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# Flow of Unsteady Dusty Fluid Under Varying Pulsatile Pressure Gradient in Anholonomic Co-ordinate System

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**Abstract:** An analytical study of unsteady viscous dusty fluid flow with uniform distribution of dust particles between two infinite parallel plates has been studied by taking into the account of the influence of pulsatile pressure gradient. The flow analysis is carried out using differential geometry techniques and analytical solutions of the problem is obtained with the help of Laplace Transform technique and which are discussed with the help of graphs.

*Keywords: Frenet frame field system; laminar flow, dusty gas; velocity of dust gas and fluid phase, unsteady dusty fluid, pulsatile pressure gradient, relaxation zone and density.*
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## 1. Introduction

A dusty fluid is a mixture of fluid and fine dust particles. Its study is important in areas like environmental pollution, smoke emission from vehicles, emission of effluents from industries, cooling effects of air conditioners, flying ash produced from thermal reactors and formation of raindrops, etc. Also it is useful in the study of lunar ash flow which explains many features of lunar soil.

P.G.Saffman [15] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Liu [11] has studied the Flow induced by an oscillating infinite plat plate in a dusty gas. Michael and Miller[12] investigated

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the motion of dusty gas with uniform distribution of the dust particles occupied in the semi-infinite space above a rigid plane boundary. Samba Siva Rao [16] have obtained the analytical solutions for the dusty fluid flow through a circular tube under the influence of constant pressure gradient, using appropriate boundary conditions. Later T.M.Nabil [13] studied the Effect of couple stresses on pulsatile hydromagnetic poiseuille flow, N.Datta [5] obtained the solutions for Pulsatile flow of heat transfer of a dusty fluid through an infinitely long annular pipe. A.Eric [6] have studied the Quantitative Assessment of Steady and Pulsatile Flow Fields in a Parallel Plate Flow Chamber.

Some researchers like Kanwal [10], Trusdell [17], Indrasena [9], Purushotham [14], Bagewadi, Shantharajappa and Gireesha [1, 2, 3] have applied differential geometry techniques to investigate the kinematical properties of fluid flows in the field of fluid mechanics. Further, the authors [2, 3] have studied two-dimensional dusty fluid flow in Frenet frame field system. Recently the authors [7, 8] have studied the flow of unsteady dusty fluid under varying different pressure gradients like constant, periodic and exponential. The present work is on the laminar flow of a dusty fluid between two infinite stationary parallel plates with a pulsatile pressure gradient in anholonomic co-ordinate system. Further by considering the fluid and dust particles are at rest initially, the analytical expressions are obtained for velocities of fluid and dust particles. The changes in the velocity profiles at different times are shown graphically.

## 2. Equations of Motion

The equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are given by [15]:

For fluid phase

$$\nabla.\overrightarrow{u} = 0 \qquad \text{(Continuity)} \tag{1}$$

$$\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u}.\nabla)\overrightarrow{u} = -\rho^{-1}\nabla p + \upsilon\nabla^2\overrightarrow{u} + \frac{kN}{\rho}(\overrightarrow{v} - \overrightarrow{u}) \tag{2}$$

(Linear Momentum)

For dust phase

$$\nabla.\overrightarrow{v} = 0 \qquad \text{(Continuity)} \tag{3}$$

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v}.\nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}) \qquad \text{(Linear Momentum)} \tag{4}$$

We have following nomenclature:

$\overrightarrow{u}$−velocity of the fluid phase, $\overrightarrow{v}$−velocity of dust phase, $\rho$−density of the gas, $p$−pressure of the fluid, $N$−number of density of dust particles, $\upsilon$−kinematic viscosity, $k = 6\pi a\mu$−Stoke's resistance (drag coefficient), $a$−spherical radius of dust particle, $m$−mass of the dust particle, $\mu$−the co-efficient of viscosity of fluid particles, $t$−time.

Let $\overrightarrow{s}, \overrightarrow{n}, \overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively, Geometrical relations are given by Frenet formulae [4]

$$\begin{aligned}
&i) \quad \frac{\partial \overrightarrow{s}}{\partial s} = k_s \overrightarrow{n}, \quad \frac{\partial \overrightarrow{n}}{\partial s} = \tau_s \overrightarrow{b} - k_s \overrightarrow{s}, \quad \frac{\partial \overrightarrow{b}}{\partial s} = -\tau_s \overrightarrow{n} \\
&ii) \quad \frac{\partial \overrightarrow{n}}{\partial n} = k'_n \overrightarrow{s}, \quad \frac{\partial \overrightarrow{b}}{\partial n} = -\sigma'_n \overrightarrow{s}, \quad \frac{\partial \overrightarrow{s}}{\partial n} = \sigma'_n \overrightarrow{b} - k'_n \overrightarrow{n} \\
&iii) \quad \frac{\partial \overrightarrow{b}}{\partial b} = k''_b \overrightarrow{s}, \quad \frac{\partial \overrightarrow{n}}{\partial b} = -\sigma''_b \overrightarrow{s}, \quad \frac{\partial \overrightarrow{s}}{\partial b} = \sigma''_b \overrightarrow{n} - k''_b \overrightarrow{b} \\
&iv) \quad \nabla . \overrightarrow{s} = \theta_{ns} + \theta_{bs}; \quad \nabla . \overrightarrow{n} = \theta_{bn} - k_s; \quad \nabla . \overrightarrow{b} = \theta_{nb}
\end{aligned} \tag{5}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity ) lines, principal normal and binormal. The functions $(k_s, k'_n, k''_b)$ and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsion of the above curves and $\theta_{ns}$ and $\theta_{bs}$ are normal deformations of these spatial curves along their principal normal and binormal respectively.

## 3. Formulation and Solution of the Problem

In the present problem we consider unsteady laminar flow of an incompressible viscus fluid with uniform distribution of dust particles between two infinite stationary parallel plates separated by a distance h in the absence of body force. The flow is due to the influence of pulsatile pressure gradient with respect to time. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction and the velocities are varies along binormal direction and with time $t$, since we extended the fluid to infinity in the principal normal direction.

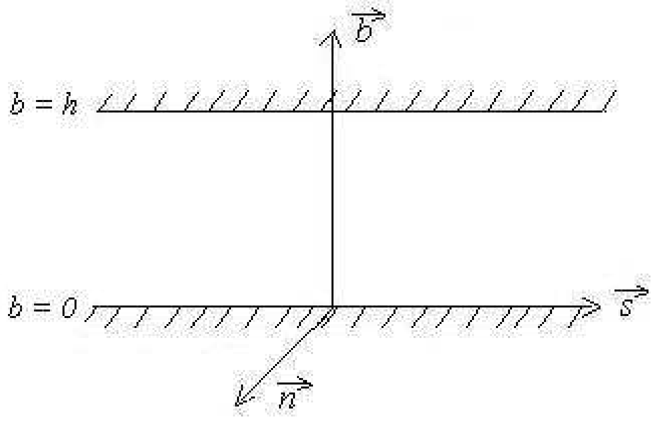

**Fig. 1** Geometry of the flow

By virtue of system of equations (5) the intrinsic decomposition of equations (2) and (4) give the following forms;

$$\frac{\partial u_s}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \nu\left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right] + \frac{kN}{\rho}(v_s - u_s) \tag{6}$$

$$2u_s^2 k_s = -\frac{1}{\rho}\frac{\partial p}{\partial n} + \nu\left[2\sigma_b''\frac{\partial u_s}{\partial b} - u_s k_s^2\right] \tag{7}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial b} + \nu\left[u_s k_s \tau_s - 2k_b''\frac{\partial u_s}{\partial b}\right] \tag{8}$$

$$\frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s) \tag{9}$$

$$2v_s^2 k_s = 0 \tag{10}$$

where $C_r = (\sigma_n'^2 + k_n'^2 + k_b''^2 + \sigma_b''^2)$ is called curvature number [3].

From equation (10) we see that $v_s^2 k_s = 0$ which implies either $v_s = 0$ or $k_s = 0$. The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0$, which shows that the flow doesn't exist. Hence $k_s = 0$, it suggests that the curvature of the streamline along tangential direction is zero. Thus no radial flow exists.

Equation (6) and (9) are to be solved subject to the initial and boundary conditions;

$$\left\{\begin{array}{ll}\text{Initial condition;} & \text{at } t = 0;\ u_s = 0, v_s = 0 \\ \text{Boundary condition;} & \text{for } t > 0;\ u_s = 0,\ \text{at } b = 0 \text{ and } b = h\end{array}\right\} \tag{11}$$

Since we have assumed that a pulsatile pressure gradient is impressed on the system for $t > 0$, we can write

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} = c + \alpha cos(\beta t)$$

where $c$ and $\alpha$ are constants and $\beta$ is the frequency of oscillation.

We define Laplace transformations of $u_s$ and $v_s$ as

$$U = \int_0^\infty e^{-st} u_s dt \quad \text{and} \quad V = \int_0^\infty e^{-st} v_s dt \tag{12}$$

Applying the Laplace transform to equations (6), (9) and to boundary conditions, then by using initial conditions one obtains

$$sU = \frac{c}{s} + \frac{\alpha s}{(s^2 + \beta^2)} + \nu\left[\frac{\partial^2 U}{\partial b^2} - C_r U\right] + \frac{L}{\tau}(V - U) \tag{13}$$

$$sV = \frac{1}{\tau}(U - V) \tag{14}$$

$$U = 0,\ \text{at } b = 0 \text{ and } b = h \tag{15}$$

where $L = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$. Equation (14) implies

$$V = \frac{U}{1+s\tau} \tag{16}$$

Eliminating $V$ from (13) and (16) we obtain the following equation

$$\frac{d^2U}{db^2} - Q^2U = -\left[\frac{c}{\nu s} + \frac{\alpha s}{\nu(s^2+\beta^2)}\right] \tag{17}$$

where $Q^2 = \left(C_r + \frac{s}{\nu} + \frac{sL}{\nu(1+s\tau)}\right)$.

The velocities of fluid and dust particle are obtained by solving the equation (17) subjected to the boundary conditions ((15)) as follows

$$U = \frac{1}{\nu Q^2}\left[\frac{c}{s} + \frac{\alpha s}{s^2+\beta^2}\right]\left\{\frac{sinhQ(b-h) - sinh(Qb)}{sinh(Qh)} + 1\right\}$$

Using $U$ in (16) we obtain $V$ as

$$V = \frac{1}{(\nu Q^2)(1+s\tau)}\left[\frac{c}{s} + \frac{\alpha s}{s^2+\beta^2}\right]\left\{\frac{sinhQ(b-h) - sinh(Qb)}{sinh(Qh)} + 1\right\}$$

By taking inverse Laplace transform to $U$ and $V$, one can obtain

$$\begin{aligned}
u_s = {} & \frac{c}{\nu\lambda^2}\left(\frac{sinh(\lambda(b-h)) - sinh(\lambda b)}{sinh(\lambda h)} + 1\right) \\
& + \frac{4c}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1} sin\left(\frac{2n+1}{h}\pi b\right)\left(\frac{e^{x_1 t}(1+x_1\tau)^2}{((1+x_1\tau)^2+L)} + \frac{e^{x_2 t}(1+x_2\tau)^2}{((1+x_2\tau)^2+L)}\right) \\
& + \frac{\alpha}{\nu}\left(\frac{(AE+BF)M_1 - (BE-AF)M_2}{\left[(y_1y_2-\beta^2)^2 + (\beta y_1 + \beta y_2)^2\right](E^2+F^2)}\right) + \frac{4\alpha}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1} sin\left(\frac{2n+1}{h}\pi b\right) \\
& \times\left[\frac{x_1 e^{x_1 t}(1+x_1\tau)^2}{(x_1^2+\beta^2)((1+x_1\tau)^2+L)} + \frac{x_2 e^{x_2 t}(1+x_2\tau)^2}{(x_2^2+\beta^2)((1+x_2\tau)^2+L)}\right]
\end{aligned}$$

$$\begin{aligned}
v_s = & \frac{c}{\nu\lambda^2}\left(\frac{sinh(\lambda(b-h)) - sinh(\lambda b)}{sinh(\lambda h)} + 1\right) \\
& + \frac{4c}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1} sin\left(\frac{2n+1}{h}\pi b\right)\left[\frac{e^{x_1 t}(1+x_1\tau)}{((1+x_1\tau)^2+L)} + \frac{e^{x_2 t}(1+x_2\tau)}{((1+x_2\tau)^2+L)}\right] \\
& + \frac{\alpha}{\nu}\left(\frac{(M_1A - M_2B)(E - F\beta\tau) + (M_2A + M_1B)(E\beta\tau + F)}{[(y_1y_2-\beta^2)^2 + (\beta y_1 + \beta y_2)^2]\,(E^2+F^2)(1+\beta^2\tau^2)}\right) \\
& + \frac{4\alpha}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1} sin\left(\frac{2n+1}{h}\pi b\right) \\
& \times\left[\frac{x_1 e^{x_1 t}(1+x_1\tau)}{(x_1^2+\beta^2)((1+x_1\tau)^2+L)} + \frac{x_2 e^{x_2 t}(1+x_2\tau)}{(x_2^2+\beta^2)((1+x_2\tau)^2+L)}\right]
\end{aligned}$$

*where*

$$\begin{aligned}
x_1 = & -\frac{1}{2\tau}\left(1 + L + \nu C_r\tau + \nu\tau\frac{n^2\pi^2}{h^2}\right) \\
& + \frac{1}{2\tau}\sqrt{\left(1 + L + \nu C_r\tau + \nu\tau\frac{n^2\pi^2}{h^2}\right)^2 - 4\tau\nu\left(C_r + \frac{n^2\pi^2}{h^2}\right)} \\
x_2 = & -\frac{1}{2\tau}\left(1 + L + \nu C_r\tau + \nu\tau\frac{n^2\pi^2}{h^2}\right) \\
& - \frac{1}{2\tau}\sqrt{\left(1 + L + \nu C_r\tau + \nu\tau\frac{n^2\pi^2}{h^2}\right)^2 - 4\nu\tau\left(C_r + \frac{n^2\pi^2}{h^2}\right)} \\
y_1 = & -\frac{1}{2\tau}\left(1 + L + \nu C_r\tau\right) + \frac{1}{2\tau}\sqrt{\left(1 + L + \nu C_r\tau\right)^2 - 4C_r\nu\tau} \\
y_2 = & -\frac{1}{2\tau}\left(1 + L + \nu C_r\tau\right) - \frac{1}{2\tau}\sqrt{\left(1 + L + \nu C_r\tau\right)^2 - 4C_r\nu\tau} \\
A = & \, sinh(\alpha_1(b-h))cos(\beta_1(b-h)) - sinh(\alpha_1 b)cos(\beta_1 b) + sinh(\alpha_1 h)cos(\beta_1 h) \\
B = & \, cosh(\alpha_1(b-h))sin(beta_1(b-h)) - cosh(\alpha_1 b)sin(\beta_1 b) + cosh(\alpha_1 h)sin(\beta_1 h) \\
M_1 = & \, (cos\beta t - \beta\tau sin\beta t)(y_1y_2 - \beta^2) + (sin\beta t + \beta\tau cos\beta t)(\beta y_1 + \beta y_2) \\
M_2 = & \, (sin\beta t - \beta\tau cos\beta t)(y_1y_2 - \beta^2) + (cos\beta t + \beta\tau sin\beta t)(\beta y_1 + \beta y_2) \\
E = & \, sinh(\alpha_1 h)cos(\beta_1 h), \quad F = cosh(\alpha_1 h)sin(\beta_1 h) \\
\delta_1 = & \, \frac{y_1y_2 - \beta^2 - \beta^2\tau(y_1+y_2)}{\nu(1+\beta^2\tau^2)}, \quad \delta_2 = \frac{\beta^2\tau - \beta(y_1+y_2) - y_1y_2\beta\tau}{\nu(1+\beta^2\tau^2)} \\
\alpha_1 = & \, \frac{\sqrt{\delta_1 + \sqrt{\delta_1^2+\delta_2^2}}}{2} \quad and \quad \beta_1 = \frac{\sqrt{-\delta_1 + \sqrt{\delta_1^2+\delta_2^2}}}{2}, \quad \lambda = \sqrt{\frac{x_1x_2}{\nu}}
\end{aligned}$$

# 4. Conclusion

The figures 2 and 3 represents the velocity profiles for the fluid and dust particles respectively, which are parabolic in nature. It is observed that velocity of fluid particles is parallel to velocity of dust particles and velocity decreases with increase in time $t$. Further one can observe that if the dust is very fine i.e., mass of the dust particles is negligibly small then the relaxation time of dust particle decreases and ultimately as $\tau \to 0$ the velocities of fluid and dust particles will be the same. Also we see that the fluid particles will reach the steady state earlier than the dust particles. This difference is due to the fact that pulsatile pressure gradient is directly exerted on the fluid.

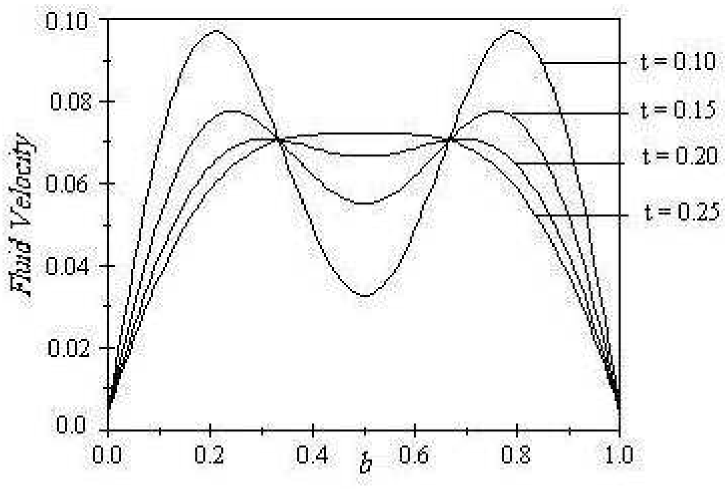

**Fig. 2** Variation of fluid velocity with $b$

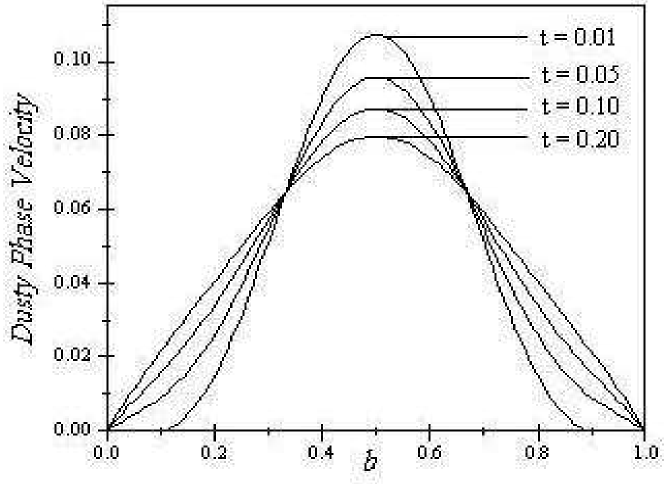

**Fig. 3** Variation of dust phase velocity with $b$

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