

Considering Relativistic Symmetry as the First Principle of Quantum Mechanics

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Abstract: On the basis of the relativistic symmetry of Minkowski space, we derive a Lorentz invariant equation for a spread electron. This equation slightly differs from the Dirac equation and includes additional terms originating from the spread of an electron. Further, we calculate the anomalous magnetic moment based on these terms. These calculations do not include any divergence; therefore, renormalization procedures are unnecessary. In addition, the relativistic symmetry existing among coordinate systems will provide a new prospect for the foundations of quantum mechanics like the measurement process.

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1. Introduction

There are many problems associated with a relativistic quantum field theory. In particular, the issue of infinity accompanied by radiative correction is troublesome. Renormalization methods allow most of the divergence to be eliminated; however, it is difficult to accept this method as the final solution. In addition, much work has been done on the study of Dirac particles [1]. Nevertheless, even today, quantum field theory continues to be problematic with regard to its relationship with the theory of relativity. Therefore, apart from the conventional approach, we will directly derive a Lorentz invariant equation for an electron on the basis of the symmetry of Minkowski space. Thus, we assume two fundamental principles, instead of the usual rules of quantum mechanics, as follows:

- (i) An electron has an inherent relativistic symmetry, i.e., the behavior of an electron is described as the function of only an invariant parameter in Minkowski space.

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- (ii) An electron has a finite size as the world length that is proportional to the reciprocal of the inertial mass.

These principles imply that an electron identifies the Minkowski space as one-dimensional; this must be a cause of the quantum behavior of an electron.

In Sec.2, we extract a relativistic invariant parameter in Minkowski space. In Sec.3, based on these principles, we derive a relativistic equation for a spread electron. This equation slightly differs from the Dirac equation and includes additional terms originating from the spread of an electron. These terms are interpreted as an enhanced Pauli term, which is related to the anomalous magnetic moment [2]. Up to now, the Pauli term has been disregarded because it makes renormalization impossible. Nevertheless, in Sec.4, we calculate the corrections in magnetic moment based on these terms, without renormalization. In addition, in Sec.5, the measurement process in quantum theory is discussed based on the relativistic symmetry existing among coordinate systems.

2. Extraction of a Relativistic Invariant Parameter

In this paper, we use Einstein's summation convention for indices μ, ν , and ξ ($= 0, 1, 2, 3$). Using the Minkowski metric $g_{\mu\nu} = \text{diag}(+, -, -, -)$, we define the relation between a covariant vector a_μ and a contravariant vector a^ν as follows:

$$a_\mu = g_{\mu\nu} a^\nu. \quad (1)$$

Further, we substitute $\hbar = c = 1$ as a rule.

According to the special theory of relativity, world length squared δs^2 is a Lorentz invariant. For any inertial coordinate system, the following identity holds between a world length δs and coordinate intervals δx^μ :

$$\delta s^2 = g_{\mu\nu} \delta x^\mu \delta x^\nu, \quad \text{where} \quad x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (t, \mathbf{x}). \quad (2)$$

For convenience, we designate the origin of the inertial coordinate system as $s = 0$; thus, the quadratic form (2) becomes

$$s^2 = g_{\mu\nu} x^\mu x^\nu. \quad (3)$$

In order to extract the relativistic invariant parameter s , we take the square root of (3) in the linear form. Now, we assume that the quadratic form (3) is decomposed as follows:

$$s = \gamma_\mu x^\mu. \quad (4)$$

Let us square both sides of Eq.(4):

$$s^2 = \sum_{\mu>\nu} (\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu) x^\mu x^\nu + \frac{1}{2} \sum_{\mu=\nu} (\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu) x^\mu x^\nu. \quad (5)$$

On the other hand, the quadratic form (3) can be rewritten as follows:

$$s^2 = \sum_{\mu>\nu} 2g_{\mu\nu} x^\mu x^\nu + \frac{1}{2} \sum_{\mu=\nu} 2g_{\mu\nu} x^\mu x^\nu. \quad (6)$$

Expressions (5) and (6) are equivalent when γ_μ satisfies the following relation:

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}. \quad (7)$$

It implies that γ_μ are isomorphic forms of the Dirac γ -matrices.

We now introduce the rule of raising and lowering indices of the γ -matrices as well as the vectors:

$$\gamma^0 = \gamma_0, \quad \gamma^k = -\gamma_k \quad (k = 1, 2, 3). \quad (8)$$

Similar to Eq.(4), the following identical equation holds:

$$\frac{d}{ds}\Psi = \gamma^\mu \partial_\mu \Psi, \quad \text{where} \quad \partial_\mu \equiv \frac{\partial}{\partial x^\mu} \equiv \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right) \equiv \left(\frac{\partial}{\partial t}, \nabla \right). \quad (9)$$

When Ψ is a function only of argument s ,

$$\frac{d\Psi(s)}{ds} = \frac{\partial x^\mu}{\partial s} \frac{\partial \Psi(s)}{\partial x^\mu} = \gamma^\mu \partial_\mu \Psi, \quad \text{where} \quad \frac{\partial x^\mu}{\partial s} = \left(\frac{\partial s}{\partial x^\mu} \right)^{-1} = \gamma_\mu^{-1} = \gamma^\mu. \quad (10)$$

In this case, Eq.(9) is valid. However, since the identical equation (9) contains γ -matrices in the operator, we consider Ψ as a four-component function.

3. Derivation of Equations for a Spread Electron

In this section, we derive a relativistic difference equation for a spread electron. Further, we derive a wave equation from this difference equation. The wave equation agrees with the Dirac equation except for the existence of additional terms originating from the spread of an electron.

3.1 Derivation of the difference equation

Based on the principles referred to in Sec.1, we identify the space-time behavior of an electron using the following equation:

$$\rho(s + \delta s) = \rho(s), \quad (11)$$

where ρ denotes the density scalar and δs denotes the world length of the electron. This equation implies the conservation law for the existing probability of a spread electron under a proper time evolution. We transform Eq.(11) into an equation for any inertial coordinate system as follows:

We introduce an adjoint of Ψ , $\bar{\Psi} \equiv \Psi^\dagger \gamma^0$, and define the density scalar ρ as follows:

$$\rho(s) \equiv \bar{\Psi}(s)\Psi(s). \quad (12)$$

Here, Ψ is a wave function of an electron, which will be clarified later. From definition (12), $\bar{\Psi}\Psi$, and not Ψ , evidently relates to the existing probability of an electron. By substituting (12) in Eq.(11), we obtain

$$\bar{\Psi}(s + \delta s)\Psi(s + \delta s) = \bar{\Psi}(s)\Psi(s). \quad (13)$$

$\Psi(s + \delta s)$ is expressed as the Maclaurin series:

$$\Psi(s + \delta s) \rightarrow \exp\left(\delta s \frac{d}{ds}\right)\Psi(s). \quad (14)$$

Since Ψ depends only on s , (9) can be substituted in (14):

$$\exp\left(\delta s \frac{d}{ds}\right)\Psi(s) \rightarrow \exp(\delta s \gamma^\mu \partial_\mu)\Psi. \quad (15)$$

Similar to Eqs.(14) and (15), using the relation $\gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$, we obtain

$$\begin{aligned} \bar{\Psi}(s + \delta s) &\equiv \Psi^\dagger(s + \delta s)\gamma^0 \rightarrow \bar{\Psi} \exp(\delta s \gamma^\mu \overleftarrow{\partial}_\mu), \\ \text{where } \bar{\Psi}\gamma^\mu \overleftarrow{\partial}_\mu &\equiv \partial_\mu \bar{\Psi}\gamma^\mu. \end{aligned} \quad (16)$$

By introducing the 4×4 square matrix U of parameter δs , we assume the following equation:

$$\exp(\delta s \gamma^\mu \partial_\mu)\Psi = U(\delta s)\Psi. \quad (17)$$

Then, an adjoint equation of Eq.(17) is expressed as follows:

$$\bar{\Psi} \exp(\delta s \gamma^\mu \overleftarrow{\partial}_\mu) = \bar{\Psi}\gamma^0 U^\dagger(\delta s)\gamma^0. \quad (18)$$

Multiplying each side of Eq.(18) from the left with the corresponding sides of Eq.(17), we obtain the following expression:

$$\bar{\Psi} \exp(\delta s \gamma^\nu \overleftarrow{\partial}_\nu) \exp(\delta s \gamma^\mu \partial_\mu)\Psi = \bar{\Psi}\gamma^0 U^\dagger(\delta s)\gamma^0 U(\delta s)\Psi. \quad (19)$$

Therefore, matrix U should satisfy the relation:

$$\gamma^0 U^\dagger(\delta s)\gamma^0 U(\delta s) = I, \quad (20)$$

such that Eq.(19) is equivalent to Eq.(13). Here, we assume that matrix $U(\delta s)$ is continuous for δs and is expressed as follows by the introduction of the 4×4 square matrix M :

$$U = \exp(i \delta s M) \quad (21a)$$

$$\text{and } U^\dagger = \exp(-i \delta s M^\dagger). \quad (21b)$$

Matrix U^\dagger can be expressed as

$$U^\dagger(\delta s) = \exp(-i \delta s M^\dagger) = \lim_{n \rightarrow \infty} \left(I - \frac{i \delta s}{n} M^\dagger \right)^n. \quad (22)$$

If matrix M is chosen to satisfy the relation:

$$\gamma^0 M^\dagger \gamma^0 = M, \quad (23)$$

we get

$$\begin{aligned}
 \gamma^0 U^\dagger \gamma^0 U &= \lim_{n \rightarrow \infty} \gamma^0 \left(\mathbf{I} - \frac{i \delta s}{n} M^\dagger \right)^n \gamma^0 U \\
 &= \gamma^0 \left(\mathbf{I} - \frac{i \delta s}{n} M^\dagger \right) \gamma^0 \gamma^0 \left(\mathbf{I} - \frac{i \delta s}{n} M^\dagger \right) \gamma^0 \dots U \\
 &= \left(\mathbf{I} - \frac{i \delta s}{n} M \right) \left(\mathbf{I} - \frac{i \delta s}{n} M \right) \dots U \\
 &= \lim_{n \rightarrow \infty} \left(\mathbf{I} - \frac{i \delta s}{n} M \right)^n = \exp(-i \delta s M) U \\
 &= U^{-1} U = \mathbf{I},
 \end{aligned} \tag{24}$$

then matrix U satisfies (20).

Now, operating γ^0 from the left-hand side of (23), we have

$$M^\dagger \gamma^0 = \gamma^0 M. \tag{25}$$

On the other hand,

$$M^\dagger \gamma^0 = M^\dagger (\gamma^0)^\dagger = (\gamma^0 M)^\dagger. \tag{26}$$

Hence, relation (23) is equivalent to the condition that $\gamma^0 M$ is hermitian. Since $\gamma^0 \gamma^\mu$ is hermitian, we adopt a linear combination of γ^μ as M :

$$M = -e \gamma^\mu A_\mu, \tag{27}$$

where e is the electron charge and A_μ represents the four-vector potential of an electromagnetic field. Then, Eq.(17) becomes

$$\exp(\delta s \gamma^\mu \partial_\mu) \Psi = \exp(-i \delta s e \gamma^\mu A_\mu) \Psi. \tag{28}$$

We consider Eq.(28) to be the fundamental equation for a spread electron in an electromagnetic field.

3.2 Derivation of the wave equation

In Eq.(28), we substitute

$$\begin{cases} X \equiv \delta s \gamma^\mu \partial_\mu, \\ Y \equiv i \delta s e \gamma^\mu A_\mu. \end{cases} \tag{29}$$

Then,

$$\exp(X) \Psi = \exp(-Y) \Psi. \tag{30}$$

It follows that

$$\{\exp(X) \exp(Y)\} \exp(-Y) \Psi = \exp(-Y) \Psi. \tag{31}$$

According to the Campbell-Hausdorff formula:

$$\begin{aligned} \exp(X) \exp(Y) &= \exp(Z), \quad \text{where} \\ Z &= X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}\{[[X, Y], Y] - [[X, Y], X]\} + \dots, \end{aligned} \quad (32)$$

equation (31) becomes

$$\{\exp(Z) - I\} \exp(-Y)\Psi = 0. \quad (33)$$

We can expand $\{\exp(Z) - I\}$ into the infinite product of a sine function as follows:

$$\begin{aligned} &\exp(Z) - I \\ &= \exp\left(\frac{Z}{2}\right) \left\{ \exp\left(\frac{Z}{2}\right) - \exp\left(-\frac{Z}{2}\right) \right\} \\ &= -2i \exp\left(\frac{Z}{2}\right) \sin\left(\frac{iZ}{2}\right) \\ &= \exp\left(\frac{Z}{2}\right) Z \prod_{n=1}^{\infty} \left\{ \left(1 - \frac{iZ}{2n\pi}\right) \exp\left(\frac{iZ}{2n\pi}\right) \right\} \left\{ \left(1 + \frac{iZ}{2n\pi}\right) \exp\left(-\frac{iZ}{2n\pi}\right) \right\}. \end{aligned} \quad (34)$$

Thus, the equation that Ψ should satisfy is

$$(iZ - 2n\pi) \exp(-Y)\Psi = 0; \quad n = 0, \pm 1, \pm 2, \dots. \quad (35)$$

Using the expansion

$$\phi \exp(i\omega) = \exp(i\omega) \left\{ \phi + i[\phi, \omega] - \frac{1}{2}[[\phi, \omega], \omega] + \dots \right\}, \quad (36)$$

and substituting

$$\begin{cases} \phi \equiv iZ - 2n\pi, \\ \omega \equiv iY, \end{cases} \quad (37)$$

we can obtain the following wave equation:

$$\left(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu + V(\delta s) - \frac{2n\pi}{\delta s} \right) \Psi = 0, \quad (38)$$

where δs is determined as the Compton wavelength $\lambda_c (= 2\pi/m)$ of the electron and n is selected as 1 such that the mass term of the electron is specified correctly.

In this case, wave equation (38) agrees with the Dirac equation except for the existence of $V(\delta s)$. Here, $V(\delta s)$ is represented as a power series of δs as follows:

$$V(\delta s) = V_1 + V_2 + \dots, \quad (39)$$

$$V_1 \equiv +\frac{1}{2} \delta s e [\gamma^\mu \partial_\mu, \gamma^\nu A_\nu], \quad (39a)$$

$$V_2 \equiv -\frac{1}{12} i \delta s^2 e [[\gamma^\mu \partial_\mu, \gamma^\nu A_\nu], \gamma^\xi (i\partial_\xi + eA_\xi)]. \quad (39b)$$

In addition, if δs is the infinitesimal, $V(\delta s) \rightarrow 0$ and the mass term corresponds to the infinite bare electron mass. Then, the usual Dirac equation is reproduced when the mass term is renormalized. Therefore, the Dirac equation is inherently relativistic invariant. However, we assume another standpoint because $V(\delta s)$ has the physical significance as shown below.

4. Calculation of the Anomalous Magnetic Moment

In this section, by considering V_n as the n th order correction of the Dirac equation, we evaluate the corrections in magnetic moment using the Foldy-Wouthuysen transformation [3] (FW transformation). The result obtained is in good agreement with the QED calculation.

4.1 FW transformation of the Dirac equation

We begin with the FW transformation of the Dirac equation:

$$(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\Psi = 0. \quad (40)$$

Note that $\beta \equiv \gamma^0$, $\beta^2 = 1$, $\boldsymbol{\alpha} \equiv \beta\boldsymbol{\gamma}$, $\partial_\mu \equiv (\partial/\partial t, \boldsymbol{\nabla})$, and $A_\mu \equiv (\phi, -\mathbf{A})$. Thus, multiplying the left-hand side of Eq.(40) with the β matrix, we obtain a time-independent Dirac Hamiltonian:

$$\mathcal{H} = \beta m + \varepsilon + o \quad (41)$$

$$\text{with the even operator } \varepsilon = e\phi \quad (41a)$$

$$\text{and the odd operator } o = \boldsymbol{\alpha} \cdot \boldsymbol{\pi}; \quad (41b)$$

$$\boldsymbol{\pi} \equiv \mathbf{p} - e\mathbf{A} \equiv -i\boldsymbol{\nabla} - e\mathbf{A}.$$

Performing the FW transformation eliminates the odd operator from \mathcal{H} :

$$\mathcal{H}^{\text{FW}} \simeq \beta m + \varepsilon + \frac{1}{2m}\beta o^2 - \frac{1}{8m^2}[o, [o, \varepsilon]]. \quad (42)$$

Using the identity

$$(\boldsymbol{\alpha} \cdot \mathbf{a})(\boldsymbol{\alpha} \cdot \mathbf{b}) = \mathbf{a} \cdot \mathbf{b} + i\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b}), \quad (43)$$

where \mathbf{a} and \mathbf{b} are arbitrary vectors and $\boldsymbol{\sigma}$ denotes the 4×4 Dirac spin matrix, we can obtain the explicit form of Eq.(42) as follows:

$$\begin{aligned} \mathcal{H}^{\text{FW}} \simeq & \frac{1}{2m}\beta\boldsymbol{\pi}^2 + e\phi + \beta m \\ & - \frac{e}{2m} \frac{\mathbf{g}}{2} \beta \boldsymbol{\sigma} \cdot \mathbf{B} - \frac{1}{2} \frac{e}{2m} \frac{1}{m} \boldsymbol{\sigma} \cdot (\mathbf{E} \times \boldsymbol{\pi}) + \frac{e}{8m^2} \boldsymbol{\nabla}^2 \phi, \end{aligned} \quad (44)$$

where $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$, $\mathbf{E} = -\boldsymbol{\nabla} \phi$,

and the gyromagnetic ratio \mathbf{g} of an electron described by the Dirac equation is only 2.

4.2 Effect of V_1

We evaluate the alteration in \mathcal{H}^{FW} by adding V_1 to the Dirac equation. V_1 is rewritten as follows:

$$\begin{aligned} V_1 = & +\kappa \frac{e}{4m} \{ \sigma^{\mu\nu} F_{\mu\nu} - 2\sigma^{\mu\nu} (A_\mu \partial_\nu - A_\nu \partial_\mu) \}, \\ \text{where } & \kappa \equiv -i \delta s m = -2\pi i, \end{aligned} \quad (45)$$

$$\sigma^{\mu\nu} \equiv \frac{i}{2} [\gamma^\mu, \gamma^\nu], \text{ and } F_{\mu\nu} \equiv \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}.$$

This expression contains the Pauli term $\kappa(e/4m)\sigma^{\mu\nu}F_{\mu\nu}$; therefore, it appears to be related to the magnetic moment. Further, V_1 can also be expressed as follows:

$$V_1 = -\kappa\frac{e}{2m}\left\{(\boldsymbol{\sigma}\cdot\mathbf{B} - i\boldsymbol{\alpha}\cdot\mathbf{E}) - 2\left\{\boldsymbol{\sigma}\cdot(\mathbf{A}\times\nabla) - i\boldsymbol{\alpha}\cdot\left(\phi\nabla + \mathbf{A}\frac{\partial}{\partial t}\right)\right\}\right\}, \quad (46)$$

where alterations in (41a) and (41b) are

$$\delta\varepsilon = +\kappa\frac{e}{2m}\beta\boldsymbol{\sigma}\cdot(\mathbf{B} - 2\mathbf{A}\times\nabla) \quad (46a)$$

$$\text{and } \delta o = -\kappa\frac{e}{2m}\beta i\boldsymbol{\alpha}\cdot\left\{\mathbf{E} - 2\left(\phi\nabla + \mathbf{A}\frac{\partial}{\partial t}\right)\right\}. \quad (46b)$$

We then calculate the alterations in \mathcal{H}^{FW} by using identity (43),

$$\begin{aligned} \delta\left\{\frac{1}{2m}\beta o^2\right\} &\simeq \frac{1}{2m}\beta(o\delta o + \delta o o) \\ &= +\kappa\frac{e}{4m^2}i\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E}) + \kappa\frac{e}{4m^2}(\nabla\cdot\mathbf{E} + \mathbf{E}\cdot\nabla) \\ &\quad -\kappa\frac{e}{2m}\beta\boldsymbol{\sigma}\cdot(\mathbf{B} - 2\mathbf{A}\times\nabla)\left\{\frac{\beta}{m}\left(i\frac{\partial}{\partial t} - e\phi\right)\right\} \\ &\quad -\kappa\frac{e^2}{2m^2}\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{E}) \end{aligned} \quad (47)$$

and

$$\begin{aligned} \delta\left\{-\frac{1}{8m^2}[o, [o, \varepsilon]]\right\} \\ \simeq -\frac{1}{8m^2}\{[\delta o, [o, \varepsilon]] + [o, [\delta o, \varepsilon]] + [o, [o, \delta\varepsilon]]\} \\ \simeq -\kappa\frac{e^2}{4m^2}i\mathbf{A}\cdot\mathbf{E}\left\{\frac{\beta}{m}\left(i\frac{\partial}{\partial t} - e\phi\right)\right\}. \end{aligned} \quad (48)$$

Consequently, the alteration in \mathcal{H}^{FW} due to V_1 can be expressed as follows:

$$\begin{aligned} \delta\mathcal{H}^{\text{FW}} &\simeq \delta\varepsilon + \delta\left\{\frac{1}{2m}\beta o^2\right\} + \delta\left\{-\frac{1}{8m^2}[o, [o, \varepsilon]]\right\} \\ &\simeq +\kappa\frac{e}{4m^2}i\boldsymbol{\sigma}\cdot(\nabla\times\mathbf{E}) + \kappa\frac{e}{4m^2}(\nabla\cdot\mathbf{E} + \mathbf{E}\cdot\nabla) \\ &\quad -\kappa\frac{e}{2m}\beta\boldsymbol{\sigma}\cdot(\mathbf{B} - 2\mathbf{A}\times\nabla)\left\{\frac{\beta}{m}\left(i\frac{\partial}{\partial t} - e\phi - \beta m\right)\right\} \\ &\quad -\kappa\frac{e^2}{2m^2}\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{E}) - \kappa\frac{e^2}{4m^2}i\mathbf{A}\cdot\mathbf{E}\left\{\frac{\beta}{m}\left(i\frac{\partial}{\partial t} - e\phi\right)\right\}. \end{aligned} \quad (49)$$

We now assume the following conditions:

- The external electromagnetic field is sufficiently small and static.
- The kinetic energy is sufficiently smaller than the rest energy of the electron.

Then, in Eq.(49), $\nabla\times\mathbf{E} = 0$ and $i(\partial/\partial t) - e\phi \simeq \beta m$. In addition, since the scalar potential ϕ is time-independent, it commutes with \mathcal{H}^{FW} , i.e.,

$$0 = \frac{d\phi}{dt} = i[\mathcal{H}^{\text{FW}}, \phi] \simeq \beta\frac{i}{2m}(p^2\phi - \phi p^2) = \beta\frac{i}{2m}(\nabla\cdot\mathbf{E} + \mathbf{E}\cdot\nabla). \quad (50)$$

Therefore, Eq.(49) becomes

$$\delta\mathcal{H}^{\text{FW}} \simeq -\kappa\frac{e^2}{2m^2}\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{E}) - \kappa\frac{e^2}{4m^2}i\mathbf{A}\cdot\mathbf{E}. \quad (51)$$

Since $\mathbf{A}\times\mathbf{E}$ and $\mathbf{A}\cdot\mathbf{E}$ are sufficiently small, $\delta\mathcal{H}^{\text{FW}}$ can be neglected as compared to \mathcal{H}^{FW} . Hence, Eq.(38) roughly agrees with the Dirac equation.

4.3 Self-energy influence

Here, we assume that there exist no external electric charges. According to classical electromagnetics, the electron obtains its self-energy in the form of electrostatic energy. In addition, the electrostatic energy $e\phi$ in the Dirac Hamiltonian differs from m by a factor of the β matrix:

$$\mathcal{H} \simeq \beta m + e\phi = \beta(m + \beta e\phi). \quad (52)$$

Hence, the self-energy can be defined as $\delta m \equiv \beta e\phi$ such that δm may behave as a part of m .

We now evaluate the alteration $\delta\mathcal{H}^{\text{FW}}$ by taking the self-energy into consideration. Here, the electric field generated by the rest electron is $\mathbf{E} = -(\phi/r^2)\mathbf{r}$, and the vector potential for a constant magnetic field is $\mathbf{A} = (1/2)\mathbf{B}\times\mathbf{r}$, where \mathbf{r} is the position vector from the charge. Then, the second term in (51) becomes zero, since $\mathbf{A}\cdot\mathbf{E} = -(1/2)(\phi/r^2)(\mathbf{B}\times\mathbf{r})\cdot\mathbf{r} = 0$. On the other hand, $(\mathbf{B}\times\mathbf{r})\times\mathbf{r} = -\mathbf{B}(\mathbf{r}\cdot\mathbf{r}) + \mathbf{r}(\mathbf{B}\cdot\mathbf{r}) = -\mathbf{B}r^2$, since the mean value of $\mathbf{B}\cdot\mathbf{r}$ becomes zero due to the spherical symmetry of \mathbf{r} . Therefore, the first term in (51) becomes

$$\begin{aligned} -\kappa\frac{e^2}{2m^2}\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{E}) &= -\frac{1}{2}\left(\kappa\frac{\beta e\phi}{m}\right)\frac{e}{2m}\frac{\beta\boldsymbol{\sigma}\cdot\{-(\mathbf{B}\times\mathbf{r})\times\mathbf{r}\}}{r^2} \\ &= -\frac{1}{2}\left(\kappa\frac{\delta m}{m}\right)\frac{e}{2m}\beta\boldsymbol{\sigma}\cdot\mathbf{B}. \end{aligned} \quad (53)$$

It is observed that (53) gives a correction in the magnetic moment that is proportional to δm .

4.4 Self-energy estimation

In this study, we assumed that an electron has a time-like size δs as the world length. We then interpret $0.5\delta s (= \pi/m)$ as the four-dimensional radius r_0 of the electron, which is the same degree of Zitterbewegung amplitude being expected from the Dirac equation.

The classical calculation provides a good approximation of the self-energy since the quantum effects are insignificant when $r_0 > 1/m$, even if these effects take part in. In order to estimate the self-energy of an electron that is spread in four dimensions, we extend the definition of the electric field, as shown below, by applying Gauss's law to the surface of a four-dimensional sphere. The area of the four-dimensional sphere is $2\pi^2r^3$

and the electric charge on the sphere is e multiplied by r_0 , hence, the four-dimensional electric field $\tilde{\mathbf{E}}$ shall be defined as

$$\tilde{\mathbf{E}} = \frac{r_0 e}{2\pi^2 \epsilon_0 r^3} \frac{\mathbf{r}}{r}, \quad (54)$$

where ϵ_0 denotes the dielectric constant of vacuum.

Thus, the four-dimensional self-energy $\widetilde{\delta m}$ of an electron can be estimated by an analogy with the classical electrostatic energy as

$$r_0 \delta m = \widetilde{\delta m} = \frac{\epsilon_0}{2} \int \tilde{\mathbf{E}}^2 d^4 r. \quad (55)$$

Therefore,

$$\begin{aligned} \delta m &= \frac{\epsilon_0}{2r_0} \int \tilde{\mathbf{E}}^2 d^4 r = \frac{\epsilon_0}{2r_0} \int_{r_0}^{\infty} \left(\frac{r_0 e}{2\pi^2 \epsilon_0 r^3} \right)^2 2\pi^2 r^3 i dr \\ &= i \frac{e^2}{8\pi^2 \epsilon_0 r_0}, \end{aligned} \quad (56)$$

where spatial integration is performed with respect to the imaginary radius ir , since an electron has a time-like spread. Thus, the self-energy of an electron with a time-like spread becomes an imaginary number and it is not observable.

Nevertheless, by substituting (56) into (53), we can obtain the first-order correction of the magnetic moment:

$$-\frac{1}{2} \left(\kappa \frac{\delta m}{m} \right) \frac{e}{2m} \beta \boldsymbol{\sigma} \cdot \mathbf{B} = -\frac{1}{2} \left(\frac{\alpha}{\pi} \right) \frac{e}{2m} \beta \boldsymbol{\sigma} \cdot \mathbf{B}, \quad (57)$$

where $\alpha (\equiv e^2/4\pi\epsilon_0)$ denotes the fine structure constant. Accordingly, the correction in the gyromagnetic ratio can be expressed as

$$\frac{g-2}{2} = \frac{1}{2} \left(\frac{\alpha}{\pi} \right); \quad (58)$$

this expression agrees with the calculation by J. Schwinger (1948) [4].

4.5 Higher-order correction in the magnetic moment

Finally, we calculate the α^2 -order correction in the magnetic moment. We use the symbol $\delta^{(2)}$ to denote second-order variations and omit calculations that do not directly contribute to the magnetic moment. We now expand (39b) as follows:

$$V_2 = \kappa^2 \frac{e}{12m^2} i [[\gamma^\mu \partial_\mu, \gamma^\nu A_\nu], \gamma^\xi (i\partial_\xi + eA_\xi)], \quad (59)$$

where

$$\begin{aligned}
& [[\gamma^\mu \partial_\mu, \gamma^\nu A_\nu], \gamma^\xi (i\partial_\xi + eA_\xi)] \\
& = \\
(a) \quad & +2 \gamma^\xi \gamma^\nu (\partial_\xi A_\nu) (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu) \\
(b) \quad & +4A_\nu \gamma^\mu (i\partial_\mu - eA_\mu) \partial^\nu \\
(c) \quad & -4i \gamma^\nu A_\nu \gamma^\xi \partial_\xi \gamma^\mu \partial_\mu \\
(d) \quad & +4e \gamma^\nu A_\nu \gamma^\xi A_\xi \gamma^\mu \partial_\mu \\
(e) \quad & -2i \gamma^\mu (\partial_\nu A_\mu) \partial^\nu \\
(f) \quad & -4e \gamma^\nu (\partial_\mu A_\nu) A^\mu \\
(g) \quad & +6e \gamma^\mu (\partial_\mu A_\nu) A^\nu.
\end{aligned} \tag{60}$$

In the following, we evaluate each term in (60). Note that $e\gamma^0 A_0 = \beta e\phi = \delta m$. Thus, we have

$$\begin{aligned}
(a): \quad & +2 \gamma^\xi \gamma^\nu (\partial_\xi A_\nu) (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu) \\
& \simeq +2i \boldsymbol{\sigma} \cdot \mathbf{B} (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu) + 2 \boldsymbol{\alpha} \cdot \mathbf{E} (i\gamma^0 \partial_0 - e\gamma^0 A_0) \\
& \simeq +2i \boldsymbol{\sigma} \cdot \mathbf{B} m - 2i \beta \boldsymbol{\alpha} \cdot \mathbf{E} \partial_0 - 2 \boldsymbol{\alpha} \cdot \mathbf{E} \delta m.
\end{aligned} \tag{61}$$

The first term in (61) results in the following alteration in \mathcal{H}^{FW} :

$$\delta^{(2)} \varepsilon_{a1} = -\beta \kappa^2 \frac{e}{12m^2} i (+2i \boldsymbol{\sigma} \cdot \mathbf{B} m) = +\frac{1}{3} \kappa^2 \frac{e}{2m} \beta \boldsymbol{\sigma} \cdot \mathbf{B}. \tag{62}$$

Although (62) contributes to the magnetic moment, it will be counterbalanced by another correction term that will be calculated later in (69). The second term in (61) is unrelated to the magnetic moment since the $\boldsymbol{\sigma}$ matrix does not appear in the result of the FW transformation. The third term in (61) contributes to the magnetic moment; hence, it will be evaluated below together with the (g) term.

The terms (b), (c), and (d) might contribute to the magnetic moment through the variations in \mathbf{p} and \mathbf{A} of (41b). It should be noted that $i\gamma^0 \partial_0 \simeq m + \delta m$ and $e\gamma^0 A_0 = \delta m$. Thus, we have

$$\begin{aligned}
(b): \quad & +4A_\nu \gamma^\mu (i\partial_\mu - eA_\mu) \partial^\nu \\
& \simeq +\frac{4i}{e} \sum_k e\gamma^0 A_0 \gamma^k (i\partial_k + eA_k) i\gamma^0 \partial_0 - \frac{4i}{e} e\gamma^0 A_0 \gamma^0 (i\partial_0 - eA_0) i\gamma^0 \partial_0 \\
& \simeq -\frac{4i}{e} \sum_k \gamma^k (-i\partial_k - eA_k) \delta m (m + \delta m) - \frac{4i}{e} \delta m m (m + \delta m),
\end{aligned} \tag{63}$$

$$\begin{aligned}
(c): \quad & -4i \gamma^\nu A_\nu \gamma^\xi \partial_\xi \gamma^\mu \partial_\mu \\
& \simeq -\frac{4i}{e} \sum_k e\gamma^k A_k i\gamma^0 \partial_0 i\gamma^0 \partial_0 + \frac{4i}{e} e\gamma^0 A_0 i\gamma^0 \partial_0 i\gamma^0 \partial_0 \\
& \quad -\frac{4i}{e} \sum_k e\gamma^0 A_0 (\gamma^0 \gamma^k + \gamma^k \gamma^0) \partial_0 \partial_k \\
& \simeq +\frac{4i}{e} \sum_k \gamma^k (-eA_k) (m + \delta m)^2 + \frac{4i}{e} \delta m (m + \delta m)^2,
\end{aligned} \tag{64}$$

$$\begin{aligned}
\text{(d): } & + 4e \gamma^\nu A_\nu \gamma^\xi A_\xi \gamma^\mu \partial_\mu \\
& \simeq -\frac{4i}{e} \sum_k e \gamma^0 A_0 e \gamma^0 A_0 i \gamma^k \partial_k - \frac{4i}{e} e \gamma^0 A_0 e \gamma^0 A_0 i \gamma^0 \partial_0 \\
& + \frac{4i}{e} \sum_k (\gamma^0 \gamma^k + \gamma^k \gamma^0) e^2 A_0 A_k i \gamma^0 \partial_0 \\
& \simeq + \frac{4i}{e} \sum_k \gamma^k (-i \partial_k) \delta m^2 - \frac{4i}{e} \delta m^2 (m + \delta m).
\end{aligned} \tag{65}$$

After collecting the terms (b), (c), and (d), we get the following two terms:

$$+ \frac{4i}{e} \sum_k \gamma^k (-e A_k) m^2 + \frac{4i}{e} \sum_k \gamma^k (i \partial_k - e A_k) m \delta m. \tag{66}$$

These terms result in the following alterations in \mathcal{H} :

$$\begin{aligned}
\delta^{(2)} O_{bcd} & \simeq -\beta \kappa^2 \frac{e}{12m^2} i \frac{4i}{e} \sum_k \gamma^k (-e A_k) m^2 \\
& - \beta \kappa^2 \frac{e}{12m^2} i \frac{4i}{e} \sum_k \gamma^k (i \partial_k - e A_k) m \delta m \\
& = + \frac{1}{3} \kappa^2 (-e \boldsymbol{\alpha} \cdot \mathbf{A}) + \frac{1}{3} \kappa^2 \frac{\delta m}{m} \boldsymbol{\alpha} \cdot (-\mathbf{p} - e \mathbf{A}).
\end{aligned} \tag{67}$$

Then, vector potential \mathbf{A} in (41b) is corrected by the first term in (67) as

$$\mathbf{A} \rightarrow \left(1 + \frac{1}{3} \kappa^2\right) \mathbf{A}. \tag{68}$$

Accordingly, the magnetic moment in (44) is corrected as

$$-\frac{e}{2m} \beta \boldsymbol{\sigma} \cdot \mathbf{B} \rightarrow -\left(1 + \frac{1}{3} \kappa^2\right) \frac{e}{2m} \beta \boldsymbol{\sigma} \cdot \mathbf{B}. \tag{69}$$

The variation in (69) is counterbalanced by $\delta^{(2)} \varepsilon_{a1}$, which was previously calculated. In the second term of (67), corrections in the magnetic moment due to variations in \mathbf{p} and \mathbf{A} cancel each other out in the result of the FW transformation.

The (e) and (f) terms can be neglected under the conditions assumed in Subsection 4.2.

$$\begin{aligned}
\text{(g): } & + 6e \gamma^\mu (\partial_\mu A_\nu) A^\nu \\
& \simeq + 6 \sum_k \gamma^0 \gamma^k (-\partial_k A_0) e \gamma^0 A_0 \simeq + 6 \boldsymbol{\alpha} \cdot \mathbf{E} \delta m.
\end{aligned} \tag{70}$$

This term contributes to the magnetic moment as well as the third term of (a).

In any case, we obtain the alteration in \mathcal{H} related to the magnetic moment by adding (g) and the third term of (a) as follows:

$$\delta^{(2)} O_{a3+g} = -\beta \kappa^2 \frac{e}{12m^2} i (-2 \boldsymbol{\alpha} \cdot \mathbf{E} \delta m + 6 \boldsymbol{\alpha} \cdot \mathbf{E} \delta m) = -\frac{1}{3} \kappa^2 \frac{\delta m}{m} \frac{e}{m} i \beta \boldsymbol{\alpha} \cdot \mathbf{E}. \tag{71}$$

This term results in the following alteration in \mathcal{H}^{FW} :

$$\begin{aligned} \delta^{(2)} \left\{ \frac{1}{2m} \beta o^2 \right\}_{a3+g} &\simeq \frac{1}{2m} \beta \{ o \delta^{(2)} o_{a3+g} + \delta^{(2)} o_{a3+g} o \} \\ &\simeq -\frac{1}{3} \kappa^2 \frac{\delta m}{m} \frac{e}{2m^2} i \beta \{ (-e \boldsymbol{\alpha} \cdot \mathbf{A}) (\beta \boldsymbol{\alpha} \cdot \mathbf{E}) + (\beta \boldsymbol{\alpha} \cdot \mathbf{E}) (-e \boldsymbol{\alpha} \cdot \mathbf{A}) \} \\ &= +\frac{1}{3} \left(\kappa \frac{\delta m}{m} \right) \left\{ \kappa \frac{e^2}{m^2} \boldsymbol{\sigma} \cdot (\mathbf{A} \times \mathbf{E}) \right\} = +\frac{1}{3} \left(\frac{\alpha}{\pi} \right)^2 \frac{e}{2m} \beta \boldsymbol{\sigma} \cdot \mathbf{B}. \end{aligned} \quad (72)$$

Consequently, the correction in the gyromagnetic ratio up to the order of α^2 becomes

$$\frac{\mathbf{g} - 2}{2} = \frac{1}{2} \left(\frac{\alpha}{\pi} \right) - \frac{1}{3} \left(\frac{\alpha}{\pi} \right)^2, \quad (73)$$

whereas the corresponding correction calculated in QED is [5, 6]

$$\frac{\mathbf{g}_{\text{qed}} - 2}{2} = \frac{1}{2} \left(\frac{\alpha}{\pi} \right) - 0.3285 \left(\frac{\alpha}{\pi} \right)^2. \quad (74)$$

Both these results agree within the error margin of the order of α^3 .

4.6 Vacuum polarization effect

In the previous sections, the effect of vacuum polarization is not taken into account. Hence, we consider that the marginal error between (73) and (74) can be attributed to vacuum polarization due to the electron pair creation. The size of the error can be approximated by the following expression:

$$\frac{\mathbf{g}_{\text{qed}}}{2} - \frac{\mathbf{g}}{2} = \left\{ -0.3285 + \frac{1}{3} \right\} \left(\frac{\alpha}{\pi} \right)^2 \simeq +\frac{1}{12} \left(\frac{\alpha}{4\pi} \right)^2. \quad (75)$$

Accordingly, we assume that the alteration in \mathcal{H}^{FW} due to the electron pair creation is given by the following formula:

$$\delta \mathcal{H}^{\text{FW}}_{e^+e^-} \simeq -\frac{1}{12} \left(\frac{\alpha}{4\pi} \right)^2 \frac{e}{2m} \beta \boldsymbol{\sigma} \cdot \mathbf{B}. \quad (76)$$

Then, the correction in the gyromagnetic ratio for the α^2 order is recalculated as

$$-\left\{ +\frac{1}{3} - \frac{1}{12} \left(\frac{1}{4} \right)^2 \right\} \left(\frac{\alpha}{\pi} \right)^2 \simeq -0.3281 \left(\frac{\alpha}{\pi} \right)^2. \quad (77)$$

In fact, this value is almost in agreement with that of the QED calculation and differs from the experimental value by only $1.5 (\alpha/\pi)^3$ [7].

The above assumption is found to be appropriate by estimating the muon magnetic moment in which the influence of vacuum polarization is more significant. The vacuum polarization due to the muon pair creation yields a correction similar to that observed in Eq.(76):

$$\delta \mathcal{H}^{\text{FW}}_{\mu^+\mu^-} \simeq -\frac{1}{12} \left(\frac{\alpha}{4\pi} \right)^2 \frac{e}{2m_\mu} \beta \boldsymbol{\sigma} \cdot \mathbf{B}, \quad (78)$$

where m_μ denotes the muon mass. In addition, the effect of electron pair creation exists. With regard to the muon magnetic moment, we simply assume that the effect of electron pair creation is the same as that given by Eq.(76):

$$\delta\mathcal{H}^{\text{FW}}_{e^+e^-} \simeq -\frac{1}{12}\left(\frac{\alpha}{4\pi}\right)^2\frac{e}{2m}\beta\boldsymbol{\sigma}\cdot\mathbf{B} = -\frac{1}{12}\left(\frac{\alpha}{4\pi}\right)^2\left(\frac{m_\mu}{m}\right)\frac{e}{2m_\mu}\beta\boldsymbol{\sigma}\cdot\mathbf{B}, \quad (79)$$

where the mass ratio m_μ/m is around 206.8.

Then, the correction in the muon gyromagnetic ratio for the α^2 order is obtained by adding (78), (79), and (72) for the muon mass:

$$-\left\{+\frac{1}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^2\left(1 + \frac{m_\mu}{m}\right)\right\}\left(\frac{\alpha}{\pi}\right)^2 \simeq 0.75\left(\frac{\alpha}{\pi}\right)^2. \quad (80)$$

This value agrees with that obtained by the QED calculation [8, 9].

Therefore, the second-order correction (73) is also considered to be an appropriate result when the effect of vacuum polarization is not taken into account.

5. Many-Coordinate Systems Interpretation

We assumed that an electron has an inherent relativistic symmetry. In other words, all the inertial coordinate systems in Minkowski space are symmetric and superpositioned from a viewpoint of a free electron. In this context, the measurement process is also explained as symmetry breaking caused by the observation from a specific inertial coordinate system. This results in a nonlocal stochastic process because for an electron, the measurement implies an unpredictable selection of a specific coordinate system in which the observation is performed.

For example, quantum entanglement, i.e., the EPR correlation [10] of pair particles with opposite helicity, is prepared by the superposition of a right- and a left-handed coordinate system, which correspond to either of the eigenstates of helicity. The observation of helicity in one particle concurrently fixes the state of another particle through the selection of either of the coordinate systems.

The many-coordinate systems interpretation presented here is similar to the many-worlds interpretation of Everett [11] *et al.* They propose the existence of many worlds corresponding to the superpositioned eigenstates. However, it is not the worlds but the coordinate systems that will branch because of the observation. Special relativity guarantees that all the coordinate systems that may branch exist in a Minkowski space. In addition, we consider that the material particle in classical mechanics is a substance in which relativistic symmetry is almost lost due to the coupling of a large number of elementary particles.

6. Conclusion

In this study, we assumed that the quantum behavior of an electron lies in its relativistic symmetry. Based on this idea, we derived a Lorentz invariant equation for the

spread electron and demonstrated the validity of the equation by calculating the anomalous magnetic moment without renormalization. In addition, based on the same idea, we consistently explained the measurement process in quantum theory. The calculation method in the present paper is not practical since the electromagnetic interaction is not the minimal one and is not gauge invariant. However, an inherent relativistic symmetry holds true also for the dimensionless electron described by the unrenormalized Dirac equation. We conclude that the foundations of quantum mechanics will be understood only in relation to relativistic symmetry; this is the only manner in which the foundations of both theories can be bridged within a conventional Minkowski space.

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