

On Inflation Potentials in Randall-Sundrum Braneworld Model

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Abstract: We study the inflationary dynamics of the universe in the Randall-Sundrum typeII Braneworld model. We consider both an inverse-power law and exponential potentials and apply the Slow-Roll approximation in high energy limit to derive analytical expression of relevant inflationary quantities. An upper bound for the coupling constant was also obtained and a numerical value of perturbation spectrum is calculated in good agreement with observation.

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1. Introduction

Recently a great interest has been devoted to study cosmological inflation [1] in the framework of braneworld scenario; for a review see [2]-[6]. This issue has been motivated by observations on accelerating universe [9] and dark energy [10] as well as results on interpretations of these phenomena in terms of scalar field dynamics [11, 12]. In this regards higher dimensional cosmological models have been built and new solutions were

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obtained [4]. Of particular interest is the Randall-Sundrum (RS) braneworld picture based on considering a type IIB D3-brane embedded in AdS_5 [5], where the conventional $4d$ gravity is recovered in low energy limit despite that fifth extra dimension is non compact. As shown in [6], the RS braneworld model is described by a $4d$ effective gravity induced on the world volume of the D3-brane embedded in $5d$ Einstein gravity. The $5d$ Planck scale M_5 is assumed considerably smaller than the corresponding $4d$ effective Planck scale, $M_4 = 1.2 \times 10^{19} GeV$ opening an issue for new perspectives in cosmology at low energies. Since RS development, diverser inspired RS brane inflationary cosmological models have been constructed [7, 8] and new insights have been gained. Like in standard inflation, most of these cosmological models rest on a single scalar field rolling in a given inflaton field potential $V(\phi)$.

In this paper, we consider inflation dynamics in braneworld cosmology, first with an inverse-power low potential $V(\phi) = \mu^{\alpha+4}/\phi^\alpha$ and then with an exponential one; namely $V(\phi) = V_0 e^{-\beta\phi}$; the μ, α and β moduli will be computed later on. These two types of potential are interesting in cosmology since they are used to modeling tachyonic and quintessential matter [12, 13]. Using slow roll approximation, we compute, in the high energy limit, the inflaton time evolution $\phi = \phi(t)$ and determine the corresponding scale factor $a = a(t)$ and other cosmological quantities. We also derive an upper bound for the coupling constant and give a numerical value of perturbation spectrum, which is in good agreement with observation.

The presentation of this work is as follows: In section 2, we present the basic equations of braneworld inflation assuming the Randall-Sundrum's second model, and recall some known results, especially for chaotic inflation. In section 3, we present our results on scalar field dynamics in braneworld inflation, for both inverse power law and exponential potentials. In this context, we derive the time evolution of the scalar field and scale factor. Various spectral quantities are also calculated and an upper value of coupling constant is obtained. Our results are compared to those obtained in standard inflationary cosmology and are shown to be in good agreement with the observable quantities [14]. We end this paper by a conclusion.

2. Inflation In Randall-Sundrum Braneworld Model

2.1 Modified Einstein-Friedmann Equations

We start this section by recalling briefly some fundamentals on Randall-Sundrum type II braneworld model[6]. We first give the braneworld cosmological Einstein-Friedmann equations; then we discuss resulting modifications of Friedmann equations, slow roll parameters and perturbation spectrum.

Starting from $5d$ Einstein equation with cosmological constant Λ , and by supposing that matter fields is confined on D3-brane, Shiromizu *et al.*[15] have shown that $4d$

Einstein equation induced on the brane reads as

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left(\frac{8\pi}{M_4^2}\right) T_{\mu\nu} + \left(\frac{8\pi}{M_5^3}\right)^2 \pi_{\mu\nu} - E_{\mu\nu}. \quad (1)$$

In this relation, $T_{\mu\nu}$ is the energy-momentum tensor of matter on the brane, $\pi_{\mu\nu}$ is a tensor quadratic in $T_{\mu\nu}$ and $E_{\mu\nu}$ is a projection of the five-dimensional Weyl tensor describing the effect of bulk graviton degrees of freedom on brane dynamics. The effective cosmological constant Λ_4 on the brane is determined by the five-dimensional bulk cosmological constant Λ and the 3-brane tension λ as shown below,

$$\Lambda_4 = \frac{4\pi}{M_5^3} \left(\Lambda + \frac{4\pi}{3M_5^3} \lambda^2 \right). \quad (2)$$

Recall also that 4d and 5d Planck scales M_4 and M_5 are related as,

$$M_4 = \sqrt{\frac{3}{4\pi}} \left(\frac{M_5^2}{\sqrt{\lambda}} \right) M_5, \quad (3)$$

with λ as before. In braneworld cosmological model where the metric projected onto the brane is a spatially flat Friedmann-Robertson-Walker model with scale factor $a(t)$, Friedmann equation on the brane reads as [16],

$$H^2 = \frac{\Lambda_4}{3} + \left(\frac{8\pi}{3M_4^2}\right) \rho + \left(\frac{4\pi}{3M_5^3}\right)^2 \rho^2 + \frac{\mathcal{E}}{a^4}, \quad (4)$$

where \mathcal{E} is an integration constant arising from $E_{\mu\nu}$ and thus transmitting bulk graviton influence onto the brane. This term appears as a form of “dark radiation” and may be fixed by observation [16, 17]. However, during inflation this term is rapidly diluted, so we will neglect it. We will also assume that, in the early universe, the bulk cosmological constant is $\Lambda \sim -4\pi\lambda^2/3M_5^3$ so that Λ_4 is negligible. With above assumptions, braneworld Friedmann equation(4) reduces to,

$$H^2 = \frac{8\pi}{3M_4^2} \rho \left[1 + \frac{\rho}{2\lambda} \right]. \quad (5)$$

Note that the crucial correction to standard inflation is given by the density quadratic term ρ^2 . Brane effect is then carried here by the deviation factor $\frac{\rho}{2\lambda}$ with respect to unity. This deviation has the effect of modifying the dynamics of the universe for densities $\rho \gtrsim \lambda$. Note also that in the limit $\lambda \rightarrow \infty$, we recover standard four-dimensional general relativistic results (neglecting \mathcal{E}). Note moreover that in inflationary theory with inflaton potential $V(\phi)$, energy density $\rho = \rho(\phi)$ and pressure $p = p(\phi)$ are given by the following relations,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi); \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (6)$$

where ϕ is the inflaton field and the dot stands for the derivative with respect time t . The potential $V(\phi)$ is the initial vacuum energy responsible of inflation. Along with

these equations, one also has the second inflation Klein-Gordon equation governing the dynamic of the scalar field ϕ

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0. \quad (7)$$

This is a second order evolution equation which follows from conservation condition of energy-momentum tensor $T_{\mu\nu}$ on 3-brane dominated by a scalar field ϕ with a self-interaction potential $V(\phi)$. To calculate physical quantities like scale factor or perturbation spectrum, one has to solve eqs(5,7) for some specific potentials $V(\phi)$. To do so, the following approximations are needed.

2.2 Slow-roll Approximation and Perturbation Spectrum on Brane

Inflationary dynamics requires that inflaton field ϕ driving inflation moves away from the false vacuum and slowly rolls down to the minimum of its effective potential $V(\phi)$ [18]. In this scenario, the initial value $\phi_i = \phi(t_i)$ of the inflaton field and the Hubble parameter H are supposed large and the scale factor $a(t)$ of the universe growth rapidly. Using Friedman equation, the inflation condition $\ddot{a} > 0$ allows us to derive the following bound on pressure,

$$\ddot{a} > 0 \quad \Rightarrow \quad p < -\frac{\lambda + 2\rho}{3(\lambda + \rho)}\rho. \quad (8)$$

In the limit $\rho/\lambda \rightarrow \infty$, this condition reduces to $p < -\frac{2}{3}\rho$; this is a more restrictive constraint relation than the corresponding one in standard inflation relation which requires $p < -\frac{\rho}{3}$. Applying the slow roll approximation, $\dot{\phi}^2 \ll V$ and $\ddot{\phi} \ll V'$, to brane field equations (5,7), we obtain:

$$H^2 \simeq \frac{8\pi V}{3M_4^2} \left(1 + \frac{V}{2\lambda}\right), \quad \dot{\phi} \simeq -\frac{V'}{3H}. \quad (9)$$

The presence of the factor $(1 + \frac{V}{2\lambda})$ carries the brane-modification with respect to the standard slow-roll expression recovered by taking the limit $\lambda \rightarrow \infty$. Note that slow roll approximation puts a constraint on the slope and the curvature of the potential; this is clearly seen on the field expressions of ϵ and η parameters given by [7],

$$\epsilon = -\frac{\dot{H}}{H^2} \equiv \frac{M_4^2}{4\pi} \left(\frac{V'}{V}\right)^2 \left[\frac{\lambda(\lambda + V)}{(2\lambda + V)^2}\right], \quad (10)$$

$$\eta = \frac{V''}{3H^2} \equiv \frac{M_4^2}{4\pi} \left(\frac{V''}{V}\right)^2 \left[\frac{\lambda}{2\lambda + V}\right]. \quad (11)$$

Slow-roll approximation takes place if these parameters are such that $\max\{\epsilon, |\eta|\} \ll 1$ and inflationary phase ends when ϵ and $|\eta|$ are equal to one. The other important quantity related to inflation is the number N of e-folding, indicating the growing of the size of universe. Using slow roll approximation, this N number reads in present case as follows,

$$N \simeq -\frac{8\pi}{M_4^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} \left(1 + \frac{V}{2\lambda}\right) d\phi. \quad (12)$$

Before proceeding it is interesting to comment low and high energy limits of these parameters. Note that at low energies where $V \ll \lambda$, the slow-roll parameters take the standard form. At high energies $V \gg \lambda$, the extra contribution to the Hubble expansion dominates and the new factors in square brackets of eqs(10-11) become of order λ/V . The number of e-folding in the limit $V \gg \lambda$,

$$N \simeq -\frac{4\pi}{\lambda M_4^2} \int_{\phi_i}^{\phi_f} \frac{V^2}{V'} d\phi, \quad (13)$$

where ϕ_i and ϕ_f stand for initial and final value of inflaton.

To test inflation model, one must compute the spectrum of perturbations produced by quantum fluctuations of fields around their homogeneous background values. Using slow-roll equations and following [7], the scalar amplitude A_s^2 of density perturbation, evaluated by neglecting back-reaction due to metric fluctuation in fifth dimension ($E_{\mu\nu} = 0$), is given by

$$A_s^2 \simeq \left(\frac{512\pi}{75M_4^6} \right) \frac{V^3}{V'^2} \left[\frac{2\lambda + V}{2\lambda} \right]^3 \Big|_{k=aH}. \quad (14)$$

Note that for a given positive potential, the A_s^2 amplitude is increased in comparison with the standard result. Note also that in high energy limit this quantity behaves as,

$$A_s^2 \simeq \frac{64\pi}{75\lambda^3 M_4^6} \frac{V^6}{V'^2}. \quad (15)$$

On the other hand, using eqs(10-11), one can compute the perturbation scale-dependence described by the spectral index $n_s \equiv 1 + d(\ln A_s^2)/d(\ln k)$. We find,

$$n_s - 1 \simeq 2\eta - 6\epsilon, \quad (16)$$

Note that at high energies λ/V , the slow-roll parameters are both suppressed; and the spectral index is driven towards the Harrison-Zel'dovich spectrum, $n_s \rightarrow 1$ as $V/\lambda \rightarrow \infty$.

In what follows, we shall apply the braneworld formalism that we have described above by singling out two specific kinds of inflaton potentials. These are the inverse power law potential and the exponential one that have gained revival interest in recent literature in connection with dark matter and quintessence cosmology [13].

3. Scalar Field Dynamics in Braneworld Scenario

To begin, recall that chaotic inflationary model, which was first introduced by Linde [18], has been reconsidered recently by several authors in the context of braneworld scenario [7, 20]. In present work, we are interested by coupling constants for inverse power-law potential $V(\phi) = \frac{\mu^{\alpha+4}}{\phi^\alpha}$ and exponential one $V(\phi) = V_0 \exp(-\beta\phi)$.

3.1 Inverse-Power Law Potential

In the braneworld cosmology high energy limit where $V \gg \lambda$, brane effect becomes important and the Friedmann equations are simplified as,

$$H^2 \simeq \left(\frac{4\pi}{3M_5^3} \right) V^2, \quad \dot{\phi} \simeq -\frac{V'}{V} \left(\frac{M_5^3}{4\pi} \right). \quad (17)$$

and Slow-Roll parameters (10-11) reduces to:

$$\epsilon \equiv \frac{M_4^2}{16\pi} \left(\frac{V'}{V} \right)^2 \left[\frac{4\lambda}{V} \right], \quad \eta \equiv \frac{M_4^2}{8\pi} \left(\frac{V''}{V} \right) \left[\frac{2\lambda}{V} \right]. \quad (18)$$

Consider the inverse power law potential given by,

$$V = \frac{\mu^{\alpha+4}}{\phi^\alpha}, \quad (19)$$

where μ is the inflaton coupling constant and α some critical exponent. This potential has been studied in various occasions; in particular in connection with quintessence in brane inflation [13] and tachyonic inflation [21]. One of the interesting results in this matter, and which is due to Huey and Lidsey[13], is that inflation is generated for the range $\alpha > 2$. Combining results on universe observation and Slow-Roll approximation, we want to show that is possible to express the inverse power law potential coupling constant μ in terms of α and M_5 . To that purpose, consider the field expression of the Slow-Roll parameter ϵ namely,

$$\epsilon \simeq \frac{3M_5^6}{16\pi^2} \left(\frac{\alpha^2}{\mu^{\alpha+4}} \right) \frac{1}{\phi^{2-\alpha}}. \quad (20)$$

Then using the constraint relation $\epsilon \sim 1$ characterizing end of inflation, one can invert above relation as follows,

$$\phi_{end}^{2-\alpha} = \frac{3M_5^6 \alpha^2}{16\pi^2 \mu^{\alpha+4}}. \quad (21)$$

Similarly, we can compute the e-folding number by help of eq(13). We find,

$$N \simeq \frac{16\pi^2 \mu^{\alpha+4}}{3M_5^6} \frac{1}{\alpha(2-\alpha)} [\phi_f^{2-\alpha} - \phi_i^{2-\alpha}], \quad (22)$$

where ϕ_i and ϕ_f stand for initial and end inflaton field values. From these equations (21-22), we can deduce the expression of ϕ_i in term of the e-folding number N ,

$$\phi_i^{2-\alpha} = \frac{3M_5^6 \alpha}{16\pi^2 \mu^{\alpha+4}} [\alpha - N(2-\alpha)]. \quad (23)$$

Now, using observation data giving $N_{cobe} \approx 55$ [22] and identifying the initial and final inflaton field values of inflation interval as $\phi_i = \phi_{cobe}$ and $\phi_f = \phi_{end}$, we get,

$$\phi_{cobe}^{2-\alpha} = \frac{3M_5^6 \alpha}{16\pi^2 \mu^{\alpha+4}} [56\alpha - 110]. \quad (24)$$

Putting back into eq(15) after substituting equation (19), we deduce the following expression of scalar amplitude,

$$A_S \simeq \frac{64\pi(\mu^{\alpha+4})^2}{45M_5^9\alpha}\phi_{cobe}^{1-2\alpha}. \quad (25)$$

Using the observed numerical value of A_S from COBE namely $A_S \sim 2.10^{-5}$, we can invert above identity to fix the inflaton coupling constant μ in term of $5d$ Planck mass M_5 and inflaton field exponent α as shown below,

$$\mu^{\alpha+4} = \left[\frac{90 \cdot 10^{-5} \alpha M_5^9}{64\pi} \right]^{\frac{2-\alpha}{3}} \left[\frac{8\pi^2}{\alpha M_5^6 (84\alpha - 165)} \right]^{\frac{1-2\alpha}{3}}. \quad (26)$$

Note that to get sufficient inflation ($N = 55$) one can obtain from eqs(24,26) a constraint on the initial value of the field which depends on α . For $\alpha = 4$, for example, we get $\phi_i < 162.2 M_4$.

Following the same method, it is not difficult to check, by help of eqs(11,19), that the spectral index n_S reads as,

$$n_S - 1 \simeq -6\epsilon + 2\eta = \frac{1 - 2\alpha}{28\alpha - 55}, \quad (27)$$

or equivalently $n_S = \frac{26\alpha - 55}{28\alpha - 55}$ whose positivity condition requires $\alpha > 2 + \frac{3}{26}$ in agreement with Huey and Lidsey prediction [13]. Fixing a value of the inflaton field exponent α , we can compute the inflaton field coupling constant μ and the spectral index n_S . For $\alpha = 3$ for instance, we get $n_S = 0.83$ and for large α , we have 0.92 in perfect agreement with observation [22] In what follows, we consider the case of inflation exponential potential.

3.2 Exponential Inflation on Brane

Here we consider our second brane inflation model with an exponential potential [23]. This potential has been used to study tachyonic inflation [12] and quintessence [24]. In the present work, we give a quantitative study and compute physical quantities relevant to observation; in particular the perturbation spectrum. To that purpose consider the exponential potential,

$$V(\phi) = V_0 e^{-\beta\phi} \quad (28)$$

where V_0 is a constant and β is the coupling strength of the scalar field. By integrating eqs(9), one can derive the time evolution of scalar field. We find $\phi(t) = C + \frac{M_5^3}{4\pi}\beta t$, where C is an integration constant. To obtain the scale factor $a(t)$, one have to integrate the Friedmann equation (17); we get,

$$a(t) = a_i \exp \left(-\frac{V_0}{3} \frac{16\pi^2}{\beta^2 M_5^6} \exp [-\beta(C + M_5^3 \beta t)] \right). \quad (29)$$

From the study of the inflection point of this relation, one may compute t_{end} , the end time of inflation; this is given by

$$t_{end} = \frac{4\pi}{\beta^2 M_5^3} \left[-\beta C - \ln \left(\frac{3M\beta_5^6}{16\pi^2 V_0} \right) \right]. \quad (30)$$

Putting back into $\phi_{end}(t) = C + \frac{M_5^3}{4\pi}\beta t_{end}$, we determine the value of the scalar field at the end of inflation,

$$\phi_{end} = \frac{1}{\beta} \ln \left(\frac{16\pi^2 V_0}{3\beta^2 M_5^6} \right). \quad (31)$$

Using eqs(13), we can compute the number N of e-folding for the exponential potential,

$$N = -\frac{16\pi^2 V_0}{3\beta^2 M_5^6} (e^{-\beta\phi_f} - e^{-\beta\phi_i}), \quad (32)$$

where ϕ_i is the value of the scalar field at the beginning of inflation,

$$\phi_i \simeq -\frac{1}{\beta} \ln \left(\frac{21\beta^2 M_5^6}{2\pi^2 V_0} \right). \quad (33)$$

We assume now that the number of e foldings before the end of inflation at which observable perturbations are generated corresponds to $N = 55$ [22] and setting $\phi_f = \phi_{end}$, $\phi_i = \phi_{cobe}$, and $A_S^2 = 2.10^{-5}$, we can give an estimation of the coupling constant β of the scalar field. Straightforward computation leads to,

$$\beta = 1.07 \times 10^{-2} M_5^{-1}. \quad (34)$$

Since $M_5 < M_4$, one can deduce an upper bound limit of the coupling constant β as shown below,

$$\beta > 0.877 \times 10^{-21} \quad (35)$$

This will give a constraint on the initial value of the scalar field and should be compared with standard inflation result for same potential for which $\beta_0 = 708,9 \times 10^{-2} M_4^{-1}$. Note that like for chaotic inflation, we have here also a very weakly coupled scalar field on the brane compared to that of standard inflation. Using high energy limit and following same analysis as for inverse power law potential, we can compute spectral index n_s for the exponential case. We find,

$$n_s = 0.92 \quad (36)$$

which is in good agreement with observation for wich

$$0.8 < n_s < 1.05. \quad (37)$$

As far as this result is concerned, it is interesting to note that a similar value was also obtained in [25] for tachyonic inflation.

4. Conclusion

In this paper, we have studied aspects of inflationary dynamics in the framework of braneworld cosmology. It has been shown that in this scenario, the Friedmann equations governing the dynamics of scalar field in the universe are modified by an extra term which depend on the ratio of the density of matter ρ and the brane tension λ . In Slow-Roll approximation, this ratio depend only on the potential V and the tension λ .

In this short letter, we have studied brane effects on inflation for both the inverse power law and exponential potentials. In this context, we have calculated the scale factor for the potential(28) in braneworld formalism and shown that it has an exponential form, as it should be in inflation theory. We have also calculated a numerical value of the perturbation spectrum for potentials (19,28) in good agreement with observation and an upper bound for the coupling constant for exponential potential was obtained. This work may be applied to a concret physical problems such as tachyonic inflation and dark matter.

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