$v$_i^2$ where m$_i$ is the mass of the heat carriers and v=α_i c. The constants α_i are the coupling constants: α_1=1/137 for electromagnetic interaction ,α_2 =0.14 for strong interaction and and c is the speed of light in vacuum.
In monograph [1] it was shown that the absorption of thermal energy generated by ultrashort laser pulses is quantized. For the quantum of energy *heaton* the equation holds

$$E_h = p_h v_h \tag{1}$$

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where E_h is the *heaton* energy , p_h -the *heaton* momentum, $v_h = \alpha c$ and $\alpha = 1/137$ is the fine structure constant for the electromagnetic interaction .The relativistic formula for the energy reads:

$$E_h = ((p_h v_h)^2 + (m_h v_h^2)^2 \tag{2}$$

Comparing Eq(1) to Eq(2) we conclude that m_h=0

2. The model of the ultra-short laser pulses-matter interaction

Let us consider the interaction of the ultra-short laser pulse with matter. In monograph [1] the damped Klein – Gordon equation heat transport was developed

$$\frac{1}{v^2}\frac{\partial^2 T}{\partial t^2} + \frac{m}{\hbar}\frac{\partial T}{\partial t} + \frac{2Vm}{\hbar^2}T = \nabla^2 T \tag{3}$$

where T is the temperature of the system, m is the mass of heat carrier, V is the potential energy and v is the speed of heat transport and τ is the relaxation time for thermal excitation, Equation (1) is the *quantum hyperbolic* heat transport equation. The $\hbar$/m is the quantum diffusion coefficient, and v is the heat propagation velocity. For electromagnetic interaction v=$\alpha_1 c$ $\approx 10^6$ m/s and for strong interaction v=α_2c=10^7 m/s. For c→ ∞, v1 ,v2→ ∞ and Eq.(4) :

$$\frac{m}{\hbar}\frac{\partial T}{\partial t} + \frac{2Vm}{\hbar^2}T = \nabla^2 T \tag{4}$$

is the Fourier diffusion equation with the potential energy V and diffusion coefficient D=h/m. It is interesting to observe that only hyperbolic equation (2) is in agreement with special relativity theory, for the speeds $v_{1,2}$ are finite.

The solution of equation (1) can be written as (for 1D)

$$T(x,t) = e^{-t/2\tau}u(x,t) \tag{5}$$

After substituting formula (5) into (4) we obtain new equation

$$(\Box + q^2)u(x,t) = 0, \qquad \Box = \frac{1}{v^2}\frac{\partial u^2}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} \tag{6}$$

Equation (6) is the Klein-Gordon equation in Minkowski spacetime *(vt,x),*where mass term q^2, has the form

$$q^2 = \frac{2Vm}{\hbar^2} - \left(\frac{mv}{2\hbar}\right)^2 \tag{7}$$

As can be seen from formulae (5) and (7) for $q^2 = 0$ equation (4) is the undamped wave equation

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \tag{8}$$

with

$$V\tau = \frac{\hbar}{8} \tag{9}$$

It is interesting to observe that the equation

$$q^2 = m_h^2 v^2/h^2 = 0 \tag{10}$$

can be recognized formally as the condition for the massless fermions which obeys the wave equation.

In the seminal paper [2] Manchester Group of professor A. K. Geim showed that in graphene the electrons behave as the massless fermions. It occurs that the electrons fulfill the relation

$$E_e = p_e v_e \tag{11}$$

where the E_e is the energy of electrons, p_e is the momentum and v_e the speed of electrons. Eq.(8) is the dispersion relation for the relativistic particles with velocity $v_e \approx 10^6$ m/s. Let us consider the *heatons* temperature fields for the Cauchy problem (2D): $-\infty \leq x \leq \infty, \ -\infty \leq y \leq \infty$

$$\begin{aligned} u(x,t) &= f(x,y) \quad \text{at} \quad t=0 \\ \tfrac{\partial u}{\partial t} &= g(x,y) \quad \text{at} \quad t=0 \end{aligned} \tag{12}$$

2D solution of the equation (7) for Cauchy problem (11) has the form:

$$\begin{aligned} &\text{For } q^2v^2 < 0 \\ &w(x,y,t) = \tfrac{1}{2\pi v}\tfrac{\partial}{\partial t} \iint\limits_{\rho\leq vt} f(\zeta,\eta)\frac{\cosh\left(q\sqrt{(vt)^2-\rho^2}\right)}{\sqrt{(vt)^2-\rho^2}}d\zeta d\eta \\ &+ \ \tfrac{1}{2\pi v} \iint\limits_{\rho\leq vt} g(\zeta,\eta)\frac{\cosh\left(q\sqrt{(vt)^2-\rho^2}\right)}{\sqrt{(vt)^2-\rho^2}}d\zeta d\eta \\ &\rho = \sqrt{(x-\zeta)^2+(y-\eta)^2} \end{aligned} \tag{13}$$

$$\begin{aligned} &\text{For } q^2v^2 > 0 \\ &u(x,y,t) = \tfrac{1}{2\pi v}\tfrac{\partial}{\partial t} \iint\limits_{\rho\leq vt} f(\zeta,\eta)\frac{\cos\left(q\sqrt{(vt)^2-\rho^2}\right)}{\sqrt{(vt)^2-\rho^2}}d\zeta d\eta \\ &+ \ \tfrac{1}{2\pi v} \iint\limits_{\rho\leq vt} g(\zeta,\eta)\frac{\cos\left(q\sqrt{(vt)^2-\rho^2}\right)}{\sqrt{(vt)^2-\rho^2}}d\zeta d\eta \\ &\rho = \sqrt{(x-\zeta)^2+(y-\eta)^2} \end{aligned} \tag{14}$$

Conclusions

Klein-Gordon equation (5) is the master equation for the ultra-short thermal phenomena with finite transport speed . It can describe the thermal energy propagation in electron and nucleon gases. Both gases have the same formula for the relaxation time, but with different values of the mass for heat carriers and coupling constants. It means that the heat transport on the electron and nucleon scales can treated on the same footing.

For very long time periods, i.e. , when Δ t> τ the Klein- Gordon equation gives the same results as the Fourier equation Considering equations (13) and (14) we argue that the mode of the thermal motion changes as the function of the sign of the $(qv)^2$.

References

[1] M. Kozłowski, J. Marciak – Kozłowska, Thermal processes using attosecond laser pulses, Springer 2006.

[2] K.S. Novosilov et al., Nature 438 (2005) 197