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Petrov classification of the conformal tensor

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Abstract: We exhibit a flux diagram in its tensorial and Newman-Penrose representations for the Petrov classification.

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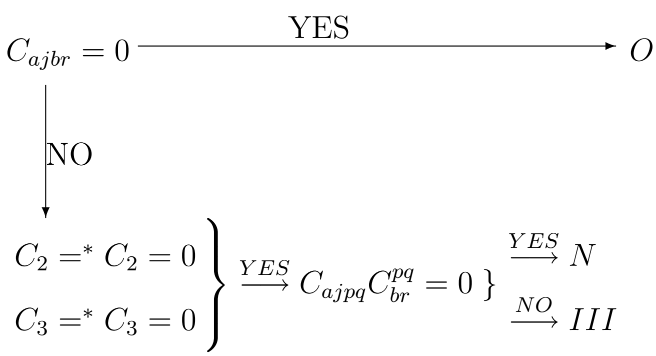
1. Introduction

We shall use the quantities and notation stated in [1]. The Petrov classification (PC) [1-8] of the Weyl tensor has a great importance in general relativity, thus it is attractive to account with an efficient procedure to determine the Petrov type of an arbitrary gravitational field. Peres [9] has worked out the tensorial version of the Petrov's matrix approach [2-4] to perform the PC. In the following figure we illustrate a flux diagram that simplifies the application of the Peres algorithm:



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↓ NO

$$\left.\begin{array}{l}(C_3)^2-({}^*C_3)^2=\frac{C_2}{12}[(C_2)^2-3({}^*C_2)^2]\\ {}^*C_3C_3=\frac{{}^*C_2}{24}[3(C_2)^2-({}^*C_2)^2]\end{array}\right\}\xrightarrow{\text{NO}} I$$

↓ YES

$$\left.\begin{array}{l}C_{ajpq}C^{pq}_{br}+2\Re(\lambda)C_{ajbr}-2\Im(\lambda)^*C_{ajbr}+\\ +\frac{{}^*C_2}{12}\eta_{ajbr}-\frac{C_2}{12}\left(g_{ab}g_{jr}-g_{ar}g_{jb}\right)=0\end{array}\right\}\begin{array}{l}\xrightarrow{\text{NO}}\text{II}\\ \xrightarrow{\text{YES}}\text{D}\end{array}$$

where $C_{ajbr}$is the Weyl tensor [1,6] associated to the metric g_{ab}, and η_{ajbr} represents the Levi-Civita tensor, besides:

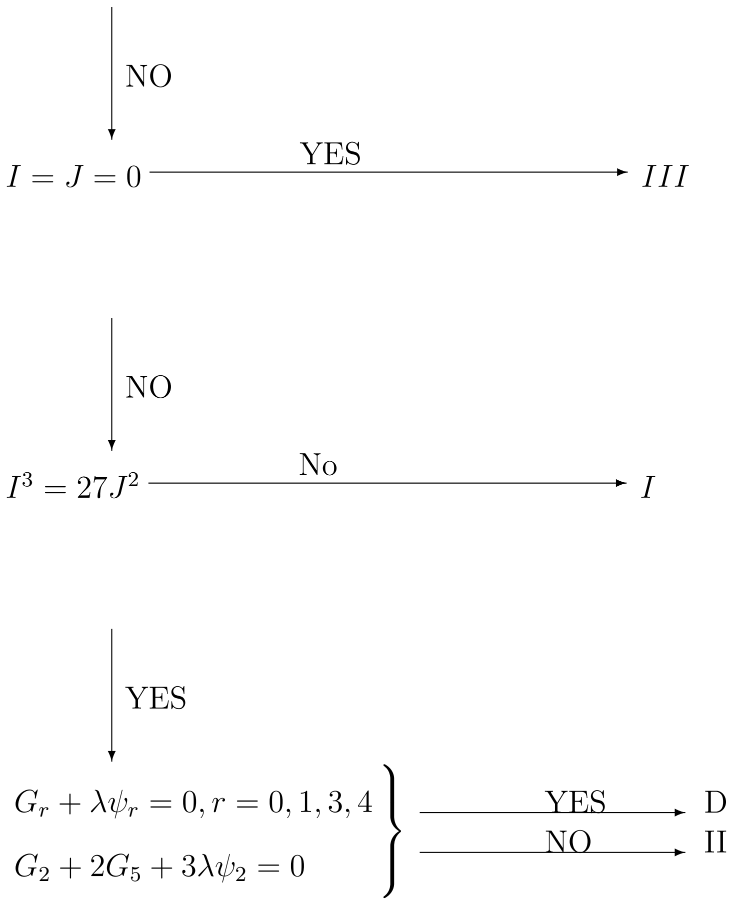
$$\begin{array}{cc}{}^*C_{ajbr}=\frac{1}{2}\eta_{ajpq}C^{pq}_{br}, & \lambda=\Re(\lambda)+i\Im(\lambda),\\ C_2=C_{ajpq}C^{ajpq}, & C_3=C_{ajbr}C^{brpq}C^{aj}_{pq},\\ {}^*C_2=*C_{ajbr}C^{ajbr}, & {}^*C_3=*C_{ajbr}C^{brpq}C^{aj}_{pq},\\ \lambda^2=\frac{1}{48}\left(C_2+i*C_2\right), & \lambda^3=-\frac{1}{96}\left(C_3+i^*C_3\right).\end{array}$$

D'Inverno-Russel Clark [10] employed the Newman-Penrose (NP) formalism [1,11-13], Lorentz rotations [14]-[15] and Debever-Penrose vectors [1,6,16-26] to deduce a 4^{th} order algebraic equation [27] and the study of its roots implies a procedure for the PC. Here we show a different approach to obtain the Petrov type without the use of Debever-Penrose null principal directions and Lorentz transformations. In fact, the flux diagram for the Peres tensorial method is now expressed by using the NP technique, resulting thus the following algorithm [28] which involves fewer computations than the D'Inverno-Russell Clark's approach.

$$\psi_r=0,\quad r=0,\cdots,4\xrightarrow{\text{YES}} O$$

↓ NO

$$G_r=0,\quad r=0,\cdots,5\xrightarrow{\text{YES}} N$$



where

$$G_0 = 2\left(\psi_0\psi_2 - \psi_1^2\right), \qquad G_1 = \psi_0\psi_3 - \psi_1\psi_2,$$

$$G_2 = \psi_2^2 + \psi_0\psi_4 - 2\psi_1\psi_3, \quad G_3 = \psi_1\psi_4 - \psi_2\psi_3,$$

$$G_4 = 2\left(\psi_2\psi_4 - \psi_3^2\right), \qquad G_5 = 2\left(\psi_1\psi_3 - \psi_2^2\right),$$

$$I = G_2 - G_5, \qquad J = -\psi_3 G_1 + \tfrac{1}{2}\left(\psi_2 G_5 + \psi_4 G_0\right),$$

$$\lambda^2 = \tfrac{I}{3}, \qquad \lambda^3 = -J$$

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