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Spinning of Particles in Schwarzschild-de-Sitter and Schwarzschild-Anti-de-Sitter Space-Times with 'Effective Cosmological Constant'

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Abstract: Spinning of particles in **SdS** and **SAdS** space-times with effective cosmological constant is discussed in details. It is shown that the equilibrium conditions are independent of the spin of the test particles and are satisfied only for particular conditions relating the Einstein's cosmological constant with the ultra-light masses implemented in the theory from supergravities arguments and non-minimal coupling.

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In a recent paper [**1**], we have investigated the cosmological model with complex scalar self-interacting inflation field non-minimally coupled to gravity and based on supergravity arguments. It was shown that in the case of non-minimal coupling between the scalar curvature and the density of the scalar field like $L = -\xi\sqrt{g}R\phi^*\phi, \xi = 1/6$ (*R being the scalar curvature or the Ricci scalar*) and for a particular scalar complex potential field $V(\phi\phi^*) = 3/4m^2(\omega\phi^2\phi^{*2} - 1)$, ω being a tiny parameter, inspired from supergravity inflation theories, ultra-light masses "m" are implemented naturally in Einstein field equations (**EFE**), leading to a cosmological constant "Λ" in accord with observations. The metric tensor of the space-time is treated as a background and the Ricci scalar in the non-minimal coupling term, regarded as an "*external parameter*", was found to be $=4\bar{\Lambda}$. We call $\bar{\Lambda} = \Lambda - 3/4m^2$ the "*effective supergravity non-minimal cosmological constant*[2]". That is, we have considered the two contributions to the cosmological constant, the first is

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[2] In [**1**], $8\pi G \equiv \kappa$ was set equal to unity and $\hbar = c = 1$.

"Λ" and the second one is "m^2". These ultra-light masses are in fact too low ($m \approx H_0; H_0$ *being the present Hubble constant*) but they may have desirable feature for the description of the accelerated universe. In fact, scalars of such low squared mass $\approx O(H^2)$give rise to long-range forces when coupled to ordinary matter in a certain space-time background. Remember that ultra-light fermions such modulinos and dilatinos with only gravitational strength interactions resembles that of gravitinos. In all the way, one possible candidate field equation of our theory takes the form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \left(\Lambda - \frac{3}{4}m^2\right)g_{\mu\nu} = 0 \tag{1}$$

In this work which in fact is an application of the results obtained by Stuchlik [**2**], we consider the two contributions to the cosmological constant, "Λ" and "m^2", in order to investigate on the equilibrium conditions of spinning of particles in two special space-times, in particular Schwarzschild-de-Sitter (**SdS)** and Schwarzschild-Anti-de-Sitter (**SAdS**).

(1) In **SdS** space-time: $\bar{\Lambda} = \Lambda - 3/4m^2$ and $V(\phi\phi^*) = 3/4m^2(\omega\phi^2\phi^{*2} - 1)$

(2) In **SAdS** space-time: $\bar{\Lambda} = \Lambda + 3/4m^2$, that is we assume a negative complex potential $V(\phi\phi^*) = 3/4m^2(1 - \omega\phi^2\phi^{*2})$or $m^2 < 0$

Case 1: $\bar{\Lambda} = \Lambda - 3/4m^2$

It is well known that in the standard Schwarzschild backgrounds the equilibrium of test particles is impossible, because only gravitational attraction is present there. However, recent observations indicate that the cosmological constant is positive [**3,4**]. Due to the presence of the cosmological repulsion, the equilibrium of uncharged particles is possible in the **SdS** space-times and the standard one with mass parameter is determined by the line element [**5**]:

$$ds^2 = -\exp\left(2A\left(r\right)\right)dt^2 + \exp\left(2B\left(r\right)\right)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{2}$$

After applying the field equation (1) to (2), we find:

$$\exp\left(2A\left(r\right)\right) = \exp\left(-B\left(r\right)\right) = 1 - \frac{2M}{r} - \left(\frac{\Lambda}{3} - m^2\right)r^2 \tag{3}$$

M being the "*macroscopic*" mass parameter, that is the total mass of the spherically symmetric object. Making use of an orthonormal frame of differential forms and their dual forms of tetrad, the Riemann curvature tensor can be easily derived using the standard methods of differential forms and the Cartan structure equations [**2,6**]. The first and second Cartan structure equations, the tetrad components of the Riemann tensor and their coordinate components as well as their non-zero components are given by:

$$0 = de^{(a)} + \omega^{(a)}_{(b)} \wedge \omega^{(b)}_{(a)} \tag{4}$$

$$\Omega^{(a)}_{(b)} = d\omega^{(a)}_{(b)} + \omega^{(a)}_{(c)} \wedge \omega^{(c)}_{(b)} \tag{5}$$

$$\Omega^{(a)(b)} = R^{(a)(b)}_{|(c)(d)|}\omega^{(c)} \wedge \omega^{(d)} \tag{6}$$

$$R^{(\alpha\beta)} = e^{\alpha}_{(a)} + e^{\beta}_{(b)} + e^{(a)}_{\gamma} R^{(a)(b)}_{(c)(d)} \tag{7}$$

$$R^{tr}_{tr} = R^{(t)(r)}_{(t)(r)} = R^{\theta\phi}_{\theta\phi} = R^{(\theta)(\phi)}_{(\theta)(\phi)} = \frac{2M}{r^3} + \frac{\Lambda}{3} - m^2 \tag{8}$$

$$R^{t\theta}_{t\theta} = R^{(r)(\theta)}_{(r)(\theta)} = R^{t\phi}_{t\phi} = R^{(t)(\phi)}_{(t)(\phi)} = R^{r\theta}_{r\theta} = R^{(r)(\varphi)}_{(r)(\varphi)} = -\frac{M}{r^3} + \frac{\Lambda}{3} - m^2 \tag{9}$$

where $\omega^{(a)}_{(b)}$ are the connection forms, $\Omega^{(a)}_{(b)}$ are the curvature forms, d is the exterior derivative and ˆ the exterior product.

$|(c)\,(d)|$ means that the sum over (c) and (d) is restricted to $(c)\wedge(d)$, so each index pair occurs only once. The equations of motion could be easily studied using the Papapetrou equations of motion for a spinning test particle in an arbitrary gravitational field [**2**,**7**]:

$$m\frac{Du^{\alpha}}{d\tau} = -\varepsilon^{\alpha\mu\nu\beta}\frac{D^2u^{\beta}}{d\tau^2}S_{\mu}u_{\nu} + \frac{1}{2}\varepsilon^{\lambda\mu\rho\sigma}R^{\alpha\nu}_{\lambda\mu}u_{\nu}u_{\sigma}S_{\rho} \tag{10}$$

$\varepsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric Leci-Civita tensor, $D/d\tau$ the covariant derivative, S_{σ} the covariant spin vector defined by $S_{\alpha} = 1/2\varepsilon_{\rho\mu\nu\sigma}u^{\rho}S^{\mu\nu}$ added to Pirani spin condition $S^{\alpha\beta}u_{\beta} = 0$[**8**]. Following [**2**], the dynamics of the spin vector is given by the Fermi-Walker transport equation:

$$\frac{DS_{\alpha}}{d\tau} = u_{\alpha}\frac{Du^{\beta}}{d\tau}S_{\beta} \tag{11}$$

The equilibrium conditions are easily derived after finding the conditions which guarantee that equations (10), (11) and the orthogonal condition $S_{\alpha}u^{\alpha} = 0$ are simultaneously satisfied for the four-vector u^{α}corresponding to a stationary particle in the **SdS** manifold. The spinning test particle in the static **SdS** space-time must have their spin vector fixed, so that for the stationary particles:

$$\frac{D^2u_{\alpha}}{d\tau^2} = u_{\alpha,\beta\gamma}u^{\beta}u^{\gamma} + u_{\alpha,\beta}u^{\beta}_{,\gamma}u^{\gamma} - \Gamma^{\mu}_{\alpha\beta,\gamma}u_{\mu}u^{\beta}u^{\gamma} - 2\Gamma^{\mu}_{\alpha\beta}u_{\mu,\nu}u^{\gamma}u^{\beta}$$

$$-\Gamma^{\mu}_{\alpha\beta}u_{\mu,\nu}u^{\gamma}u_{\mu} + \Gamma^{\mu}_{\alpha\gamma}\Gamma^{k}_{\mu\beta}u_{k}u^{\gamma}u^{\beta} = -u^{t}\Gamma^{r}_{\alpha t}\Gamma^{t}_{rt} \tag{12}$$

The symbol $\Gamma^{\gamma}_{\alpha\beta}$denotes the coefficient of the affine connection of the **SdS** space-time. The only non-zero components is:

$$\frac{D^2u_t}{d\tau^2} = -\frac{\left(M - \left(\frac{\Lambda}{3} - m^2\right)r^3\right)^2}{\sqrt{r^7\left(r - 2M - \left(\frac{\Lambda}{3} - m^2\right)r^3\right)}} \tag{13}$$

Because of completely antisymmetric Levi-Civita tensor, this component can not enter the equation of motion and therefore it is not influenced by the spin vector of the stationary spin vector. For the stationary particles, we find $\Gamma^{r}_{tt}\left(u^{t}\right)^2 = 0$which implies that:

$$r^3_{equilibrium} = \frac{3M}{\Lambda - 3m^2} \tag{14}$$

with $\Lambda > 3m^2$. If $0 < \Lambda < 3m^2$, the equilibrium is not possible. For the particular value $\Lambda = 9m^2/2 \quad (\bar{\Lambda} > 0)$, we find in natural units, the interesting relation:

$$r^3_{equilibrium} = 2\frac{MG}{c^2}\left(\frac{\hbar}{mc}\right)^2 = R_{Schwarszchild}\lambda^2_{Compton} \tag{15}$$

where "R_S" is the Schwarzschild radius and "λ_C" being the Compton wavelength. In this way at equilibrium, the micro-world and the macro-world are coupled together in one equation and this later could be an important perspective about the quantum effects occurring in a black hole and the unified theory. As it is expected, the stationary equilibrium of a spinning test particle in the **SdS** space-time is independent of the mass and spin of the test particle [**2**], and gratefully the presence of the "m^2" term didn't affect this important result.

Case 2: $\bar{\Lambda} = \Lambda + 3/4m^2$and $V(\phi\phi^*) = 3/4m^2(1 - \omega\phi^2\phi^{*2})$

In this case:

$$\exp\left(2A\left(r\right)\right) = \exp\left(-B\left(r\right)\right) = 1 - \frac{2M}{r} - \left(\frac{\Lambda}{3} + m^2\right)r^2 \tag{16}$$

$$\frac{D^2 u_t}{d\tau^2} = -\frac{\left(M - \left(\frac{\Lambda}{3} + m^2\right)r^3\right)^2}{\sqrt{r^7\left(r - 2M - \left(\frac{\Lambda}{3} + m^2\right)r^3\right)}} \tag{17}$$

$$r^3_{equilibrium} = \frac{3M}{\Lambda + 3m^2} \tag{18}$$

For the particular value $\Lambda = -3m^2/2 \quad (\bar{\Lambda} < 0)$, we find again in natural units, the interesting relation:

$$r^3_{equilibrium} = 2\frac{MG}{c^2}\left(\frac{\hbar}{mc}\right)^2 = R_{Schwarszchild}\lambda^2_{Compton} \tag{19}$$

$\Lambda < (>)\, 0$ for $m^2 > (<)\, 0$.

As a conclusion, we have found that the equilibrium conditions depend on the sign of the complex potential or the ultra-light masses implemented on the theory. In the **AdS** space-times with $m^2 > 0$ and positive complex scalar potential, the equilibrium is forbidden. While in **SdS** space-times, the equilibrium is possible unless $\Lambda > 3m^2$ while for the particular value $\Lambda = 9m^2/2$, $r^3_{equilibrium} \approx R_{Schwarszchild}\lambda^2_{Compton}$ and this relation combines the macro with the micro world and could have important cosmological consequences. This shows the whole important role playing by the ultra-light masses in relativistic astrophysics.

Further details and investigations of this model will be dealt in a future paper.

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