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Spinning of Particles in Schwarzschild-de-Sitter and Schwarzschild-Anti-de-Sitter Space-Times with ‘Effective Cosmological Constant’

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Abstract: Spinning of particles in **SdS** and **SAdS** space-times with effective cosmological constant is discussed in details. It is shown that the equilibrium conditions are independent of the spin of the test particles and are satisfied only for particular conditions relating the Einstein’s cosmological constant with the ultra-light masses implemented in the theory from supergravities arguments and non-minimal coupling.

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Keywords: SdS and SAdS spacetimes, effective cosmological constant, equilibrium conditions.
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In a recent paper [1], we have investigated the cosmological model with complex scalar self-interacting inflation field non-minimally coupled to gravity and based on supergravity arguments. It was shown that in the case of non-minimal coupling between the scalar curvature and the density of the scalar field like $L = -\xi\sqrt{g}R\phi^*\phi, \xi = 1/6$ (R being the scalar curvature or the Ricci scalar) and for a particular scalar complex potential field $V(\phi\phi^*) = 3/4m^2(\omega\phi^2\phi^{*2} - 1)$, ω being a tiny parameter, inspired from supergravity inflation theories, ultra-light masses “ m ” are implemented naturally in Einstein field equations (**EFE**), leading to a cosmological constant “ Λ ” in accord with observations. The metric tensor of the space-time is treated as a background and the Ricci scalar in the non-minimal coupling term, regarded as an “external parameter”, was found to be $=4\bar{\Lambda}$. We call $\bar{\Lambda} = \Lambda - 3/4m^2$ the “effective supergravity non-minimal cosmological constant²”. That is, we have considered the two contributions to the cosmological constant, the first is

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² In [1], $8\pi G \equiv \kappa$ was set equal to unity and $\hbar = c = 1$.

" Λ " and the second one is " m^2 ". These ultra-light masses are in fact too low ($m \approx H_0$; H_0 being the present Hubble constant) but they may have desirable feature for the description of the accelerated universe. In fact, scalars of such low squared mass $\approx O(H^2)$ give rise to long-range forces when coupled to ordinary matter in a certain space-time background. Remember that ultra-light fermions such modulinos and dilatinos with only gravitational strength interactions resembles that of gravitinos. In all the way, one possible candidate field equation of our theory takes the form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \left(\Lambda - \frac{3}{4}m^2\right)g_{\mu\nu} = 0 \quad (1)$$

In this work which in fact is an application of the results obtained by Stuchlik [2], we consider the two contributions to the cosmological constant, " Λ " and " m^2 ", in order to investigate on the equilibrium conditions of spinning of particles in two special space-times, in particular Schwarzschild-de-Sitter (**SdS**) and Schwarzschild-Anti-de-Sitter (**SAdS**).

(1) In **SdS** space-time: $\bar{\Lambda} = \Lambda - 3/4m^2$ and $V(\phi\phi^*) = 3/4m^2(\omega\phi^2\phi^{*2} - 1)$

(2) In **SAdS** space-time: $\bar{\Lambda} = \Lambda + 3/4m^2$, that is we assume a negative complex potential $V(\phi\phi^*) = 3/4m^2(1 - \omega\phi^2\phi^{*2})$ or $m^2 < 0$

Case 1: $\bar{\Lambda} = \Lambda - 3/4m^2$

It is well known that in the standard Schwarzschild backgrounds the equilibrium of test particles is impossible, because only gravitational attraction is present there. However, recent observations indicate that the cosmological constant is positive [3,4]. Due to the presence of the cosmological repulsion, the equilibrium of uncharged particles is possible in the **SdS** space-times and the standard one with mass parameter is determined by the line element [5]:

$$ds^2 = -\exp(2A(r))dt^2 + \exp(2B(r))dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (2)$$

After applying the field equation (1) to (2), we find:

$$\exp(2A(r)) = \exp(-2B(r)) = 1 - \frac{2M}{r} - \left(\frac{\Lambda}{3} - m^2\right)r^2 \quad (3)$$

M being the "*macroscopic*" mass parameter, that is the total mass of the spherically symmetric object. Making use of an orthonormal frame of differential forms and their dual forms of tetrad, the Riemann curvature tensor can be easily derived using the standard methods of differential forms and the Cartan structure equations [2,6]. The first and second Cartan structure equations, the tetrad components of the Riemann tensor and their coordinate components as well as their non-zero components are given by:

$$0 = de^{(a)} + \omega_{(b)}^{(a)} \wedge \omega_{(a)}^{(b)} \quad (4)$$

$$\Omega_{(b)}^{(a)} = d\omega_{(b)}^{(a)} + \omega_{(c)}^{(a)} \wedge \omega_{(b)}^{(c)} \quad (5)$$

$$\Omega^{(a)(b)} = R_{|(c)(d)}^{(a)(b)} \omega^{(c)} \wedge \omega^{(d)} \quad (6)$$

$$R^{(\alpha\beta)} = e_{(a)}^\alpha + e_{(b)}^\beta + e_\gamma^{(a)} R_{(c)(d)}^{(a)(b)} \quad (7)$$

$$R_{tr}^{tr} = R_{(t)(r)}^{(t)(r)} = R_{\theta\phi}^{\theta\phi} = R_{(\theta)(\phi)}^{(\theta)(\phi)} = \frac{2M}{r^3} + \frac{\Lambda}{3} - m^2 \quad (8)$$

$$R_{t\theta}^{t\theta} = R_{(r)(\theta)}^{(r)(\theta)} = R_{t\phi}^{t\phi} = R_{(t)(\phi)}^{(t)(\phi)} = R_{r\theta}^{r\theta} = R_{(r)(\varphi)}^{(r)(\varphi)} = -\frac{M}{r^3} + \frac{\Lambda}{3} - m^2 \quad (9)$$

where $\omega_{(b)}^{(a)}$ are the connection forms, $\Omega_{(b)}^{(a)}$ are the curvature forms, d is the exterior derivative and \wedge the exterior product.

$|(c)(d)|$ means that the sum over (c) and (d) is restricted to $(c)\wedge(d)$, so each index pair occurs only once. The equations of motion could be easily studied using the Papapetrou equations of motion for a spinning test particle in an arbitrary gravitational field [2,7]:

$$m \frac{Du^\alpha}{d\tau} = -\varepsilon^{\alpha\mu\nu\beta} \frac{D^2 u^\beta}{d\tau^2} S_\mu u_\nu + \frac{1}{2} \varepsilon^{\lambda\mu\rho\sigma} R_{\lambda\mu}^{\alpha\nu} u_\nu u_\sigma S_\rho \quad (10)$$

$\varepsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric Levi-Civita tensor, $D/d\tau$ the covariant derivative, S_σ the covariant spin vector defined by $S_\alpha = 1/2 \varepsilon_{\rho\mu\nu\sigma} u^\rho S^{\mu\nu}$ added to Pirani spin condition $S^{\alpha\beta} u_\beta = 0$ [8]. Following [2], the dynamics of the spin vector is given by the Fermi-Walker transport equation:

$$\frac{DS_\alpha}{d\tau} = u_\alpha \frac{Du^\beta}{d\tau} S_\beta \quad (11)$$

The equilibrium conditions are easily derived after finding the conditions which guarantee that equations (10), (11) and the orthogonal condition $S_\alpha u^\alpha = 0$ are simultaneously satisfied for the four-vector u^α corresponding to a stationary particle in the **Sds** manifold. The spinning test particle in the static **Sds** space-time must have their spin vector fixed, so that for the stationary particles:

$$\begin{aligned} \frac{D^2 u_\alpha}{d\tau^2} &= u_{\alpha,\beta\gamma} u^\beta u^\gamma + u_{\alpha,\beta} u_{,\gamma}^\beta u^\gamma - \Gamma_{\alpha\beta,\gamma}^\mu u_\mu u^\beta u^\gamma - 2\Gamma_{\alpha\beta}^\mu u_{\mu,\nu} u^\nu u^\beta \\ &\quad - \Gamma_{\alpha\beta}^\mu u_{\mu,\nu} u^\nu u_\mu + \Gamma_{\alpha\gamma}^\mu \Gamma_{\mu\beta}^k u_k u^\gamma u^\beta = -u^t \Gamma_{\alpha t}^r \Gamma_{rt}^t \end{aligned} \quad (12)$$

The symbol $\Gamma_{\alpha\beta}^\gamma$ denotes the coefficient of the affine connection of the **Sds** space-time. The only non-zero components is:

$$\frac{D^2 u_t}{d\tau^2} = -\frac{(M - (\frac{\Lambda}{3} - m^2) r^3)^2}{\sqrt{r^7 (r - 2M - (\frac{\Lambda}{3} - m^2) r^3)}} \quad (13)$$

Because of completely antisymmetric Levi-Civita tensor, this component can not enter the equation of motion and therefore it is not influenced by the spin vector of the stationary spin vector. For the stationary particles, we find $\Gamma_{tt}^r (u^t)^2 = 0$ which implies that:

$$r_{equilibrium}^3 = \frac{3M}{\Lambda - 3m^2} \quad (14)$$

with $\Lambda > 3m^2$. If $0 < \Lambda < 3m^2$, the equilibrium is not possible. For the particular value $\Lambda = 9m^2/2$ ($\bar{\Lambda} > 0$), we find in natural units, the interesting relation:

$$r_{equilibrium}^3 = 2 \frac{MG}{c^2} \left(\frac{\hbar}{mc} \right)^2 = R_{Schwarzschild} \lambda_{Compton}^2 \quad (15)$$

where " R_S " is the Schwarzschild radius and " λ_C " being the Compton wavelength. In this way at equilibrium, the micro-world and the macro-world are coupled together in one equation and this later could be an important perspective about the quantum effects occurring in a black hole and the unified theory. As it is expected, the stationary equilibrium of a spinning test particle in the **SdS** space-time is independent of the mass and spin of the test particle [2], and gratefully the presence of the " m^2 " term didn't affect this important result.

Case 2: $\bar{\Lambda} = \Lambda + 3/4m^2$ and $V(\phi\phi^*) = 3/4m^2(1 - \omega\phi^2\phi^{*2})$

In this case:

$$\exp(2A(r)) = \exp(-B(r)) = 1 - \frac{2M}{r} - \left(\frac{\Lambda}{3} + m^2 \right) r^2 \quad (16)$$

$$\frac{D^2 u_t}{d\tau^2} = - \frac{(M - (\frac{\Lambda}{3} + m^2) r^3)^2}{\sqrt{r^7 (r - 2M - (\frac{\Lambda}{3} + m^2) r^3)}} \quad (17)$$

$$r_{equilibrium}^3 = \frac{3M}{\Lambda + 3m^2} \quad (18)$$

For the particular value $\Lambda = -3m^2/2$ ($\bar{\Lambda} < 0$), we find again in natural units, the interesting relation:

$$r_{equilibrium}^3 = 2 \frac{MG}{c^2} \left(\frac{\hbar}{mc} \right)^2 = R_{Schwarzschild} \lambda_{Compton}^2 \quad (19)$$

$\Lambda < (>) 0$ for $m^2 > (<) 0$.

As a conclusion, we have found that the equilibrium conditions depend on the sign of the complex potential or the ultra-light masses implemented on the theory. In the **AdS** space-times with $m^2 > 0$ and positive complex scalar potential, the equilibrium is forbidden. While in **SdS** space-times, the equilibrium is possible unless $\Lambda > 3m^2$ while for the particular value $\Lambda = 9m^2/2$, $r_{equilibrium}^3 \approx R_{Schwarzschild} \lambda_{Compton}^2$ and this relation combines the macro with the micro world and could have important cosmological consequences. This shows the whole important role playing by the ultra-light masses in relativistic astrophysics.

Further details and investigations of this model will be dealt in a future paper.

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How S-S' di Quark Pairs Signify an Einstein Constant Dominated Cosmology, and Lead to New Inflationary Cosmology Physics

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Abstract: We review the results of a model of how nucleation of a new universe occurs, assuming a di quark identification for soliton-anti soliton constituent parts of a scalar field. Initially, we employ a false vacuum potential system; however, when cosmological expansion is dominated by the Einstein cosmological constant at the end of chaotic inflation, the initial di quark scalar field is not consistent w.r.t a semi classical consistency condition we analyze as the potential changes to the chaotic inflationary potential utilized by Guth . We use Scherrer's derivation of a sound speed being zero during initial inflationary cosmology, and obtain a sound speed approaching unity as the slope of the scalar field moves away from a thin wall approximation. All this is to aid in a data reconstruction problem of how to account for the initial origins of CMB due to dark matter since effective field theories as presently constructed require a cut off value for applicability of their potential structure. This is often at the cost of, especially in early universe theoretical models, of clearly defined baryogenesis, and of a well defined mechanism of phase transitions.

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1. Introduction

As of June 2005, an effort was made to combine reconstruction of data gathering techniques with the requirement of the JDEM dark matter-dark energy search for the origins of dark matter in the early universe [1]. This has, among other things, lead to methodologies being presented which could shed light as to the initial formation of scalar potentials

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which could contribute to CMB background radiation. In doing so, it was noted that initial dimensions, as postulated by Quin, Pen, and Silk [2] presented evidence as to how three extra dimensions play a role in explaining how at very short distances gravity would have a r^{-5} spatial behavior dependence in force calculations. We do believe that in the initial stages of cosmic inflation, that space, indeed had additional dimensions and that the dimensions play a role as far as nucleation of a new universe.

We have, therefore, written up how to reconstruct potentials, using the methodology presented by Kadota et al [3], but we also think that it is important to pick out properties of the potential in question with respect to early universe models, since CMB data as presently configured is too imprecise to get anything other than the standard FRW flat space metric, 1000 or so years after the big bang. So being the case, we have constructed a list of properties of what an early universe potential system, composed of di quark constituents in order to help researchers investigate CMB data more accurately for very early universe configurations.

2. Organization Of The Paper

Appendix I, parts A and B highlights what can be said about typical data reconstruction for early universe potential systems. **Appendix II** initiates setting up three regimes for an evolving potential model leading to chaotic inflation, in line with Dr. Guths quartic potential system. **Appendix III** presents what is done with instanton models of what is called in the literature, QCD balls [4], for initially stable di quark pairs which we believe are the building blocks for a false vacuum nucleation of initial baryonic states of matter. Afterwards, in the main text of this document, I examine the break down of what Bunyi and Hsu of the U. of Oregon call a semi classical approximation [5], but which I call a consistency condition for di quark pair contributions to Guth style chaotic inflation. In making this consistency evaluation, I then refer to in **Appendix IV, part A and B** how and why the initial wave functionals used in forming this semi classical evaluation are formed. This uses the results of two accepted world press scientific articles, one published in IJMPB [6], and also another accepted already for publication in Modern Physics Letters B [7], which describe necessary and sufficient conditions for a false vacuum based construction of Gaussian wave functionals, which then have, due to the very short distances involved, a discrete state presentation which is then used in an inner product evaluation of potential systems.

It is interesting to note that the semi classical (consistency) condition so outlined in the main text works best initially for a modified driven washboard potential system, which is integrated over six dimensions, in line with Silks presentation as to the importance of higher dimensions being very significant for extremely small spatial dimensions. This is given more structure in **Appendix V**. I do believe that this is no accident, and is congruent with the Calabi Yau conjecture in string theory with the curling up of higher spatial dimensions, which after a certain phase in inflation no longer contribute significantly to the chaotic inflation paradigm presented by Guth [8].

How does this bend in with more observational techniques as given by astrophysics researchers? Early universe nucleation is too small a region of space for typical action integral arguments to be effectual. So I have presented an alternative, as given in **Appendix VII**, which I do believe is able to give a new structure as to how to consider the flux of particles from a cosmic nucleation stand point [9]. In addition, the initial configuration of matter states would not be treatable by the Einstein cosmological constant. But that the evolution of di quark states can, after the onset of inflation, evolve into an Einstein cosmological constant dominated epoch, as given by an argument based upon a modified Scherrer k essence argument. This argument is important, and is in the main text of this document. All of this has been presented in PANIC 2005, and will be included in the AIP proceedings of that conference [10].

This last step depends upon a break down of a thin wall approximation. I do believe that this is consistent with the quantum fluctuations of momentum discussed in the paper written by R. Aloisio[11] et al, about deformed special relativity and its relations to a supposed quantum gravitational background.

Finally, I make direct connection with Venzianos[12] postulates as to the links between Planck scale length, a scalar field term, and a wavelength approximately in sync with the initial scale of a nucleating universe. I suggest here that the initial cosmic nucleation diameter was of the order of Planck Length, and subsequently radically expanded afterwards, in a result consistent with cosmic inflation.

The initial impetus for making this effort was due to the following conundrum. As is commonly known in cosmology circles, one would expect a flat Friedman – Walker universe after 60 e-foldings, but beforehand one could expect sharp deviations as to flat space geometry. The moment one would expect to have deviations from the flat space geometry would closely coincide with Rocky Kolb’s model for when degrees of freedom would decrease from over 100 degrees of freedom to roughly ten or less during an abrupt QCD phase transition [13]. As was mentioned by Joe Lykken , the CMB model should yield a distinct ‘signal’ which is lending toward a non flat cosmological metric space potential which can be seen to be initiating a phase transition at about the end of the 60 e-folding regime of cosmological expansion [14]. My own model is useful for such QCD phase transitions; while Kenji Kadoka’s potential reconstruction scheme is not specific as to a **UNIQUE** potential structure. It would be enough in itself to try to combine the two techniques as to go before the thousand year mark Kenji mentioned as to data sets permitting potential reconstruction, and to find evidence as to CMB background as to the initial phases of CMB generation leading to the datum Kolb mentioned as to the decrease in cosmic microwave radiation to its present value as a result of a QCD phase transition in the expansion of the early universe.

3. Brief Re Cap Of Qins Extra Dimensions From Dark Matter Article, Plus The Evolving Potential System Accomodating Di Quark Scalar Fields

As mentioned, Quinn’s article [2] gives a new force law, with respect to distances at or below $1nm$ in length. As presented in the article [10], this appears to be a verification of the existence of small but non infinitesimal extra dimensions. The key assumption which was used in their paper was a force law of the general form for distances $r \ll R$:

$$F = \alpha \cdot \frac{GMm}{r^{2+n}} \quad (3.1)$$

Here, α is a constant with dimensions $[length]^n$, G is the gravitational constant, and M and m are the masses of the two particles and $\alpha \equiv R^n$ was set, while the value of n was, partly to fit with an argument given by Volt and Wannier [15] that the quantum mechanical cross section for collision is twice the corresponding classical value, if one assumes a central force field dependence of r^{-5} . This all together, if one assumes that initially r is of the order of magnitude of Planck’s length l_P would lead to extremely strong pressure values upon the domain walls of a nucleated scalar field initial states, which I claim would lead to a quite necessary collapse of the thin wall approximation. This collapse of the thin wall approximation set the stage for an Einstein constant dominated regime in inflation, if one adheres to a version of Scherrer’s K essence theory [16] results for modeling the di quark pairs used as an initial starting point for soliton-anti soliton pairs(S-S’) in the beginning of quantum nucleation of our universe [10].

We should note that **Appendix I** as given gives a necessary and sufficient condition for constructing a potential system, initially in the false vacuum mode of potential, due to a pop up of a di quark state [10]. Here, for reasons of scale, we set M_P as a Planck mass, and the 2^{nd} mass, m , as considerably smaller. The scalar field term ϕ is constructed in terms of di quarks, in line with the soliton- anti soliton (S-S’) used in the two accepted articles using similar constructions in IJMPB, etc [6, 7], and ϕ^* is, here, picked in terms of the limits of quantum fluctuations of a scalar field, in line with Guths model of chaotic inflation [17]. Furthermore we write the potentials V_1 , V_2 , and V_3 in terms of S-S’ di quark pairs nucleating and then contributing to a chaotic inflationary scalar potential system [10].

$$V_1(\phi) = \frac{M_P^2}{2} \cdot (1 - \cos(\phi)) + \frac{m^2}{2} \cdot (\phi - \phi^*)^2 \quad (3.2a)$$

$$V_2(\phi) \approx \frac{(1/2) \cdot m^2 \phi^2}{(1 + A \cdot \phi^3)} \quad (3.2b)$$

$$V_3(\phi) \approx (1/2) \cdot m^2 \phi^2 \quad (3.2c)$$

The difference between these potentials becomes extraordinarily important in considering how the nucleating universe system evolves in time from the onset of the big bang itself. Furthermore, as a convenience, I have bench marked the ϕ^* term via the following procedure. We consider if and when we have classical and quantum fluctuations approximately

giving the same value for a phase value of [17]

$$\phi^* \equiv \left(\frac{3}{16 \cdot \pi} \right)^{\frac{1}{4}} \cdot \frac{M_P^{3/2}}{m^{\frac{1}{2}}} \cdot M_P \rightarrow \left(\frac{3}{16 \cdot \pi} \right)^{\frac{1}{4}} \cdot \frac{M_P^{3/2}}{m^{\frac{1}{2}}} \quad (3.3)$$

where we have set M_P as the typical Planck mass which we normalized to being unity in this paper for the hybrid false vacuum – inflaton field cosmology example, as well as having set the general evolution of our scalar field as having the form of [17]

$$\phi \equiv \tilde{\phi}_0 - \frac{m}{\sqrt{12 \cdot \pi \cdot G}} \cdot t \quad (3.4)$$

This then permits us to look at how consistent having a di quark model, with a thin wall approximation is, with the evolving potential system, given above. It is important, since we are finding that having the additional dimensions specified in the beginning permits us to have a more physically consistent picture of how the phase transition to an Einstein constant dominated cosmology occurs in the first place. This is especially relevant from going from the 1st to the 3rd potential given above. **Appendix III** presents what is done with instanton models of what is called in the literature, QCD balls⁴, for initially stable di quark pairs, which is what we are assuming with this construction of ϕ , and we can use to obtain S-S' type pairs which are then used to construct wave functional representation of early universe states. This is in part based upon **Appendix IV, part A and B** on how and why the initial wave functional used in forming this semi classical evaluation are formed. This uses the results of two accepted world press scientific articles, one published in IJMPB [6], and also another accepted already for publication in Modern Physics Letters B [7]. The important thing to consider here, though is that we are looking at understanding the existence of the phase transformation from the first to the third potential occurs, and what it says about the formation of conditions relevant toward an Einstein constant dominated cosmology

4. Criteria Used By Bunyi And Hsu, Which We Call A Consistency Condition Reflecting The Occurance Of A Phase Transition.

Let us first consider an elementary definition of what constitutes a semi classical state. As visualized by Bunyi and Hsu, [5] it is of the form $|a\rangle$ which has the following properties:

i) Assume $\langle a | 1 | a \rangle = 1$

(Where 1 is an assumed identity operator, such that $1|a\rangle = |a\rangle$)

ii) We assume that $|a\rangle$ is a state whose probability distribution is peaked about a central value, in a particular basis, defined by an operator Z

a) Our assumption above will naturally lead, for some n values

$$\langle a | Z^n | a \rangle \equiv (\langle a | Z | a \rangle)^n \quad (4.1)$$

Furthermore, this will lead to, if an operator Z obeys Eq. (30) that if there exists another operator, call it Y which does not obey Eq. (30), that usually we have non commutativity

$$[Y, Z] \neq 0 \quad (4.2)$$

Buniy and Hsu [5] speculate that we can, in certain cases, approximate a semi classical evolution equation of state for physical evolution of cosmological states with respect to classical physics operators. This well may be possible for post inflationary cosmology; however, in the initial phases of quantum nucleation of a universe, it does not apply. We do this with a potential system, with S-S' di quark constituents we model via using [10]

$$\phi \equiv \pi \cdot [\tanh b(x - x_a) + \tanh b(x_b - x)] \quad (4.3)$$

We can, in this give an approximate wave function as given by a discretized version of the wave functional given for the first potential system as in Appendix IV, B:

$$\psi \cong c_1 \cdot \exp(-\tilde{\alpha} \cdot \phi(x)) \quad (4.4)$$

Then we can look to see if we have[5, 10]

$$\left(\int_{x_a}^{x_b} \psi \cdot V_i \cdot \psi \cdot 4\pi \cdot x^2 \cdot dx \right)^N \equiv \int_{x_a}^{x_b} \psi \cdot [V_i]^N \cdot \psi \cdot 4 \cdot \pi \cdot x^2 \cdot dx \Big|_{i=1,2,3} \quad (4.5)$$

This was later generalized, in the initial phases of nucleation for the 1st potential system as being, in six initial dimensions

$$\left(\int_{x_a}^{x_b} \psi \cdot V_i \cdot \psi \cdot 4\pi \cdot x^5 \cdot dx \right)^N \equiv \int_{x_a}^{x_b} \psi \cdot [V_i]^N \cdot \psi \cdot 4 \cdot \pi \cdot x^5 \cdot dx \Big|_{i=1} \quad (4.6)$$

In addition, the analysis of how to work with a ratio of the values of the left and right hand sides of eq (4.5 and (4.6) as a way of looking at the consistency of what has been called the semi classical approximation would lead to analyzing

$$\Phi_{i,n,N} (ratio) \equiv \frac{\left(\int_{x_a}^{x_b} \psi \cdot V_i \cdot \psi \cdot x^{2+n} \cdot dx \right)^N}{\int_{x_a}^{x_b} \psi \cdot [V_i]^N \cdot \psi \cdot x^{2+n} \cdot dx} \Big|_{i=1,2,3} \quad (4.7)$$

The first coefficient, i , denotes which potential system is picked, and ranges in value from 1 to 3. The second coefficient, n , is either 0 or 3, depending upon what dimensionality is assumed for this problem. The third coefficient, N , is freely ranging in values from 1 up to 100. I as a convenience often worked with $N=9$. This eventually led to the calculations of **Appendix V**, which highlight the importance of higher dimensionality in the initial stages of nucleation, for the first potential system.

Assuming that this is a valid initial dimensional approximation, we did the following for the three potentials.

a. Assumed that the scalar wave functional term was decreasing in ‘*height*’ and increasing in ‘*width*’ as we moved from the first to the third potentials. ϕ also had a definite evolution of the domain wall from a ‘*near perfect*’ thin wall approximation to one which had a considerable slope existing with respect to the wall.

b. We also observed that in doing this sort of model that there was a diminishing of magnitude from unity for Eq. (4.7) for large N values, regardless if or not the thin wall approximation was weakened as we went from the first to the third potential system. In doing to, we also noted that even in Eq. (4.7) for the first potential, Eq. (4.7) had diminishing applicability as a result for decreasing b values in Eq. (4.3), which corresponded to when the thin wall approximation was least adhered to.

We also observed that for the third potential, that there was never an overlap in value between the left and right hand sides of Eq. (4.5) and Eq. (4.6), regardless of whether the thin wall approximation was adhered to. In other words, the third potential was least linkable to a semi classical approximation of physical behavior linkable to a physical system, while Eq. (4.5) and Eq. (4.6) worked best for a thin domain wall approximation to Eq. (4.4) in the driven sine Gordon approximation of a potential system. In all this, we assumed that the small perturbing term added to the $(1 - \cos(\phi))$ part of Eq. (9a) was a physical driving term to a very classical potential system $(1 - \cos(\phi))$ which had a quantum origin consistent with the interpretation of a false vacuum nucleation of the sort initially formulated by Sidney Coleman [18]. Furthermore, as we observed an expanding ‘width’ in Eq. (4.3), the alpha term in Eq (4.4) shrank in its value, corresponding to a change in the position of constituent SS’ components in the scalar field given in this model. The SS’ terms roughly corresponded to di quark pairs.

c. Chaotic inflation in cosmology is, in the sense a quartic potential portrayed by Guth,[17] a general term for models of the very early Universe which involve a short period of extremely rapid (exponential) expansion; blowing the size of what is now the observable Universe up from a region far smaller than a proton to about the size of a grapefruit (or even bigger) in a small fraction of a second. This process smoothes out space-time to make the Universe flat, but is not in the model presented linkable in the chaotic inflationary region given by the third potential to any semi classical arguments. The relative good fit of Eq. (4.7) for the first potential is in itself an argument that the thin wall approximation breaks down past the point of baryogenesis after the chaotic inflationary regime is initiated by the third potential as modeled by Guth[17].

Since we have established this, we should then attempt to consider if the higher dimensional physical state relevant to the 1st potential system are countable. Yes they are, but not by ordinary least action principal arguments [19]. I give a variant of what could be analyzed in **Appendix VII**, after stating that the earlier least action counting algorithms referenced in **Appendix VI** (summary of Garrigas work)[20] is not germane to such a small scale physical system. This then leads to to consider what the evolving state of di quark pairs says about , from a Scherrer k essence stand point of how the

evolution to Guth chaotic inflation, as given by the third potential corresponds to the rise of an Einstein constant dominated inflationary cosmology [10].

5. How Dark Matter Ties In, Using Pure Kinetic K Essence As Dark Matter Template For A Near Thin Wall Approximation Of The Domain Wall For ϕ

We define k essence as any scalar field with non-canonical kinetic terms. Following [10, 21, 22] we introduce a momentum expression via

$$p = V(\phi) \cdot F(X) \quad (5.1)$$

where we define the potential in the manner we have stated for our simulation as well as set [10, 21, 22]

$$X = \frac{1}{2} \cdot \nabla_\mu \phi \nabla^\mu \phi \quad (5.2)$$

and use a way to present F expanded about its minimum and maximum [10, 21, 22]

$$F = F_0 + F_2 \cdot (X - X_0)^2 \quad (5.3)$$

where we define X_0 via $F_X|_{X=X_0} = \frac{dF}{dX}|_{X=X_0} = 0$, as well as use a density function [10, 21, 22]

$$\rho \equiv V(\phi) \cdot [2 \cdot X \cdot F_X - F] \quad (5.4)$$

where we find that the potential neatly cancels out of the given equation of state so [10, 21, 22]

$$w \equiv \frac{p}{\rho} \equiv \frac{F}{2 \cdot X \cdot F_X - F} \quad (5.5)$$

as well as a growth of density perturbations terms factor Garriga and Mukhanov [20] wrote as

$$C_x^2 = \frac{(\partial p / \partial X)}{(\partial \rho / \partial X)} \equiv \frac{F_X}{F_X + 2 \cdot X \cdot F_{XX}} \quad (5.6)$$

where $F_{XX} \equiv d^2F/dX^2$, and since we are fairly close to an equilibrium value, we pick a value of X close to an extremal value of X_0 . [10, 21, 22]

$$X = X_0 + \tilde{\epsilon}_0 \quad (5.7)$$

where, when we make an averaging approximation of the value of the potential as very approximately a constant, we may write the equation for the k essence field as taking the form (where we assume $V_\phi \equiv dV(\phi)/d\phi$)

$$(F_X + 2 \cdot X \cdot F_{XX}) \cdot \ddot{\phi} + 3 \cdot H \cdot F_X \cdot \dot{\phi} + (2 \cdot X \cdot F_X - F) \cdot \frac{V_\phi}{V} \equiv 0 \quad (5.8)$$

as approximately

$$(F_X + 2 \cdot X \cdot F_{XX}) \cdot \ddot{\phi} + 3 \cdot H \cdot F_X \cdot \dot{\phi} \cong 0 \quad (5.9)$$

which may be re written as [10, 21, 22]

$$(F_X + 2 \cdot X \cdot F_{XX}) \cdot \ddot{X} + 3 \cdot H \cdot F_X \cdot \dot{X} \cong 0 \quad (5.10)$$

In this situation, this means that we have a very small value for the growth of density perturbations [10, 21, 22]

$$C_S^2 \cong \frac{1}{1 + 2 \cdot (X_0 + \tilde{\varepsilon}_0) \cdot (1/\tilde{\varepsilon}_0)} \equiv \frac{1}{1 + 2 \cdot \left(1 + \frac{X_0}{\tilde{\varepsilon}_0}\right)} \quad (5.11a)$$

when we can approximate the *kinetic energy* from

$$(\partial_\mu \phi) \cdot (\partial^\mu \phi) \equiv \left(\frac{1}{c} \cdot \frac{\partial \phi}{\partial \cdot t}\right)^2 - (\nabla \phi)^2 \cong -(\nabla \phi)^2 \rightarrow -\left(\frac{d}{dx} \phi\right)^2 \quad (5.11b)$$

and, if we assume that we are working with a comparatively small contribution w.r.t. time variation but a very large, in many cases, contribution w.r.t. spatial variation of phase

$$|X_0| \approx \frac{1}{2} \cdot \left(\frac{\partial \phi}{\partial x}\right)^2 \gg \tilde{\varepsilon}_0 \quad (5.11c)$$

$$0 \leq C_S^2 \approx \varepsilon^+ \ll 1 \quad (5.12)$$

And 10, 20

$$w \equiv \frac{p}{\rho} \cong \frac{-1}{1 - 4 \cdot (X_0 + \tilde{\varepsilon}_0) \cdot \left(\frac{F_2}{F_0 + F_2 \cdot (\tilde{\varepsilon}_0)^2} \cdot \tilde{\varepsilon}_0\right)} \approx 0 \quad (5.13)$$

We get these values for the phase ϕ being nearly a box, i.e. the thin wall approximation for b being very large in Eq. (4.3); this is consistent with respect to Eq. (5.13) main result, with $w \equiv \frac{p}{\rho} \cong 0 \Rightarrow$ treating the potential system given by the first potential (modified sine Gordon with small quantum mechanical driving term added) as a semi classical system leading to Eq. (4.7) nearly being unity. This also applies to the formation of SS' pair formation due to the di quarks as alluded to in Zhitinisky's [16] formulation of QCD balls with an axion wall squeezer having a 'thin wall' character.

When we observed

$$|X_0| \approx \frac{1}{2} \cdot \left(\frac{\partial \phi}{\partial x}\right)^2 \cong \frac{1}{2} [\delta_n^2(x + L/2) + \delta_n^2(x - L/2)] \quad (5.14)$$

with

$$\delta_n(x \pm L/2) \xrightarrow{n \rightarrow \infty} \delta(x \pm L/2) \quad (5.15)$$

as the slope of the SS' pair approaches a box wall approximation in line with thin wall nucleation of SS' pairs being in tandem with $b \rightarrow$ larger. Specifically, in our simulation, we had $b \rightarrow 10$ above, rather than go to a pure box style representation of SS' pairs; this could lead to an unphysical situation with respect to delta functions giving infinite values of infinity, which would force both C_s^2 and $w \equiv \frac{p}{\rho}$ to be zero for $|X \approx X_0| \cong \frac{1}{2} \cdot \left(\frac{\partial \phi}{\partial x}\right)^2 \rightarrow \infty$

if the ensemble of **SS'** pairs were represented by a pure thin wall approximation, [20] i.e., a box. If we adhere to a finite but steep slope convention to modeling both C_s^2 and $w \equiv \frac{p}{\rho}$, we get the following: When $b \geq 10$ we obtain the conventional results of

$$w \cong \frac{-1}{1 - 4 \cdot \frac{X_0 \cdot \tilde{\varepsilon}_0}{F_2}} \rightarrow -1 \quad (5.16)$$

and recover Scherrer's solution for the speed of sound [10, 21, 22]

$$C_s^2 \approx \frac{1}{1 + 4 \cdot X_0 \left(1 + \frac{X_0}{2 \cdot \tilde{\varepsilon}_0}\right)} \rightarrow 0 \quad (5.17)$$

(If an example $F_2 \rightarrow 10^3, \tilde{\varepsilon}_0 \rightarrow 10^{-2}, X_0 \rightarrow 10^3$). Similarly, we would have if $b \rightarrow 3$ in Eq. (4.5)

$$w \cong \frac{-1}{1 - 4 \cdot \frac{X_0 \cdot \tilde{\varepsilon}_0}{F_2}} \rightarrow -1 \quad (5.18)$$

and

$$C_s^2 \approx \frac{1}{1 + 4 \cdot X_0 \left(1 + \frac{X_0}{2 \cdot \tilde{\varepsilon}_0}\right)} \rightarrow 1 \quad (5.19)$$

if $F_2 \rightarrow 10^3, \tilde{\varepsilon}_0 \rightarrow 10^{-2}$. Furthermore $|X_0| \rightarrow$ a small value, which for $b \rightarrow 3$ in Eq. (5) would lead to $C_s^2 \approx 1$, i.e., when the wall boundary of a SS' pair is no longer approximated by the thin wall approximation. This eliminates having to represent the initial state as behaving like pure radiation state (as Cardone [23] postulated), i.e., we then recover the cosmological constant. When $|X_0| \approx \frac{1}{2} \cdot \left(\frac{\partial \phi}{\partial x}\right)^2 \gg \tilde{\varepsilon}_0$ no longer holds, we can have a hierarchy of evolution of the universe as being first radiation dominated, then dark matter, and finally dark energy.

If $|X \approx X_0| \cong \frac{1}{2} \cdot \left(\frac{\partial \phi}{\partial x}\right)^2 \rightarrow \infty$, neither limit leads to a physical simulation that makes sense; so, in this problem, we then refer to the contributing slope as always being large but not infinite. We furthermore have, even with $w = -1$

$$C_s^2 \equiv 1 \xrightarrow{b \rightarrow 3} 1 \quad (5.20)$$

indicating that the evolution of the magnitude of the phase $\phi \rightarrow \varepsilon^+$ corresponds with a reduction of our cosmology from a dark energy dark matter mix to the more standard cosmological constant models used in astrophysics. This coincidentally is when the semi classical evaluation involving S-S' di quark pairs breaks down, as given by Eq. (4.7) being much smaller than unity and corresponds to the b of Eq. (4.3) for $\phi \rightarrow \varepsilon^+$ being quite small. It also denotes a region where there is a dramatic reduction of the degrees of freedom of the FRW space time metric, as Kolb postulated [24, 25] so that we can then visualize cosmological dynamics being governed by the Einstein constant at the conclusion of the cosmological inflationary period

6. Conclusion

Veneziano’s model [12] gives us a neat prescription of the existence of a Planck’s length dimensionality for the initial starting point for the universe via:

$$l_P^2/\lambda_S^2 \approx \alpha_{GAUGE} \approx e^\phi \quad (6.1)$$

where the weak coupling region would correspond to where $\phi \ll -1$ and λ_S is a so called quanta of length, and $l_P \equiv c \cdot t_P \sim 10^{-33}cm$. As Veneziano implies by his 2nd figure [6], a so called scalar dilaton field with these constraints would have behavior seen by the right hand side of his figure one, with the $V(\phi) \rightarrow \varepsilon^+$ but would have no guaranteed false minimum $\phi \rightarrow \phi_F < \phi_T$ and no $V(\phi_T) < V(\phi_F)$. The typical string models assume that we have a present equilibrium position in line with strong coupling corresponding to $V(\phi) \rightarrow V(\phi_T) \approx \varepsilon^+$ but no model corresponding to potential barrier penetration from a false vacuum state to a true vacuum in line with Coleman’s presentation.[5, 20] However, FRW cosmology [26] will in the end imply

$$t_P \sim 10^{-42} \text{seconds} \Rightarrow \text{size of universe} \approx 10^{-2}cm \quad (6.2)$$

which is still huge for an initial starting point, whereas we manage to in our S-S’ ‘distance model’ to imply a far smaller but still non zero radii for the initial ‘universe’ in our model.

We find that the above formulation in Eq. (6.1) is most easily accompanied by the given SS’ di quark pair basis for the scalar field used in this paper, and that it also is consistent with the initial scalar cosmological state evolving toward the dynamics of the cosmological constant via the k essence argument built up near the end of this document. Furthermore, we also argue that the semi classical analysis of the initial potential system as given by Eq. (4.7) and its subsequent collapse is de facto evidence for a phase transition to conditions allowing for CMB to be created at the beginning of inflationary cosmology.

We are fortunate as shown in **Appendix V** that for determining the relative good fit of Eq. (4.7) that the relative domain walls slope of the initial phase given by Eq. (4.5) was not terribly significant, for the first potential system, which dove tails with Eq. (30) merely pushing out the domain walls, as a primary effect, for a driven sine Gordon type modeling of false vacuum nucleation. As mentioned earlier, this was actually heightened by the extra dimensionality as alluded to by the power law relationship in Eq. (30) making an almost perfect equality between the left and right hand sides of Eq. (4.7). That the ratio Eq. (4.7) in **Appendix V** had varying values, showing different degrees of break down of this relationship for the 2nd transitional potential, due to differences in dimensionality and slope of the scalar field as given by Eq. (4.3) is probably due to this representing the abrupt loss of numbers of degrees of freedom Rocky Kolb has mentioned as part of a phase transition. Needless to say though, as we evolve toward the Einstein cosmological constant era and chaotic inflation, as given by the 3rd potential, we should keep in mind very real limits as to the comparative sharpness of the slope of the scalar field as given by Eq. (4.3)

K essence analysis argues against making b in Eq. (4.3) too large, i.e., if we have a ‘perfect’ thin wall approximation to our S-S’ di quark pairs, we will have the unphysical

speed of sound results plus other consequences detailed in the k essence section of the document which we do not want. On the other hand, the semi classical analysis brought up in the section starting with Eq. (4.5), Eq. (4.6) and summarized by Eq. (4.7) shows us that a close to the thin wall approximation for S-S' di quark pairs gives an optimal fit for consistency in the potential with the wave functions exhibiting a thin wall approximation 'character'. It is useful to note that our kinetic model can be compared with the very interesting Chimentos [27] purely kinetic k –essence model, with density fluctuation behavior at the initial start of a nucleation process. The model indicate our density function reach $\rho = \text{constant}$ after passing through the tunneling barrier as mentioned in our nucleation of a S-S' pair ensemble. This is when the Einstein constant becomes dominant and that the semi classical approximation in Eq. (4.7) for a domain wall at the time the comparative thin wall approximation S-S' pair ceases to be relevant.

Our initial attempt here very likely should be re visited, especially if the sort of brane world objects referred to by Trodden et a [28] are used in a future calculation for initial nucleation states. However, this should all be done to re calibrate how to fill in the CMB contribution toward reconstruction of a suitable class of potentials which could shed light not only on the origins of baryogenesis, in early universe models, but also in determining how dark matter-dark energy could contribute to the formation of initial inflationary cosmology parameters. The hope is that if suitable data reconstruction methodology is obtained and refined, that one could as an example determine how the initial physical fundamental constants could be set as they are, as well understand how dark matter-dark energy contribute to the initial origins of CMB itself This also would allow us to improve upon the particle flux from nucleation argument we used, using Gongs [29] approximate construction in order to get around limitations in least action principles due to tiny spatial dimensions. Furthermore, we should note that these nucleation configurations fit in well with the following model of false vacuum nucleation. This is in line with the first specified potential as given in Eq. (9a) which we claim eventually becomes in sync with Eq. (19). Further progress in investigating this phenomenology should take into account the datum so mentioned in the text, about the original multiple dimensions in the initial phases of a nucleating universe, which subsequently are reduced as the scalar potential evolves toward the chaotic potential given in Eq. (19). This should permit us to be able to reconstruct potentials far closer to the big bang itself, than the 1000 or so year limit alluded to by Dr. Kadota in his May 2005 Pheno talk given in Madison, Wisconsin [30]. This in its own way will entail considerable additional analytical work, along the lines first specified by *Edmund J. Copelan et al.* in their ground breaking tome on potential reconstruction techniques applied to cosmology [31]. It is worthwhile to note that the orientation of my white paper was in unifying certain techniques, and methodology of what is known in the literature as QCD balls in an instanton configuration to use data reconstruction in order to obtain information on dark matter physics. In doing so, I wound up using a lot of ideas, as was done by other physicists considering early universe nucleation models, from condensed matter physics. The emphasis though of the presented concept was in setting up a template as to examine what actually constitutes dark matter. This

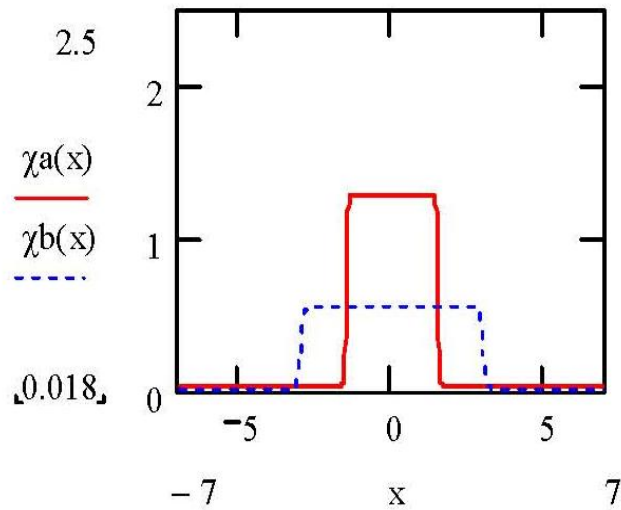


Fig. 1 a,b Evolution of the phase from a thin wall approximation to a more nuanced thicker wall approximation with increasing L between S-S' instanton components. The 'height' drops and the 'width' L increases correspond to a de evolution of the thin wall approximation. This is in tandem with a collapse of an initial nucleating 'potential' system to the standard chaotic scalar ϕ^2 potential system of Guth. As the 'hill' flattens, and the thin wall approximation dissipates, the physical system approaches standard cosmological constant behavior.

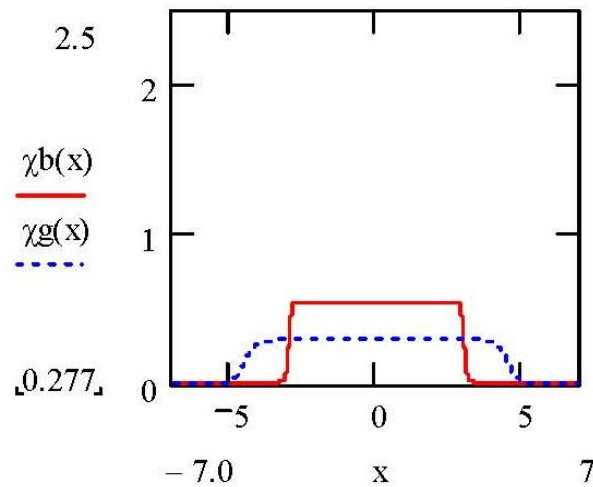


Fig. 2 a,b As the walls of the S-S' pair approach the thin wall approximation, a normalized distance, $L = 9 \rightarrow L = 6 \rightarrow L = 3$, approaches delta function behavior at the boundaries of the new nucleating phase. As L increases, the delta function behavior subsides dramatically. Here, the $L = 9 \Leftrightarrow$ conditions approaching a cosmological constant. $L = 6 \Leftrightarrow$ conditions reflecting Scherrer's dark energydark matter mix. $L = 3 \Leftrightarrow$ approaching unphysical delta function contributions due to a pure thin wall model.

should be what future inquiry should be directed toward.

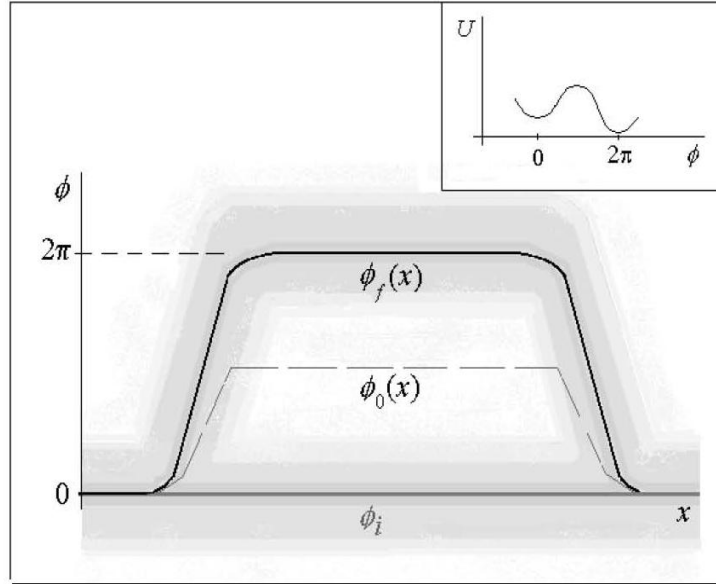


Fig. 3 Initial configuration of the domain wall nucleation potential as given by Eq. (4.4a) which we claim eventually becomes in sync with Eq. (4.4c) due to the phase transition alluded to by Dr. Edward Kolbs model of how the initial degrees of freedom declined from over 100 to something approaching what we see today in flat Euclidian space models of space time (i.e. the FRW metric used in standard cosmology)

Appendix Ia: Initial Statement Of Kadotas Potential Reconstruction Methodology

Kenji Kadota of FNAL in Pheno 2005 [30] and also in arXIV [3] talked of comparing two graphs, one with a combination of scalar potential terms $\left[3 \cdot \left(\frac{V'}{V}\right)^2 - 2 \cdot \frac{V''}{V}\right]$ against $[(\xi)]$ (Mpc) with a graph of

$m(j)$ against ***mode numbers***. Here, in this situation,

$$m(j) = \text{linear combination of } \{P(j)\} \quad (1a)$$

And when we set t_{END} = the demarcation of the end of time for the inflation, for a scale factor a leads to

$$\xi \equiv - \int_t^{t_{END}} \frac{dt'}{a(t')} \quad (1b)$$

In this situation, the $\{P(j)\}$ refer to pixel data slices which show up in

$$\left[3 \cdot \left(\frac{V'}{V}\right)^2 - 2 \cdot \frac{V''}{V}\right] \equiv \sum_i p_i \cdot B_i(\ln \xi) \quad (2a)$$

We should identify the left hand side of equation 2 with the derivative of a function $G(\xi)$, i.e.

$$\frac{dG(\xi)}{d\xi} \equiv \sum_i p_i \cdot B_i(\ln \xi) \quad (2b)$$

This is when Kadota et al defined

$$B_i(\ln(\xi)) = \begin{cases} 1 \\ \text{with a value of 1 iff } \ln \xi_i < \ln \xi < \ln \xi_{i+1} \\ 0 \end{cases} \quad (3)$$

In the most recent arXIV article, Kadota defined a procedure as to how to identify useful entries as to acceptable $\{P(j)\}$ values as to a simplified scalar potential structure which is $V(\phi) \equiv (V_0 \cdot e^{\lambda(\phi-\phi_0)}) \cdot [1 + c \cdot e^{-\nu(\phi-\phi_0)^2}]$ for a perturbation centered at $\phi \equiv \phi_0$ where this has $|\lambda|, c \ll 1, |\nu| \gg 1$, so then after Kadota defined

$$\phi \cong \lambda \cdot \ln \xi \quad (4)$$

so one could write

$$\phi_0 \equiv \lambda \cdot \ln \xi_0 \quad (5)$$

He, Kadota, obtained graphical behavior as seen in his fig 8 and fig 9 of his arXIV article[3]. An even simpler situation graphically emerged when Kadota set the left hand side of Eq. (1a) equal to a constant which permitted him, using Eq. (2) and Eq. (3) above to give constant values to the p_i pixels, which was equivalent to his figure 7 which was for a potential system leading to a constant spectral index value, n when he defined via linking $n - 1$ to the derivative with respect to k of an expression of the primordial power spectrum $P(k)$ via

$$n - 1 = \frac{dP(k)}{dk} \quad (6)$$

Here, in this situation we have that if we interpret $\vartheta(1)$ as an order of magnitude constant of about $1 < \vartheta(1) < 10$. We should also note that often $\vartheta(1)$ is often set very close to 1 itself.

$$k = \vartheta(1) \cdot a \cdot H \equiv a \cdot H \quad (7)$$

The exact particulars of the power spectra $P(k)$ are in Kadoka's well written arXIV paper, but it suffices to say that the natural logarithm of the power spectra $P(k)$ is equal to an integral over ξ values from zero to infinity, with part of the integrand involving a so called 'window function' times the power spectra $P(k)$, for $G(k)$ of equation 2.2a above. I do believe one can say the following:

Kenji Kadoka's methodology permits the general reconstruction of potentials as up to about 1000 years after the big bang. The issue at stake though is if or not reconstructive methodology using some of these same methods could be countenanced going up to the end of the 60 e-folding period commonly viewed as the demarcation between flat and curved space, with a curved space milieu being the regime of active nucleation of our universe. This would entail, among other things, finding traces in CMB data of the initial signature of the big bang itself and tying it into a QCD style phase transition.

Appendix I B: Using Jdem Analysis Of Data With This Built Up Potential System, With Kadotas Potential Reconstruction Procedures

The first step would be to refine the analytical algorithms to, give reliable data inputs into the right hand side [3, 30] of $\frac{dG(\xi)}{d\xi} \equiv \sum_i p_i \cdot B_i(\ln \xi)$, where the left hand side of this equation actually could use, in a modified format the procedure given in Eq (9a) to Eq (19) of the main text , and this done to obtain a match up of the acceptable p_i entries with CMB data.

This would entail use of Monte Carlo simulations as well as far more developed analysis of how to obtain acceptable p_i entries in a more realistic manner than the toy problem analyzed by *Kadoka's* toy problem [3, 30] example which he presented in fig 7 of his arXIV article [3].

Afterwards, once acceptable procedures are outlined as to finding acceptable p_i entries for potentials other than the potential given by Kadoka's test scalar potential given as

$$V(\phi) \equiv (V_0 \cdot e^{\lambda \cdot (\phi - \phi_0)}) \cdot [1 + c \cdot e^{-\nu \cdot (\phi - \phi_0)^2}] \quad (1)$$

The potential reconstruction I believe could be greatly aided by some of the initial effective contributions of extra dimensionality and of side effects of the baryogenesis mentioned in the formation of our early universe potential nucleation model The idea would be to find ways to obtain data sets via techniques most congruent to reliable potential reconstruction of the early inflationary cosmos. Before the 1000 or so year limit specified by Kenji Kadota in discussions I had with him at Pheno 2005 [30].

If finding acceptable match up of data sets with how to reconstruct a complicated potential beyond the one given by Eq (1) above was completed in general. Then one would face a discussion with manufacturers of the satellite used for dark matter searching as to tailor made electronics which would be acceptable for obtaining sufficient data sets. I am assuming that this investigation would be one out of many being used in the upcoming satellite mission.

Appendix II: Links To The Potential System Used For Cosmological Nucleation

$$\begin{aligned} V_1 &\rightarrow V_2 \rightarrow V_3 \\ \phi(\text{increase}) &\leq 2 \cdot \pi \rightarrow \phi(\text{decrease}) \leq 2 \cdot \pi \rightarrow \phi \approx \varepsilon^+ \\ t &\leq t_P \rightarrow t \geq t_P + \delta \cdot t \rightarrow t \gg t_P \end{aligned} \quad (1)$$

We described the potentials V_1 , V_2 , and V_3 in terms of S-S' di quark pairs nucleating and then contributing to a chaotic inflationary scalar potential system.

$$V_1(\phi) = \frac{M_P^2}{2} \cdot (1 - \cos(\phi)) + \frac{m^2}{2} \cdot (\phi - \phi^*)^2 \quad (2a)$$

$$V_2(\phi) \approx \frac{(1/2) \cdot m^2 \phi^2}{(1 + A \cdot \phi^3)} \quad (2b)$$

$$V_3(\phi) \approx (1/2) \cdot m^2 \phi^2 \quad (2c)$$

Appendix III: Including In Necessary And Sufficient Conditions For Forming A Condensate State At Or Before Planck Time t_P

For a template for the initial expansion of a scalar field leading to false vacuum inflationary dynamics in the expansion of the universe, Zhitnitsky's [4] formulation for how to form a condensate of a stable soliton style configuration of cold dark matter is a useful starting point for how an axion field can initiate forming a so called QCD ball. Zhitnitsky [4] uses quarks in a non-hadronic state of matter that, in the beginning, can be in di quark pairs. A di quark pair would permit making equivalence arguments to what is done with cooper pairs and a probabilistic representation as to find the relative 'size' of the cooper pair. We assume an analogous operation can be done with respect to di quark pairs. In doing so, calculations [4] for quarks being are squeezed by a so called QCD phase transition due to the violent collapse of an axion domain wall. The axion domain wall would be the squeezer to obtain a so called SS' configuration. This presupposes a formation of a highly stable soliton type configuration in the onset due to the growth in baryon mass

$$M_B \approx B^{8/9} \quad (1)$$

This is due to a large baryon (quark) charge B which Zhitnitsky [4] finds is smaller than an equivalent mass of a collection of free separated nucleons with the same charge. This provides criteria for absolute stability by writing a region of stability for the QCD balls dependent upon the inequality occurring for $B > B_C$ (a critical charge value)

$$m_N > \frac{\partial M_B}{\partial B} \quad (2)$$

He [19] furthermore states that stability, albeit not absolute stability is still guaranteed for the formation of meta stable states occurring with

$$1 \ll B < B_C \quad (3)$$

If we make the assumptions that there is a balance between Fermi pressure P_f and a pressure due to surface tension, with σ being an axion wall tension value [4] so that

$$\left(P_\sigma \cong \frac{2\sigma}{R} \right) \equiv \left(P_f \cong -\frac{\Omega}{V} \right) \quad (4)$$

This pre supposes that Ω is some sort of thermodynamic potential of a non interacting Fermi gas, so that one can then get a mean radius for a QCD ball at the moment of formation of the value, when assuming $\tilde{c} \approx .7$, and also setting $B \approx B_C \propto 10^{+33}$ so that

$$R \equiv R_0 \cong \left(\frac{\tilde{c} \cdot B^{4/3}}{8 \cdot \pi \cdot \sigma} \right)^{1/3} \quad (5)$$

If we wish to have this of the order of magnitude of a Planck length l_P , then the axion domain wall tension must be huge, which is not unexpected. Still though, this presupposes a minimum value of B which Zhitnitsky [4] set as

$$B_C^{\text{exp}} \sim 10^{20} \quad (6)$$

We need to keep in mind that Zhitnitsky [4] set this parameterization up to account for a dark matter candidate. I am arguing that much of this same concept is useful for setting up an initial condensate of di quark pairs as, separately SS' in the initial phases of nucleation, with the further assumption that there is an analogy with the so called color super conducting phase (CS) which would permit di quark channels. The problem we are analyzing not only is equivalent to BCS theory electron pairs but can be linked to creating a region of nucleated space in the onset of inflation which has SS' pairs. The SS' pairs would have a distance between them proportional to distance mentioned earlier, R_0 , which would be greater than or equal to the minimum Planck's distance value of l_P . The moment one would expect to have deviations from the flat space geometry would closely coincide with Rocky Kolb's model for when degrees of freedom would decrease from over 100 degrees of freedom to roughly ten or less during an abrupt QCD phase transition [4]. The QCD phase transition would be about the time one went from the first to the second potential systems mentioned above.

Appendix IV A: Wave Functionals Used In This Model And Their Analogies To Black Hole Nucleation

This idea of pair creation arose once again in a later context in an article by Dias and Lemos called 'Pair creation of black holes on a cosmic string background' where the so called 'amplitude' for the propagation from 'nothing' to a three dimensional surface boundary Σ was given by the wave function [32] (wave functional):

$$\psi(h_{ij}, A_i) = \int d[g_{uv}] \cdot d[A_u] \cdot \exp(-I(g_{uv}, A_u)) \quad (1)$$

where h_{ij} and A_i are the induced metric and electromagnetic potential on the boundary $\Sigma = \partial M$ of a compact manifold M , and $I(g_{uv}, A_u)$ is the Euclidian action, with $d[g_{uv}]$ and $d[A_u]$ measures of the metric g_{uv} and the Maxwell field A_u . Diaz and Lemos further state that a semi classical instanton approximation allows us to state that dominant contributions to the path integral come from metrics and Maxwell fields where are near the solutions which extremalize the Euclidian action and satisfy boundary conditions. So if we have this process, we may construct a wave function that that denotes the creation of a black hole via

(2a)

where B is a one loop contribution from quadratic fluctuations in the fields, $\delta^2 I$, and I_{inst} is the classical action of the gravitational instanton that mediates the pair creation of black holes. Similarly, the wave function which describes the nucleation of a dS de

Sitter space from nothing is:

$$\psi_{dS} \propto \exp(-I_{ds}) \quad (2)$$

where $I_{dS} = -\frac{3\pi}{2\Lambda}$ is the action of the S^4 gravitational instanton which according to Lemos ‘mediates’ this nucleation. So, then the nucleation probability of the dS space from nothing and then the dS space with a pair of black holes from nothing is given by $|\psi_{dS}|^2$ and $|\psi_{inst}|^2$ respectively. This then allows us to state then that if we take the ratio of these two probabilities that we obtain the pair creation rate of black holes in the dS background as

$$\Gamma \cong \eta \cdot \exp(-2 \cdot I_{inst} + 2I_{dS}) \quad (3)$$

For the Bogomol’nyi inequality approach [6, 7] we modify a de facto 1+1 dimensional problem in condensed matter physics to being one which is quasi one dimensional by making the following substitution, namely looking at the Lagrangian density ς to having a time independent behavior denoted by a sudden pop up of a S-S’ pair via the substitution of the nucleation ‘pop up’ time by [6, 7]

(5)

where t_P is the Planck’s time interval. Then afterwards, we shall use the substitution of $\hbar \equiv c \equiv 1$ so we can write

$$\psi_{\alpha c} \cdot \exp\left(-\beta \cdot \int L dx\right) \quad (4)$$

Appendix IV B: Reducing The Given Wave Functional To Having Gaussian Functional Behavior

We wish to give an argument as to how we obtain [6, 7]

$$\Psi_{i,f}[\phi(\mathbf{x})]_{\phi \equiv \phi_{ci,cf}} = c_{i,f} \cdot \exp\left\{-\int d\mathbf{x} \alpha [\phi_{ci,f}(\mathbf{x}) - \phi_0(\mathbf{x})]^2\right\}, \quad (1)$$

In both cases, we find that the coefficient in front of the wave functional in Eq. (1) is normalized due to error function integration

This is due to

We also found that in order to have a Gaussian potential in our wavefunctionals that we needed to have in both interpretations

$$\frac{(\{\})}{2} \equiv \Delta E_{gap} \equiv V_E(\phi_F) - V_E(\phi_T) \quad (2)$$

where for the Bogomol’nyi interpretation of this problem we worked with potentials (generalization of the extended Sine-Gordon model potential)[6, 7]

$$V_E \cong C_1 \cdot (\phi - \phi_0)^2 - 4 \cdot C_2 \cdot \phi \cdot \phi_0 \cdot (\phi - \phi_0)^2 + C_2 \cdot (\phi^2 - \phi_0^2)^2 \quad (3)$$

We had a Lagrangian [15] we modified to be (due to the Bogomil'nyi inequality)

$$L_E \geq |Q| + \frac{1}{2} \cdot (\phi_0 - \phi_C)^2 \cdot \{ \} \quad (4)$$

with topological charge $|Q| \rightarrow 0$ and with the Gaussian coefficient found in such a manner as to leave us with wave functionals [1, 3, 10] we generalized for charge density transport. This same Eq. (1) was more or less assumed in the Gaussian wavefunctional ansatz interpretation while

$$\begin{aligned} \Psi_f [\phi(\mathbf{x})] |_{\phi \equiv \phi_{Cf}} &= \\ c_f \cdot \exp \left\{ - \int d\mathbf{x} \alpha [\phi_{Cf}(\mathbf{x}) - \phi_0(\mathbf{x})]^2 \right\} &\rightarrow \\ c_2 \cdot \exp \left(-\alpha_2 \cdot \int d\tilde{x} [\phi_T]^2 \right) &\cong \Psi_{final}, \end{aligned} \quad (5)$$

and

$$\begin{aligned} \Psi_i [\phi(\mathbf{x})] |_{\phi \equiv \phi_{Ci}} &= \\ c_i \cdot \exp \left\{ -\alpha \int d\mathbf{x} [\phi_{Ci}(\mathbf{x}) - \phi_0]^2 \right\} &\rightarrow \\ c_1 \cdot \exp \left(-\alpha_1 \cdot \int d\tilde{x} [\phi_F]^2 \right) &\equiv \Psi_{initial}, \end{aligned} \quad (6)$$

Appendix V: Extra Dimensions And The Break Down Of The Semi - Classical Approximation. This Is An Illustration Of This Concept, And Nothing More

Here, I used equation 4.7 of the main text. For the first potential system, if we set $x_b=1$, $x_a=-1$, and $b=10$. (a sharp slope) for the scalar field boundary we have.

$$\alpha = \frac{0.373}{1} \quad (1)$$

This assumes a Gaussian wave functional of

$$\psi(x) = \exp(-\alpha \cdot \phi(x)) \quad (2)$$

As well as a power parameter of

$$\nu = 9 \quad (3)$$

Also, we are using, initially, a phase evolution parameter of

$$\phi(x) = \pi \cdot [\tanh[b \cdot (x - x_a)] - \tanh[b \cdot (x_b - x)]] \quad (4)$$

The first potential system is re scaled as

$$V1(x) = \frac{1}{2} (1 - \cos(\phi(x))) - \frac{1}{200} (\phi(x) - \pi)^2 \quad (5)$$

In addition, the following is used as a rescaling of the inner product

$$c1 = \frac{1}{\int_{-30}^{30} (\exp(-\alpha \cdot \phi(x)))^2 \frac{\pi^3}{3} \cdot x^5 dx} \quad (6)$$

$$c2 = \int_{-30}^{30} (\exp(-\alpha \cdot \phi(x)))^2 \frac{\pi^3}{3} \cdot x^5 (V1(x))^\nu \cdot |c1| dx \quad (7)$$

$$c3 = \left[\int_{-30}^{30} (\exp(-\alpha \cdot \phi(x)))^2 \frac{\pi^3}{3} \cdot x^5 V1(x) \cdot |c1| dx \right]^\nu \quad (8)$$

$$c3b = \frac{c2}{c3} \quad (9a)$$

Here,

$$c3b = 0.999 \quad (9b)$$

For the 2nd potential system, if we assume a sharp slope, i.e. b1 = b = 10, and

$$V2(x) = \frac{1}{2} \cdot \frac{(\phi a(x))^2}{1 + 0.000001 (\phi a(x))^3} \quad (10)$$

If

$$\phi a(x) = \pi \cdot [\tanh[b1 \cdot (x - xa)] - \tanh[b1 \cdot (xb - x)]] \quad (11)$$

and a modification of the ‘Gaussian width’ to be

$$\alpha1 = \frac{0.373}{30} \quad (12)$$

We do specify a denominator, due to a normalization contribution we write as

$$c1a = \frac{1}{\int_{-30}^{30} (\exp(-\alpha1 \cdot \phi a(x)))^2 \frac{\pi^3}{3} \cdot x^5 dx} \quad (13)$$

$$c4 = \int_{-30}^{30} (\exp(-\alpha1 \cdot \phi a(x)))^2 \frac{\pi^3}{3} \cdot x^5 (V2(x))^\nu \cdot |c1a| dx \quad (14)$$

In addition:

$$c5 = \left[\int_{-30}^{30} (\exp(-\alpha \cdot \phi a(x)))^2 \frac{\pi^3}{3} \cdot x^5 V2(x) \cdot |c1a| dx \right]^\nu \quad (15)$$

We then use a ratio of

$$c5b = \frac{c4}{c5} \quad (16)$$

Here, when one has the six dimensions, plus the thin wall approximation:

$$C5b = 2.926E - 3 \quad (17)$$

When one has three dimensions, plus the thin wall approximation

$$c6 = \int_{-30}^{30} (\exp(-\alpha1 \cdot \phi a(x)))^2 \frac{\pi}{0.25} \cdot x^2 (V2(x))^\nu \cdot |c1b| dx \quad (18)$$

$$c7 = \left[\int_{-30}^{30} (\exp(-\alpha \cdot \phi(x)))^2 \frac{\pi}{0.25} \cdot x^2 V2(x) \cdot |c1b| dx \right]^\nu \quad (19)$$

$$c7b = \frac{c6}{c7} \quad (20)$$

This leads to

$$c7b = 0.019 \quad (21)$$

When one has the thin wall approximation removed, via $b1 = 1.5$, one does not see a difference in the ratios obtained.

For the 3rd potential system, which is intermediate between the 1st and 2nd potentials if the $b1 = b = 10$ value is used, one obtains for when we have six dimensions

$$\alpha1 = \frac{0.373}{6} \quad (22)$$

As well as

$$V2(x) = \frac{1}{2} \cdot \frac{(\phi a(x))^2}{1 + 0.5 (\phi a(x))^3} \quad (23)$$

(When we have six dimensions)

$$C5b = 0.024 \quad (24)$$

(When we have three dimensions)

$$C7b = 0.016 \quad (25)$$

So, then one has $C5b = 0.024$, and $C7b = 0.016$ in the thin wall approximation

When $b1 = 3$ (non thin wall approximation)

$$C5b = 0.027 \quad (26)$$

(Six dimensions)

$$C7b = 0.02 \quad (27)$$

(Three dimensions)

Summarizing, if

$$V1(x) = \frac{1}{2} (1 - \cos(\phi(x))) - \frac{1}{200} (\phi(x) - \pi)^2 = V1 \quad (28)$$

$$V2(x) = \frac{1}{2} \cdot \frac{(\phi a(x))^2}{1 + 0.000001 (\phi a(x))^3} = V3 \quad (29)$$

$$V2(x) = \frac{1}{2} \cdot \frac{(\phi a(x))^2}{1 + 0.5 (\phi a(x))^3} = V2 \quad (30)$$

One finally obtains the following results, as summarized below

	b=b1=10	b1=3	b1=1
V1 (6 dim)	C3b = .999	No data	No data
V3 (6 dim)	C5b = 2.926E-3	No data	C5b = same value
V3 (3 dim)	C7b = .019	No data	C7b = same value
V2(6 dim)	C5b = .027	C5b = .024	No data
V2 (3 dim)	C7b = .02	C7b = .016	No data

Appendix VI: Decay Rates, And Cosmic Nucleation, I.E. Presenting A New Way To Obtain Initial Evolution Of The Hubble Parameter And A Rate Equation

Garriga [33], assuming a nearly flat De Sitter universe also came up with an expression for the number density of particles per unit length (time independent)

$$n \approx \frac{1}{2 \cdot \pi} \cdot \sqrt{M^2 + e \cdot \frac{E_0^2}{H^2}} \cdot \exp(-S_E) \quad (1)$$

where for our purposes we would set

$$M \leq M_P \rightarrow 1 \quad (2)$$

We prefer instead to use an estimation of a nucleation rate per Hubble volume per Hubble time [29]

$$\in (t) \equiv \lambda_0 / (H(t)^4) \approx 1 \quad (3)$$

to show the influence an evolving Hubble parameter would have , in early times, without the complexity of predicting the S_E (a Euclidian action integral) which would be in our example a D+1 dimensional space knocked down to being quasi 1 dimensional in ‘character’ . We assume, also, rescaling of Planckian length to be unity where $\hbar \equiv c \equiv G \equiv 1$.

This leads to, then

Appendix VII: Predicting How A Scale Factor Evolves In The Beginning Of Inflationary Cosmology

I wish now to look at how the scale factor, a , changes in time, in a manner we view which will enable us to delineate Hubble parameter variations in the first few moments after creation. In doing this, we can note the typical value [34] (as given by Dodelson)

$$a(t) \cong a_B \cdot \exp(H_B(t - t_B)) \quad (1)$$

with a_B , H_B and t_B being scale factor, Hubble parameter, and time values at the end of an inflationary period of expansion. Needless to say, in doing this, we are not obtaining values of what the scale factor and Hubble parameter could be at the onset of inflation, which is a situation we wish to remedy. So we set

$$\tilde{a}_0 \equiv a_B \exp(H_B(t_P - t_B)) \quad (2)$$

and afterwards approximate the evolution of phase after time t_P via use of

$$\phi \equiv \tilde{\phi}_0 - \frac{m}{\sqrt{12 \cdot \pi \cdot G}} \cdot t \cong \tilde{\phi}_i \cdot \left(\exp(-\tilde{a}_0 \cdot t/\alpha) \approx 1 - \tilde{a}_0 \cdot t/\alpha \right) \quad (3)$$

If we assume that $\tilde{\phi}_0 \cong \tilde{\phi}_i$, and that the time factors are small, we can state

$$\frac{m}{\sqrt{12 \cdot \pi \cdot G}} \cong \frac{\tilde{a}_0}{\alpha} \quad (4)$$

as an order of magnitude estimate for the initial value of our scale factor at the beginning of inflation. So being the case, we move then to obtain a value for the initial evolution of the Hubble parameter via use of conformal time, with an Einstein equation

$$\ddot{\phi} + 2 \cdot a \cdot H \cdot \dot{\phi} + a^2 \cdot m^2 (\phi - \phi^*) = 0 \quad (5a)$$

where the conformal time we write as

$$\tilde{t} \cong -\frac{1}{a(t) \cdot H} \quad (5b)$$

which may be re written using ordinary time as

$$\ddot{\phi} + 3 \cdot H \cdot \dot{\phi} + m^2 (\phi - \phi^*) = 0 \quad (5c)$$

which would lead to $\epsilon(t) \equiv \lambda_0/(H(t)^4) \approx 1$ ²⁹ implying a nucleation rate evolution along the lines of

$$H \cong \frac{1}{3} \cdot \left(\frac{\tilde{a}_0}{\alpha} \right)^{-1} + \left(\frac{\tilde{a}_0}{\alpha} \right) \frac{m^2}{3} \cdot \left(1 - \frac{\phi^*}{\phi_0} \cdot \exp\left(\frac{\tilde{a}_0}{\alpha} \cdot \tilde{t}\right) \right) \quad (6)$$

implying

$$\lambda_0(t_P + \delta \cdot t) \approx H^4(t_P + \delta \cdot t) \quad (7)$$

which for small times just past the initial value of $t \equiv t_P + \delta \cdot t$ leads to a nearly stable but increasing rate of the Hubble parameter right after a nucleation of a universe. This also leads to a phase change in behavior which I claim is motivated by the pre Planck time value of the Hubble parameter being set by (for times $t \leq t_P$)

$$H^2 \equiv \frac{8 \cdot \pi}{3} \cdot G \cdot V(\phi) \rightarrow \frac{8 \cdot \pi}{3} \cdot V(\phi) \quad (8)$$

with the potential given by a washboard potential with a small driving potential proportional to $(\phi - \phi^*)^2$ which blends into Guths chaotic inflationary model for times $t_P + \delta \cdot t$.

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Vectorial Lorentz Transformations

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Abstract: We have noticed in relativistic literature that the derivation of Lorentz Transformations (LT) usually is presented by confining the moving system O' to move along the X-axis, namely, as a particular case of a more general movement. When this movement is generalized different transformations are obtained (which is a contradiction by itself) and a hidden vectorial characteristic of time is revealed. LT have been generalized in order to solve some physical and mathematical inconsistencies, such as the dissimilar manners (transversal, longitudinal) the particle's shape is influenced by its velocity and LT's inconsistency with Maxwell equations when in its derivation the pulse of light is sent perpendicular to the displacement of the moving system O'. Unlike the canonical derivation of LT, in the undertaken development of the generalized LT, assumptions were not used. Practical applications of generalized Vectorial Lorentz Transformations (VLT) were undertaken and as outcome a new definition of Local Lorentz Transformations (LLT) of magnitudes appeared. As another consequence, a characteristic and unique scaling Lorentz factor was obtained for each magnitude. Given this, a dimensional analysis based upon these Lorentz factors came up. In addition, dynamical transformations were obtained and a new definition of mass was found.

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1. Introduction

The unexpected results obtained by Michelson and Morley in 1881, according to which Earth did not move in any direction, under the interpretation induced at that time by Ether hypothesis, motivated Dutch physicist Hendrick Antoon Lorentz and British physicist George Francis FitzGerald, almost simultaneously during 1889-1890, to look for

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a mathematical and physical way to interpret this result. They undertook this task by preserving Maxwell equations to be the same in any inertial system, and by taking into account the constancy of the speed of light by observers with different inertial movements. This led to discard the undoubted, untouchable and accepted, until that time, Galilean transformations, and to replace them by Lorentz Transformations (LT) in 1904, when they were displayed formally [1]. However, it was known at that time that Galilean transformations although privileged as of “physical common sense”, neither preserved the constancy of light speed nor were consistent with Maxwell Equations. After Einstein’s solid arguments destroyed Ether hypothesis in 1905 [2], LT became central for the Special Theory of Relativity (STR) [3]. Nevertheless, LT, as we previously indicate, have their limitations.

This work was focused in showing these limitations, in order to derive a way to eliminate them and start developing a more general approach of LT. The master key for achieving this task was the presentation of LT under a generalized configuration of an inertial stationary system and another moving system at a constant velocity, along an inclined line not coincident with any axis (**Fig. A5, Annex 1**). In this general case both systems were considered as orthogonal systems, with all their corresponding axes being parallel. It can be shown that the usual one-dimensional (extended by isotropy to other coordinates) configuration modernly used to present the LT does not allow recognizing such limitations of LT. It should be noted that Lorentz did his work based on a system moving along the X-axis by assuming the coordinates y' and z' as not depending on the Lorentz’s scaling factors, namely, $y' = y$ and $z' = z$ [1]. Almost Independently, Einstein presented this derivation more directly, but assuming the same [2]. Since then, this has become the canonical way of presenting LT.

The generalized configuration used in this work took to us to obtain transformations, different from those given by the well-known LT, without doing any type of assumptions. It also revealed to us **a vectorial presentation of time** with components **depending on spatial coordinates** within such transformations. This surprising detection was, therefore, a direct consequence of the different configuration and procedure used.

It is worth mentioning that time, having vectorial properties, has recently been proposed, as a new theory or hypothesis for solving spin problems, by several authors including E. A. B. Cole [5-7], J. Strnad [8-9], V. Barashenkov [11-13], Xiadong Cheng [14], A. P. Yefremov [15], H. Kitada [16-17], J. E. Carroll [18], among others. A common point that these authors considered is a multi-dimensional vector time, with components **not depending** on spatial coordinates, under the canonical Lorentzian scheme of presentation previously referred. In order to be consistent with currently accepted relativity concepts, channels used in these papers, to control results derived from assumptions were, firstly, they should reduce under some conditions to the results given by the four-vector space-time theory, and secondly to those of the Special Relativity Theory (SRT). Such theoretical hypotheses of time as a vector are presented as an extension of the four-dimension (4D) Minkowski space-time to six or more dimensions, the most of them consisting of the symmetrical 6D, three dimensions for time in the and three for space. Although

no experimental evidence supports these schemes, their mathematical consistency is a permanent motivation for a further discussion [7] [9]. D. Barwacz also undertakes the hypothesis of time as a vector in four-dimensions, the three spatial plus one temporal. He simplifies his task for studying them to only one spatial and one temporal, but his development is extensible to 4D [10].

Nevertheless, the present study takes a different line of presentation more general than that canonical used in LT in the sense that a stationary system and an inertial system moving along an inclined straight line are considered. The obtained results implied to work in Physics within a simple universe of three spatial dimensions (not 4D or 6D, or more). By this approach, without assumptions, we were able to derive a vectorial structure of time with spatial components t'_x, t'_y, t'_z , measured at O', in function of components t_x, t_y, t_z (measured by the observer located at the origin O of the stationary system). Thus, the deduced vector time obtained within these transformations is therefore a vector **depending** on spatial coordinates. It is also worth mentioning that Hongbao Ma briefly stated a vectorial presentation of time **depending** on spatial coordinates, without referring measurements to distinct inertial observers. Namely, by a very direct procedure he obtained the components of vector time, associated to a moving point, as the relation between the components of the radio-vector of the moving point and the magnitude of its velocity, i.e., $t_x = \frac{x}{v}$, $t_y = \frac{y}{v}$, $t_z = \frac{z}{v}$, [4].

It is also a crucial fact to keep in mind that the canonical and modern way of presenting the LT derivation (firstly focused in one-dimension, the X axis, and then extending, by the isotropy postulate, to the other coordinates) does not allow perceiving the vectorial features of time informed in this work.

A final statement condensing author's motivation for developing this work can be read in the (2004) work of Bernard Guy: "... *the relativity: This theory shows a general structure that is not uniquely linked to the properties of light: may be the photon will not have the last word...*".

"... *in a way, time a priori has a three dimensional content, and this point of view allows solving the problems arising in the standard relativity theory. **Standard relativity theory does work well only in one space dimension** with a simple duality between one space variable and one time variable. **In 3D, it does not work properly...***"[20].

The current work consists of four sections and three annexes. In section 1, is shown a modern and simple way of deriving LT following an analogous procedure like that encountered in [21], preserving the statement structure of Lorentz [1]. In section 2, by means of examples the vectorial structure of time is revealed. Based on this fact LT is generalized to Vectorial Lorentz Transformations (VLT) where assumptions were needless. In Section 3 Local Lorentz Transformations were defined and developed consistently and Section 4 is devoted to the conclusions. In Annex 1 is presented three examples revealing clearly the time as a vector. In Annex 2 is shown the consistency of VLT with Maxwell Equations, and consequently is demonstrated the inconsistency of canonical LT with Maxwell Equations. In Annex 3 is shown how the VLT can be extended to curvilinear movement.

For text and formulas Microsoft Word and its Microsoft editor of equations 3.0 were used. Pictures were done using Microsoft PowerPoint.

2. Lorentz Transformations

A simple way modernly used to arrive at LT, under the postulate that velocity of light is constant and independent of the source in any inertial frame and that the laws of physics are the same in all inertial frames, is to consider two observers, with all the equipment for doing measurements of length, time and velocities onto moving projectiles; the first one located at the origin of coordinates of a fixed system O, and the second one located at the origin of coordinates of an inertial moving system O'. System O' moves at a constant velocity v , relative to O, such that X' and X axes are on the same line. The goal is to obtain relationships between the observer's measurements, such that they must be valid for any velocity of any projectile including that of photons, c . In order to arrive at a solution taking into account these conditions, the procedure starts by taking as projectile a pulse of light. As a control, resulting transformations should be consistent with Galilean transformations for inertial systems with relative velocity $v \ll c$.

When moving origin O' coincides with fixed origin O, at $t = t' = 0$, a pulse of light is sent parallel to X-axis, (**Fig. A1a, Annex 1**). Observers measure a component x of the light pulse displacement at O and a component x' at O'. (In this case the Galilean Transformations are: $x' = x - v.t$; $y' = y = 0$; $z' = z = 0$; $t' = t$).

By doing a Galilean reasoning, but using a scaling factor k , to be calculated, for mathematically preserving the constancy of the speed of light, c , it is established the following relationship for the X-axis component of the light pulse:

$$x' = k.(x - v.t) \quad (1)$$

Under these conditions, the light pulse must fulfill the following relationships:

$$x' = c.t' \quad \Rightarrow \quad t' = \frac{x'}{c} \quad x = c.t \quad \Rightarrow \quad t = \frac{x}{c} \quad (2a)$$

Or, under the same conditions, for any projectile traveling at velocity $u_x \leq c$:

$$x' = u'_x.t' \quad \Rightarrow \quad t' = \frac{x'}{u'_x}; \quad x = u_x.t \quad \Rightarrow \quad t = \frac{x}{u_x}; \quad u_x \neq u'_x \quad (2b)$$

Substituting relations from equations (2a) into equation (1), LT for time is obtained:

$$c.t' = k. \left(c.t - v. \frac{x}{c} \right) \quad \Rightarrow \quad t' = k. \left(t - \frac{v}{c^2}.x \right) \quad (3)$$

Now, change the observer's role, namely, start considering O' fixed and O, the moving system. Under this configuration it is clear that the observer at O' will see the system O moving at velocity $-v$, namely, going in opposite sense to the light pulse. Reasoning as in the original situation, a pulse of light is sent when O and O' coincide, and a similar

relation to that of (1) is constructed through the same constant k (see **Fig. A1b, Annex 1**). Thus, from O the following relationship is constructed:

$$x = k \cdot (x' + v \cdot t') \quad (4)$$

Because these equations must be valid in any situation, from equations (1) and (4), the following relationships hold:

$$\begin{aligned} x' &= k \cdot (x - v \cdot t) = k \cdot \left(x - \frac{v \cdot x}{c}\right) = k \cdot x \cdot \left(1 - \frac{v}{c}\right) \\ x &= k \cdot (x' + v \cdot t') = k \cdot \left(x' + \frac{v \cdot x'}{c}\right) = k \cdot x' \cdot \left(1 + \frac{v}{c}\right) \end{aligned}$$

Multiplying both equations, the value of factor k is obtained:

$$x' \cdot x = k^2 \cdot x \cdot x' \cdot \left(1 - \frac{v^2}{c^2}\right) \quad \Rightarrow \quad k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5)$$

(For $v \ll c \Rightarrow k \cong 1$; $x' \cong x - v \cdot t$; $y' = y = 0$; $z' = z = 0$; $t' \cong t$. Namely, as it was expected, these transformations are reduced to those of Galileo).

In this way the set of transformation equations between both systems of coordinates are obtained. By extending the validity of these relationships to any movement of O' (see **Fig. A4, Annex 1**), “by the geometry of the problem” or by the Isotropy postulate, they become:

$$x' = \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}} \quad y' = y \quad z' = z \quad t' = \frac{t - \frac{v \cdot x}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = t \cdot \frac{1 - \frac{v}{c^2} \cdot u_x}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (6)$$

Dividing by time, velocity transformations u of light pulse are,

$$u'_x = \frac{u_x - v}{1 - \frac{v \cdot u_x}{c^2}} \quad u'_y = \frac{u_y \cdot \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v \cdot u_x}{c^2}} \quad u'_z = \frac{u_z \cdot \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v \cdot u_x}{c^2}} \quad (7)$$

First of all, it is worth mentioning that the second and third expressions in (6) do not come from equations or relationships but from *assumptions*. Again, the second and third expressions in (7) are direct consequence of such assumptions. It is important to emphasize that LT in (6) and (7) are currently accepted all over the world since the Special Theory of Relativity arrived at, a century ago.

One of the aspects that disenchant to those trying to see more inside LT is that it is impossible to find their derivation in a generalized form to any type of movement of the inertial system O'. In fact, as far as I know, in all the publications taking up this subject, the moving system O' is always confined to move on the X axis, extrapolating by “common sense” or any other postulate, to the general movement of O' [1], [2], [3], [8], [10], [16]; ¿Why? ¿Why cannot the system O' be generally presented as moving along an inclined trajectory, not coinciding with any axis? It can be understood that such configuration introduces problems for establishing the previously mentioned

assumptions, -until now untouchables and accepted-. In this work is concluded that such assumptions are unnecessary and hence, groundless. **in this work, as expected, it is presented a demonstration that LT are mathematically inconsistent with Maxwell Equations, see Annex 2.**

Obviously, Lorentz objective was to correct Galilean transformations in order to obtain general transformations that preserve the constancy of the light speed and also to be consistent with Maxwell equations so that they remained the same in any inertial system. In this work, LT are generalized by taking the moving system O' to move on an inclined line. This configuration allowed us to discover new features about time and took us to develop LT simply and consistently.

3. Is Time A Vector?

We are going to check that the answer to this question is yes. In this section we will depict the way how we were forced to arrive at the following concept of time: “Time is not only different for observers with distinct inertial movements, but additionally, it behaves between them as a vector”. As it will be observed, this concept will not result from any assumption. Instead, the vector structure will be deduced from the analysis of time's obtained expressions, for one, two, three (or more, if it were necessary) spatial dimensions. The two-dimensional case presented in **Annex 1, Fig. A3**, is a very illustrative example to clarify this concept, and it is repeated here in more detail in the next paragraphs:

Referring to figure A3, when O', moving along an inclined line, and O coincide, a light pulse is sent in any direction. By defining α , as the angle between trajectory of O' and X axis, and reasoning as in Part 1, the following equations hold:

$$\begin{aligned} x^2 + y^2 = c^2.t^2 & \quad x' = k.(x - v.t.\cos\alpha) \\ & \text{for,} \\ x'^2 + y'^2 = c^2.t'^2 & \quad y' = k.(y - v.t.\sin\alpha) \end{aligned}$$

Based on these previous relationships, by substituting, working on and grouping properly, we obtain:

$$\begin{aligned} c^2.t'^2 = x'^2 + y'^2 &= k^2.[(x - v.t.\cos\alpha)^2 + (y - v.t.\sin\alpha)^2] \\ c^2.t'^2 &= k^2.[(x^2 + y^2) + [v^2.(t.\cos\alpha)^2 + v^2.(t.\sin\alpha)^2] - 2.v.x.(t.\cos\alpha) - 2.v.y.(t.\sin\alpha)] \\ c^2.t'^2 &= k^2.[c^2.(t)^2 + v^2.(t)^2 - 2.v.[x.(t.\sin\alpha) + y.(t.\sin\alpha)]] \end{aligned}$$

Substituting: $c^2.t'^2 \equiv c^2.t^2.(sin^2\alpha + cos^2\alpha)$ and $v^2.t^2 = v^2.\frac{x^2+y^2}{c^2}$, we get:

$$\begin{aligned} c^2.t'^2 &= k^2.\{[c^2.(t.\cos\alpha)^2 - 2.v.x.(t.\cos\alpha) + v^2.\frac{x^2}{c^2}] + [c^2.(t.\sin\alpha)^2 + 2.v.y.(t.\sin\alpha) + v^2.\frac{y^2}{c^2}]\} \\ c^2.t'^2 &= k^2.[(c.t.\cos\alpha - \frac{v}{c}.x)^2 + (c.t.\sin\alpha - \frac{v}{c}.y)^2] \end{aligned}$$

From the last relationship, it is obtained the following expression for time:

$$t'^2 = k^2.[(t.\cos\alpha - \frac{v}{c^2}.x)^2 + (t.\sin\alpha - \frac{v}{c^2}.y)^2]$$

By observing carefully the right hand side of the previous expression, it reminds us the module of a vector. Thus, as it is suggested, the previous modular expression can be re-organized into its corresponding two-dimensional vectorial structure, in the following way:

$$\mathbf{t}' = k. \left[\left(t. \cos \alpha - \frac{v}{c^2}.x \right) \mathbf{i} + \left(t. \sin \alpha - \frac{v}{c^2}.y \right) \mathbf{j} \right] = k. \left[t. \cos \alpha. \mathbf{i} - \frac{v}{c^2}.x. \mathbf{i} + t. \sin \alpha. \mathbf{j} - \frac{v}{c^2}.y. \mathbf{j} \right]$$

$$\mathbf{t}' = k. \left[\left(t. \cos \alpha. \mathbf{i} + t. \sin \alpha. \mathbf{j} \right) - \frac{v}{c^2}. (x. \mathbf{i} + y. \mathbf{j}) \right] = k. \left[(t_x. \mathbf{i} + t_y. \mathbf{j}) - \frac{v}{c^2}. (x. \mathbf{i} + y. \mathbf{j}) \right]$$

$$\text{Thus, by defining: } \left\{ \begin{array}{l} t_x = t. \cos \alpha \\ t_y = t. \sin \alpha \\ \mathbf{t} = t_x \mathbf{i} + t_y \mathbf{j} \end{array} \right\} \text{ and } \left\{ \begin{array}{l} t'_x = k. \left(t_x - \frac{v}{c^2}.x \right) \\ t'_y = k. \left(t_y - \frac{v}{c^2}.y \right) \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \mathbf{t}' = k. \left(\mathbf{t} - \frac{v}{c^2}.\mathbf{r} \right) \\ \mathbf{r}' = k. \left(\mathbf{r} - v.\mathbf{t} \right) \end{array} \right\}$$

It can be realized that this vector structure of time can be easily obtained for any number of dimensions by repeating this same procedure (**see Annex 1**). So, the vector character of time is not the result of any hypothesis; it comes directly from observing vector properties clearly present inside transformations relating measurements of both inertial observers. It can also be observed that, from an epistemological point of view, time as vector forms its direction by taking it from the vector velocity v of the moving system O' , leaving such parameter with a scalar character and functioning as a scaling factor. This can be understood due to both observers are on the same inclined line, which will imply the scalar character of v . Another epistemological characteristic of vector time is its dependence on coordinates x , y and z , which means that **it is not an independent vector**, a characteristic that appears as remarkable because differs to that of four dimensions (time as a fourth independent dimension) introduced by Einstein. This also means that we are still working in three spatial dimensions in this study, and that magnitudes can continue being defined as in classical physics, but with a modern and relativistic view. From observing these **results** we can develop the following definition of time: Time is forced to behave as a vector with spatial components in each coordinate, when it appears inside the VLT, but it can appear behaving as a scalar when it is not an element of a transformation such as VLT in the way we always have known it: as a sequential meter of events. But moreover, time can also be considered as a vector in the natural way it was referred to previously in [4]. In accordance with this idea and it perfectly applies to our work, Hongbao Ma says: *“this three dimensional time concept is obtained from the mathematical conception rather than the ontological existence. Mathematical results are at the epistemological level”* [4].

By considering time as having the properties of a vector when reflected within the relation between inertial observers with different movements, let's formally obtain the vectorial version for the Lorentz transformations (VLT). So, now we will refer in general to the three-dimensional case, or for further research it could be thought in an n-dimensional case, (**see Annex 1, Fig. A5**), where system O' moves on an inclined line and both observers will measure the light pulse radio-vectors \mathbf{r} , \mathbf{r}' . Vectors from now on will be written as boldface letters. The relationships in VLT, previously seen, are easily obtained

from Fig. A5:

$$\mathbf{r} = c.\mathbf{t} \quad \mathbf{r}' = c.\mathbf{t}' \quad \mathbf{t}' = \frac{\mathbf{r}'}{c} \quad \mathbf{t} = \frac{\mathbf{r}}{c}$$

$$\left\{ \begin{array}{l} \mathbf{r}' = k(\mathbf{r} - v.\mathbf{t}) \Rightarrow c.\mathbf{t}' = k.\mathbf{t}.(c - v) \\ \mathbf{r} = k(\mathbf{r}' + v.\mathbf{t}') \Rightarrow c.\mathbf{t} = k.\mathbf{t}'.(c + v) \end{array} \right\} \Rightarrow c^2.\mathbf{t}.\mathbf{t}' = k^2.\mathbf{t}'.\mathbf{t}.(c^2 - v^2) \Rightarrow k^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\mathbf{r}' = \frac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow c.\mathbf{t}' = \frac{c.\mathbf{t} - v.\frac{\mathbf{r}}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{u}' = \frac{d\mathbf{r}'}{dt'} = \frac{d\mathbf{r} - v.d\mathbf{t}}{\left| dt - \frac{v}{c^2}.d\mathbf{r} \right|} \quad (8)$$

The following equality also holds as invariant for VLT: $c^2.t'^2 - r'^2 = c^2.t^2 - r^2$. This means that the cinematic VLT, composed by expressions, \mathbf{r}' , \mathbf{t}' and \mathbf{u}' in (8), are generally valid for a light pulse or for any projectile moving at any speed less than c . As a check, the Jacobian matrix for any value of variables, becomes symmetric and equal to one, i.e., Letting x^i , be the variables measured by O, and \bar{x}^j , be those measured by O', for $i, j = 1, 2, 3$, and for $k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, we have:

$$\begin{array}{l} \mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \quad \mathbf{r} = k.(\mathbf{r}' + v.\mathbf{t}') \\ \mathbf{t}' = k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \quad \mathbf{t} = k.(\mathbf{t}' + \frac{v}{c^2}.\mathbf{r}') \end{array} \Rightarrow \begin{array}{l} \left(\frac{\partial x^i}{\partial \bar{x}^j} \right) = \begin{bmatrix} k & -k.v \\ -k.\frac{v}{c^2} & k \end{bmatrix} \\ \left(\frac{\partial \bar{x}^j}{\partial x^i} \right) = \begin{bmatrix} k & +k.v \\ +k.\frac{v}{c^2} & k \end{bmatrix} \end{array} \quad \left(\frac{\partial x^i}{\partial \bar{x}^j} \right) = \left(\frac{\partial \bar{x}^j}{\partial x^i} \right) = 1$$

This is valid for any set of components: $\left\{ \begin{array}{l} y' = k.(y - v.t_y) \quad y = k.(y' + v.t'_y) \\ t'_y = k.(t_y - \frac{v}{c^2}.y) \quad t_y = k.(t'_y + \frac{v}{c^2}.y') \end{array} \right\}$

A final remark on the procedure previously presented: needless to say is that it was not done any assumption for obtaining the VLT presented in (8). Thus, because these are vectorial relationships, they are generally valid for any number of dimensions. It is opportune to say that the consistency of VLT with Maxwell Equations is demonstrated In Annex 2.

Let's obtain the expressions for VLT in three dimensions using spherical coordinates (**Fig. A5**). Allow β to be the angle between the inclined trajectory of O' and the plane XY; and allow α to be the angle formed by the projection of the inclined trajectory of O' on the plane XY, with the X-axis. When moving origin O' and fixed one O coincide, the light pulse is sent towards the space with generic components x, y, z , The general VLT of the vector time and that of the radio-vector of the pulse of light (or projectile), in three

dimensions, become:

$$\begin{aligned}
 x' &= \frac{x-v.t.\cos\alpha.\cos\beta}{\sqrt{1-\frac{v^2}{c^2}}} & t_x &= t.\cos\beta.\cos\alpha & x' &= \frac{x-v.t_x}{\sqrt{1-\frac{v^2}{c^2}}} \\
 y' &= \frac{y-v.t.\sin\alpha.\cos\beta}{\sqrt{1-\frac{v^2}{c^2}}} & t_y &= t.\cos\beta.\sin\alpha & y' &= \frac{y-v.t_y}{\sqrt{1-\frac{v^2}{c^2}}} \\
 z' &= \frac{z-v.t.\sin\beta}{\sqrt{1-\frac{v^2}{c^2}}} & t_z &= t.\sin\beta & z' &= \frac{z-v.t_z}{\sqrt{1-\frac{v^2}{c^2}}} \\
 t' &= \left| \frac{\mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} \right| = \sqrt{\frac{(t_x - \frac{v}{c^2} \cdot x)^2 + (t_y - \frac{v}{c^2} \cdot y)^2 + (t_z - \frac{v}{c^2} \cdot z)^2}{1-\frac{v^2}{c^2}}}
 \end{aligned}$$

The expressions for the velocities of the pulse of light or any projectile are obtained from the previous ones:

$$\begin{aligned}
 u'_x &= \frac{u_x - v.\cos\alpha.\cos\beta}{\sqrt{(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2})^2 + (\sin\alpha.\cos\beta - \frac{v.u_y}{c^2})^2 + (\sin\beta - \frac{v.u_z}{c^2})^2}} \\
 u'_y &= \frac{u_y - v.\sin\alpha.\cos\beta}{\sqrt{(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2})^2 + (\sin\alpha.\cos\beta - \frac{v.u_y}{c^2})^2 + (\sin\beta - \frac{v.u_z}{c^2})^2}} \\
 u'_z &= \frac{u_z - v.\sin\beta}{\sqrt{(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2})^2 + (\sin\alpha.\cos\beta - \frac{v.u_y}{c^2})^2 + (\sin\beta - \frac{v.u_z}{c^2})^2}}
 \end{aligned} \tag{9}$$

Now, let's particularize these general results to the conditions from where the original LT were obtained (**Fig. A4**). If we re-establish such conditions (the system O' moving along the X axis, and the light pulse sent to space), i.e., for $\alpha = \beta = 0$ we will obtain the VLT version of the original Lorentz transformations:

$$\begin{aligned}
 x' &= \frac{x-v.t}{\sqrt{1-\frac{v^2}{c^2}}} & t_x &= t \\
 y' &= \frac{y}{\sqrt{1-\frac{v^2}{c^2}}} & t_y &= 0 \\
 z' &= \frac{z}{\sqrt{1-\frac{v^2}{c^2}}} & t_z &= 0 \\
 t' &= \sqrt{\frac{(t_x - \frac{v}{c^2} \cdot x)^2 + (\frac{v}{c^2} \cdot y)^2 + (\frac{v}{c^2} \cdot z)^2}{1-\frac{v^2}{c^2}}}
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 u'_x &= \frac{u_x - v}{\sqrt{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2}} & u'_y &= \frac{u_y}{\sqrt{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2}} \\
 u'_z &= \frac{u_z}{\sqrt{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2}} & u_x'^2 + u_y'^2 + u_z'^2 &= u_x^2 + u_y^2 + u_z^2 = c^2
 \end{aligned} \tag{11}$$

Let's check the last relationship in (11), which is valid only for photons. In such equation is then implied that the velocity of light measured by any of the two observers should be the same, c (In general for any other projectile, $u'^2 \neq u^2$), i. e., on the basis of which O measures, $u_x^2 + u_y^2 + u_z^2 = c^2$, then O' will measure:

$$\begin{aligned}
 u_x'^2 + u_y'^2 + u_z'^2 &= \frac{(u_x - v)^2 + u_y^2 + u_z^2}{(1-\frac{v.u_x}{c^2})^2 + (\frac{v.u_y}{c^2})^2 + (\frac{v.u_z}{c^2})^2} = \frac{u_x^2 - 2.v.u_x + v^2 + u_y^2 + u_z^2}{1-\frac{2.v.u_x}{c^2} + \frac{v^2.u_x^2}{c^4} + \frac{v^2.u_y^2}{c^4} + \frac{v^2.u_z^2}{c^4}} \\
 &= \frac{(u_x^2 + u_y^2 + u_z^2) - 2.v.u_x + v^2}{1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^4} \cdot (u_x^2 + u_y^2 + u_z^2)} = \frac{c^2 - 2.v.u_x + v^2}{1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = \frac{c^2 \cdot (1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^2})}{1-\frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = c^2
 \end{aligned}$$

When comparing equations (10) and (11), with the original LT equations (6) and (7), the first thing we realize is that components y , z , measured by the fixed observer are different to those of y' , z' , measured by the moving observer, thus, contradicting LT's statements. Additionally, we can observe that the expression of time is completely different to that of Lorentz in (6). And of course, the obtained expressions, according to this work, for velocity components u'_x , u'_y , u'_z , are also different of those presented for LT in (7). By the way, recently J. H. Field following the canonical form of presentation of the LT in a detailed manner, shows as mathematically correct the assumptions $y' = y$ and $z' = z$ [19]. On the contrary, according to the current work these assumptions were shown to be groundless. Thus, in author's opinion, there are only two relevant possibilities for obtaining such disagreement: either isotropy postulate is not applicable for this configuration or postulate can't be applied in relativity. Further research will answer this question.

Given that our procedure to obtain the vectorial transformations did not use any type of assumptions, it sufficiently demonstrates that in LT they were needless and therefore, LT canonical procedure is reduced to be only valid for one spatial dimension. Thus, its validity cannot be extrapolated to a general configuration.

It is conceivable that when in 1905 Einstein established his remarkable concept of the variation of mass with its velocity [2], he was actually looking for the one-to-one variation of physical magnitudes between classic and relativistic physics through Lorentz factors. At that time he already "had" the relationships for length, time, velocity and mass. So, Einstein probably got to consider Lorentz transformations (*not general, as we have shown previously*) as a central part of the SRT [3]. In an author's speculative opinion, Einstein later abandons SRT due to some observed inconsistencies and limitations of LT, and may be this was one of reasons he had for developing the General Theory of Relativity (GRT) trying to avoid such type of limitations in his research.

4. Local Lorentz Transformations

As it is commonly expressed in relativistic literature, the practical consequences of LT, under conditions of simultaneity of events, or their occurrence at the same location, are those known as length contraction and time dilation, respectively.

The simultaneity of events and occurrence at the same place can be reduced to the situation of two observers measuring the same physical magnitude from the same reference. In author's opinion, Einstein probably also tried to reach this result, but he was not successful due to the observed limitations of the LT.

The question can be posed again, now under the VLT: What are those consequences of the VLT that affect in a practical way our life? And how can we manage these practical consequences?

For example, when a physicist conceives that a pulse of light lasts eight and a half minutes coming from the Sun to the Earth, and he receives such image in his eyes, he establishes that Sun is not there but 15.300 KM far apart that place. From this point of view, he is thinking in a way that instantaneously reflects a real view of the universe.

Images are never real but thoughts like the previous ones are instantaneous. In order to systematize these ideas and to apply them in a real comparison of measurements, let's establish the following two conventions:

(1) From now on, both observers will do their measurements taking the same reference.

Let the origin of the fixed observer be this reference. For example, if the fixed observer O measures the radio-vector \mathbf{r} which corresponds to the displacement of a projectile sent to the space, the observer on the moving system O' will measure a similar radio-vector \mathbf{R}' from this same reference of the fixed observer, such that \mathbf{R}' will fulfill $\mathbf{r}' = \mathbf{R}' - \mathbf{R}_0^|$, with the definitions of the variables \mathbf{R}' and $\mathbf{R}_0^|$ given below:

$$\mathbf{R}' = \frac{\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\mathbf{r} - v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} = \mathbf{r}' + \mathbf{R}_0^| \quad \text{for} \quad \mathbf{R}_0^| = \frac{v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\mathbf{r}_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

(1) The moving observer O' will not send any pulse of light (or projectile). Thus, he will measure a null displacement of the projectile. Then the radio-vector of his moving system, \mathbf{r}_0 , will be the only measurement completed. Thus, from (8) the expressions of the Local Lorentz Transformations (LLT) are obtained:

$$\mathbf{r}' = 0 \Rightarrow \mathbf{r} = v \cdot \mathbf{t} = \mathbf{r}_0; \quad \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2} \cdot v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{t}' = \mathbf{t} \cdot \sqrt{1 - \frac{v^2}{c^2}}; \quad \mathbf{R}' = \frac{\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (12)$$

In sum, the moving and fixed observers are referring their measurements to the “same time” and “same radio-vector” of system O'. As we observe in (12), time and distance vectors are related by a characteristic-scaling factor. The scaling factor (< 1) corresponding to time is a multiplier in any dimension or coordinate; for the case of distances it is a divider. In other words, a sphere expands uniformly in all directions.

However, It is important to point out that for the case of distances, the LLT are referred to the length measurements of the radio-vector that aims towards O', made by both observers **relative to the origin O**. This is completely different to what is done for VLT, where each observer does measurements relative to its own reference system. Namely, the transformations referred to LLT are different to those of VLT.

With those two conventions in mind, we will not be worried about location coincidence or simultaneity of events. The relations (12) imply that in LLT each physical magnitude, by virtue of its dependency on velocity, in a true way either contract, expand, growth or reduce, with the same scaling factor in all dimensions, independently of their image or how we see them. For example, if on the moving system the observer at O' measures a bar of length L_0 , lasting a time t_0 in his measuring, the observer on fixed system at O will measure this length as L and “time t_0 ” as t . The position of the bar in system O' is not relevant; what is important is that it is at rest for the observer at O' and moving on relative to O. Thus, the relationship between both measurements, according to LLT, will be:

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad t_0 = t \cdot \sqrt{1 - \frac{v^2}{c^2}} \quad \Rightarrow \quad L = L_0 \cdot \sqrt{1 - \frac{v^2}{c^2}}; \quad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (13)$$

This indicates that an observer in a “stationary” system O measures onto a moving bar at velocity v , a contraction from its original length L_0 to L , no matter which is the position of the bar in the system O', and a time dilation from t_0 to t , as it is shown in equations (13).

Lorentz factors in LLT act as scaling factors between measurements done at O and O', for any magnitude, no matter if this is a differential magnitude or an integral one. In other words, Lorentz factors are simply scaling factors between such measurements.

What is the real meaning of LLT expressed in equations (13)? First of all, **each component is affected by the Lorentz factor in the same way**, namely, contracting lengths. For example, If instead of a bar the observer in the moving system had had a squared bar, whose area, as we know, is the product of two lengths, then the obtained LLT of such area for observer at the fixed system, would be therefore the product of the two contracted lengths' LLT (later we will use this LLT characteristic of areas):

$$S' = S_0 = L_0^2 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{L^2}{1 - \frac{v^2}{c^2}} \Rightarrow S_0 = \frac{S}{1 - \frac{v^2}{c^2}} \Rightarrow S = S_0 \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (14)$$

A volume $V' = L_{01} \cdot L_{02} \cdot L_{03}$, measured from O' is related to the volume measured from O, $V = L_1 \cdot L_2 \cdot L_3$, by the characteristic product of the three contracted lengths given below in (15):

$$V' = V_0 = L_{01} \cdot L_{02} \cdot L_{03} = \frac{L_1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{L_2}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{L_3}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{L_1 L_2 L_3}{1 - \frac{v^2}{c^2}} \Rightarrow V_0 = \frac{V}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (15)$$

$$\Rightarrow V = V_0 \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}$$

The velocity of the origin O' is obtained by differentiating the displacement of O' respect to time and substituting known LLTs. The LLT of velocity becomes:

$$\mathbf{v}' = \frac{d\mathbf{R}'}{dt'} = \frac{\frac{d\mathbf{r}_0}{\sqrt{1 - \frac{v^2}{c^2}}}}{dt \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{r}_0}{dt}}{1 - \frac{v^2}{c^2}} \Rightarrow \mathbf{v}' = \frac{\mathbf{v}}{1 - \frac{v^2}{c^2}} \quad (16)$$

At this moment we realize that velocity of the moving system O', plays two roles: either as a scalar v , when it is inside the scaling factor, in where both observers see each other moving relative to themselves in the same line **under conditions of VLT**. Or, as a vector \mathbf{v}' , measured by the observer at O' into his own frame, but taking as reference the origin of the other system O, **under conventions of LLT**. It is important to be careful with these two different concepts!

After doing this necessary parenthesis, let's continue: LLT for acceleration is obtained in the same manner as in (16):

$$\mathbf{a}' = \frac{d\mathbf{v}'}{dt'} = \frac{\frac{d\mathbf{v}}{1 - \frac{v^2}{c^2}}}{dt \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{v}}{dt}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow \mathbf{a}' = \frac{\mathbf{a}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (17)$$

It is necessary to say at this moment that origin O' could be moving along an inertial curvilinear path following an inertial movement with variable velocity (**Annex 3, Figs. A6 and A7**). For example Earth has an undoubted inertial curvilinear movement around the Sun, and although it accelerates going to perihelion and reduce its speed after perihelion going to aphelion, we don't feel anything, buildings maintain their verticality, equilibrium of any kind is preserved, etc. So, with the transformations in equations (16) and (17), we would expect to obtain also LLT in Dynamics.

But first, let's obtain the transformation for angle between inertial systems. This magnitude emanate from the relation between curvilinear length of arc s and length of radius R . Because both magnitudes are lengths, Lorentz factors cancel out, and angle becomes invariant to LLT:

$$\alpha' = \frac{s'}{R'} = \frac{\frac{s}{\sqrt{1-\frac{v^2}{c^2}}}}{\frac{R}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{s}{R} \Rightarrow \alpha' = \alpha; \quad d\alpha' = \frac{ds'}{R'} = \frac{\frac{ds}{\sqrt{1-\frac{v^2}{c^2}}}}{\frac{R}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{ds}{R} \Rightarrow d\alpha' = d\alpha \quad (18)$$

In this way, angular velocity transforms as:

$$\omega' = \frac{d\alpha'}{dt'} = \frac{d\alpha}{dt \cdot \sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{d\alpha}{dt}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \omega' = \frac{\omega}{\sqrt{1-\frac{v^2}{c^2}}} \quad (19)$$

Let's try to obtain a dynamical transformation for Force, based on already known LLT of magnitudes. Let's suppose two masses rotating circularly around a center of mass C at the same angular velocity ω . See Figure below:

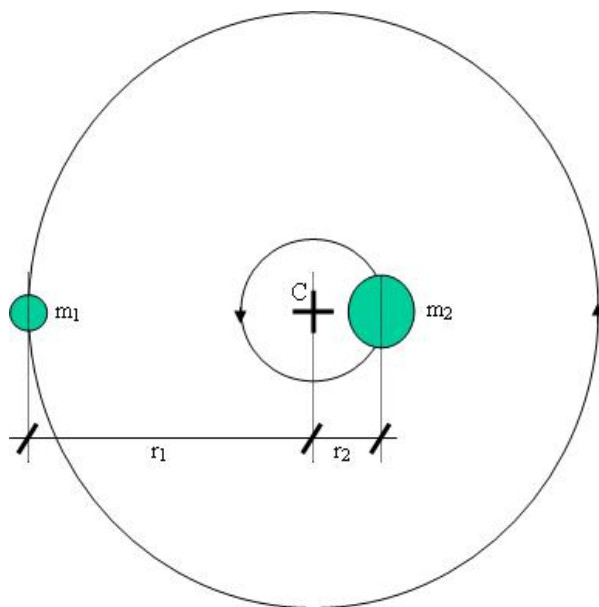


Fig. 1 Two masses rotating around a fixed center C

Because we have forced the masses to describe circular paths, it will allow us to do the following equivalent model to take over gravitational forces, i.e., only centrifugal forces

will be considered. Suppose a Hercules, located at the center of mass C, fixed, sustaining each mass through strong cords with each arm. Let there be three observers: Hercules at C, observer 1 on mass m_1 at a cord-distance r_1 from C, and observer 2 on mass m_2 at a cord-distance r_2 from C.

- (1) As a first conclusion, for Hercules to be in equilibrium, he must measure equal and opposite tensions in each arm. Thus: $m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2$.
- (2) The tension T_1 exerted at one of Hercules' arm by cord r_1 , measured by observer 1 on m_1 , will be $m'_1 \cdot \omega'^2 \cdot r'_1$, and tension T_2 exerted at Hercules' other arm by the cord r_2 , measured by observer 2 on m_2 , will be $m''_2 \cdot \omega''^2 \cdot r''_2$. Let's assume that tensions T_1 and T_2 are equal, in order to maintain, as before, Hercules in equilibrium, which would also mean that Force should be invariant to LLT. The magnitudes involved in both tensions are transformed with respect to what is measured by Hercules, the fixed observer, in the following manner (except for masses, whose transformation is unknown):

$$m'_1 \cdot \omega'^2 \cdot r'_1 = m'_1 \cdot \frac{\omega^2}{\left(1 - \frac{v_1^2}{c^2}\right)} \cdot \frac{r_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} \equiv m''_2 \cdot \omega''^2 \cdot r''_2 = m''_2 \cdot \frac{\omega^2}{\left(1 - \frac{v_2^2}{c^2}\right)} \cdot \frac{r_2}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

The only way for this relationship to always be consistent for any values of v_1 and v_2 is that masses have the following LLT:

$$m'_1 = \left(1 - \frac{v_1^2}{c^2}\right)^{\frac{3}{2}} \cdot m_1 \quad \text{and} \quad m''_2 = \left(1 - \frac{v_2^2}{c^2}\right)^{\frac{3}{2}} \cdot m_2 \quad (20)$$

In this way Lorentz factors cancel out and this would imply: $m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2$, But, as this equality was previously correctly concluded in 1), then our assumption is also correct. This can be seen in another way. For maintaining Hercules in equilibrium (first conclusion), then tensions T_1 and T_2 must be equal. Thus, these results lead to both statements imply each other, i.e.,

$$T_1 = m'_1 \cdot \omega'^2 \cdot r'_1 \equiv m_1 \cdot \omega^2 \cdot r_1 \equiv m_2 \cdot \omega^2 \cdot r_2 \equiv m''_2 \cdot \omega''^2 \cdot r''_2 = T_2.$$

Let's discuss in a deep way this equation. When observer 1 on m_1 (remember that he is fixed with respect to this mass, although the whole is a moving system) measures his mass, he measures m'_1 , which is, for him, the rest mass, $m'_1 = M_{01}$. The same applies for the other observer 2 measuring the mass where he is on: $m''_2 = M_{02}$. So, given that through this special case of circular movement we have obtained the Lorentz factors for such masses in (20), and because the LLT of a magnitude always has the same structure, we can conclude, from equation (20), with the following strong statement: **In general, an inertial mass in movement at a velocity v is related to its rest mass in the following manner:**

$$m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (21)$$

This definition differs from the well-known Einstein's mass definition: $m = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}}$. In regards with this point, it's worth mentioning that Einstein also obtained equation (21)

in his remarkable paper of 1905. He called this mass “longitudinal mass” [2], but later he discarded it from his work.

Continuing the analysis by another route to obtain the equation (21), the following is a more general way to arrive at the same result. For instance, let’s consider Earth and Sun as if they were the only bodies of the inertial solar system. We are going to consider as if the sun was the fixed system, and Earth moving around the Sun. Thus, Angular Momentum of Earth under LLT, measured by an observer from the Sun, is $m.r^2.\omega$ and its value must be constant, because there are no more forces acting around, and conservation of angular momentum holds. The “same” Angular Momentum of Earth measured by another observer, on Earth, taking Sun as his reference for measurements, is $m'.r'^2.\omega'$, which must also be constant, because the laws of nature are the same in any system of coordinates. Let’s focus our attention on the explicit transformation of the elements involved within this last expression of angular momentum except for the earth mass, whose transformation is still considered unknown:

$$m'.r'^2.\omega' = m'.\frac{r^2}{\left(1 - \frac{v^2}{c^2}\right)} \cdot \frac{\omega}{\sqrt{1 - \frac{v^2}{c^2}}} = \text{CONSTANT} \quad (22)$$

By carefully observing equation (22), we conclude that the only way for this expression being always constant, for any value of the variable v present in Lorentz factors in the denominator, is that the transformation for $m' = M_0$, cancels out the effect of such factors. For instance:

$$m' = M_0 = \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} \cdot m \quad \Rightarrow \quad m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad (23)$$

This is the same result previously obtained in equation (21). This also means that Angular Momentum is invariant under LLT (and also the force). Given that Local Lorentz factors equally influence any physical magnitude in all dimensions, we don’t have different LLT’s for the same magnitude, contrasting to which is found in the Special Theory of Relativity (remember longitudinal or transversal expressions of mass, fields, etc).

Given that LLT of mass is already known, let’s obtain other dynamical LLT.

Linear Momentum:

$$\mathbf{p}' = m'.\mathbf{v}' = m.\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} \cdot \frac{\mathbf{v}}{\left(1 - \frac{v^2}{c^2}\right)} \quad \Rightarrow \quad \mathbf{p}' = \sqrt{1 - \frac{v^2}{c^2}} \cdot \mathbf{p} \quad (24)$$

Observe this result: Linear Momentum is not invariant, as SRT states.

Angular Momentum:

$$\mathbf{L}' = \mathbf{r}' \times \mathbf{p}' = \frac{\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \times \sqrt{1 - \frac{v^2}{c^2}} \cdot \mathbf{p} \quad \Rightarrow \quad \mathbf{L}' = \mathbf{L}(\text{Invariant}) \quad (25)$$

Force:

$$\mathbf{F}' = \frac{d\mathbf{p}'}{dt'} = \frac{\sqrt{1 - \frac{v^2}{c^2}} \cdot d\mathbf{p}}{\sqrt{1 - \frac{v^2}{c^2}} \cdot dt} \Rightarrow \mathbf{F}' = \mathbf{F} \quad (\text{Invariant as expected}) \quad (26)$$

Kinetic Energy:

$$dE' = \mathbf{F}' \cdot d\mathbf{r}' = \mathbf{F} \cdot \frac{d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dE' = \frac{dE}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (27)$$

Electromagnetic or Lorentz Force: (Must be invariant, because the magnitude of any force is invariant, see equation (26) and development of equation (20))

$$\mathbf{F}' = q' \cdot (\mathbf{\Xi}' + \mathbf{v}' \times \mathbf{B}')$$

Let's discuss this relationship. Electric Charge q seems not to be influenced by the velocity. Let's assume that it is invariant under LLT. Thus, in order to preserve the invariance of Force, Electric Field $\mathbf{\Xi}$ and the product $\mathbf{v} \times \mathbf{B}$ must be invariant. ***If this assumption is false, for sure a contradiction will arise later on.*** A good property of scaling factors in LLT is that they behave as if they had magnitude, allowing dimensional analysis based on characteristic Lorentz scaling factors. From the assumed LLT invariance of q , then $\mathbf{v}' \times \mathbf{B}'$ is also invariant:

Magnetic Field Density: Applying equation (16),

$$\mathbf{B}' = \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (28)$$

Electric Field:

$$\mathbf{\Xi}' = \mathbf{\Xi} \quad (\text{Invariant}) \quad (29)$$

Electric Charge:

$$q' = q \quad (\text{Invariant}) \quad (30)$$

Electric Potential:

$$dV' = \mathbf{\Xi}' \cdot d\mathbf{r}' = \mathbf{\Xi} \cdot \frac{d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dV' = \frac{dV}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (31)$$

Let's check the assumption for Electric Charge and its effects. Let's obtain the expression for the Electric Energy. It should lead to the already obtained expression (27) for energy. In fact:

$$dE' = q' \cdot dV' = q \cdot \frac{dV}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dE' = \frac{dE}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [\text{See equation (27)}]$$

Another check: An electric charge contained in a mass m located in an uniform magnetic field, which moves describing a circular path, should lead to the LLT of angular velocity, which is already known in equation (19),

$$\omega' = -q' \cdot \frac{\mathbf{B}'}{m'} = -q \cdot \frac{\mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{m \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{-q \cdot \frac{\mathbf{B}}{m}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \omega' = \frac{\omega}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ [see equation (19)]}$$

As it is seen, our assumption for charge is consistent. Furthermore, this control ratifies mass transformation. By continuing checking:

The Magnetic Field Density B on a point at a distance R from a current $I = \frac{dq}{dt}$, given by $B = \frac{\mu \cdot I}{2\pi \cdot R} \Rightarrow \mu = \frac{2\pi \cdot R \cdot B}{I}$, leads to obtain the transformation of the Magnetic Permeability:

$$\mu' = \frac{2\pi \cdot R' \cdot B'}{I'} \Rightarrow \mu' = \mu \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (32)$$

Similarly, Electric Permittivity, ε , can be obtained from Gauss Law:

$$\oint_{S'} \boldsymbol{\Xi}' \cdot d\mathbf{S}' = \frac{q'}{\varepsilon'} \Rightarrow \oint_S \boldsymbol{\Xi} \cdot \frac{d\mathbf{S}}{\left(1 - \frac{v^2}{c^2}\right)} = \frac{q}{\varepsilon'} \Rightarrow \varepsilon' = \varepsilon \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (33)$$

Electric Displacement:

$$\mathbf{D}' = \varepsilon' \cdot \boldsymbol{\Xi}' = \varepsilon \cdot \left(1 - \frac{v^2}{c^2}\right) \cdot \boldsymbol{\Xi} \Rightarrow \mathbf{D}' = \mathbf{D} \cdot \left(1 - \frac{v^2}{c^2}\right) \quad (34)$$

Current Density:

$$\mathbf{J}' = \frac{\frac{dq'}{dt'}}{S'} = \frac{\frac{dq}{dt} \cdot \sqrt{1 - \frac{v^2}{c^2}}}{\frac{S}{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{J}' = \mathbf{J} \cdot \sqrt{1 - \frac{v^2}{c^2}} \quad (35)$$

Magnetic Field:

$$\mathbf{H}' = \frac{\mathbf{B}'}{\mu'} = \frac{\mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{\mu \cdot \left(1 - \frac{v^2}{c^2}\right)} \Rightarrow \mathbf{H}' = \mathbf{H} \text{ (Invariant)} \quad (36)$$

Magnetic Flux:

$$\phi' = \oint_{S'} \mathbf{B}' \cdot d\mathbf{S}' = \oint_{S'} \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right) \cdot \frac{d\mathbf{S}}{\left(1 - \frac{v^2}{c^2}\right)} \Rightarrow \phi' = \phi \text{ (Invariant)} \quad (37)$$

Checking:

$$\frac{\partial \boldsymbol{\Xi}'}{\partial r'} = -\frac{\partial \mathbf{B}'}{\partial t'} \Rightarrow \frac{\partial \boldsymbol{\Xi}}{\frac{\partial r}{\sqrt{1 - \frac{v^2}{c^2}}}} = -\frac{\partial \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{\partial t \cdot \sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial \boldsymbol{\Xi}}{\partial r} = -\frac{\partial \mathbf{B}}{\partial t} \text{ [Checked]}$$

$$-\frac{\partial \mathbf{B}'}{\partial r'} = -\mu' \cdot \varepsilon \cdot \frac{\partial \boldsymbol{\Xi}'}{\partial t'} \Rightarrow \frac{\partial \mathbf{B} \cdot \left(1 - \frac{v^2}{c^2}\right)}{\frac{\partial r}{\sqrt{1 - \frac{v^2}{c^2}}}} = -\mu \cdot \varepsilon \cdot \left(1 - \frac{v^2}{c^2}\right)^2 \frac{\partial \boldsymbol{\Xi}}{\partial t \cdot \sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow -\frac{\partial \mathbf{B}}{\partial r} = -\mu \cdot \varepsilon \cdot \frac{\partial \boldsymbol{\Xi}}{\partial t}$$

These results show that the relation between electric and magnetic fields holds in any reference system, as it was expected.

It can be shown that Maxwell Equations hold in any reference system under LLT. In fact, by taking into account that:

$$\nabla' = \frac{\partial}{\partial r'} = \frac{\partial}{\frac{\partial r}{\sqrt{1-\frac{v^2}{c^2}}}} \Rightarrow \nabla' = \sqrt{1-\frac{v^2}{c^2}} \cdot \nabla \quad (38)$$

$$1) \nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B} \cdot \frac{1-\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}{\partial t \cdot \sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$2) \nabla' \times \mathbf{H}' = \frac{\partial \mathbf{D}'}{\partial t'} + \mathbf{J}' \Rightarrow \sqrt{1-\frac{v^2}{c^2}} \nabla \times \mathbf{H} = \frac{\partial \mathbf{D} \cdot \frac{1-\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}{\partial t \cdot \sqrt{1-\frac{v^2}{c^2}}} + \mathbf{J} \cdot \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$3) \nabla' \cdot \mathbf{D}' = \rho' = \frac{q'}{V'} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} \nabla \cdot \mathbf{D} \cdot \left(1-\frac{v^2}{c^2}\right) = \frac{q}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow \nabla \cdot \mathbf{D} = \rho$$

$$4) \nabla' \cdot \mathbf{B}' = 0 \Rightarrow \nabla \cdot \mathbf{B} \cdot \left(1-\frac{v^2}{c^2}\right) = 0 \Rightarrow \nabla \cdot \mathbf{B} = 0$$

Pointing Theorem. For

$$P' = \frac{dE'}{dt'} = \frac{\frac{dE}{\sqrt{1-\frac{v^2}{c^2}}}}{dt \cdot \sqrt{1-\frac{v^2}{c^2}}} = \frac{P}{\left(1-\frac{v^2}{c^2}\right)} \quad (39)$$

$$P' = Re \frac{1}{2} \oint \mathbf{E}' \times \mathbf{H}' \cdot d\mathbf{S}' \Rightarrow \frac{P}{\left(1-\frac{v^2}{c^2}\right)} = Re \frac{1}{2} \oint \mathbf{E} \times \mathbf{H} \cdot \frac{d\mathbf{S}}{\left(1-\frac{v^2}{c^2}\right)} \Rightarrow P = Re \frac{1}{2} \oint \mathbf{E} \times \mathbf{H} \cdot d\mathbf{S}$$

Observe the dimensional analysis' consistency of LLTs for each magnitude. All these consistent controls seem to confirm the correctness of the LLT approach.

It is necessary to remark that Lorentz factors are only scaling factors between measurements, no matter whether they are differentials or integrals. For instance if:

$$\mathbf{p}' = \sqrt{1-\frac{v^2}{c^2}} \cdot \mathbf{p} \quad \text{Then } d\mathbf{p}' = \sqrt{1-\frac{v^2}{c^2}} \cdot d\mathbf{p} \quad \text{or } \iiint d^3\mathbf{p}' = \iiint \sqrt{1-\frac{v^2}{c^2}} \cdot d^3\mathbf{p}$$

However, a different thing is the contraction suffered by a bar going through the space with velocity v , from a known length L_0 , to L , measured by one observer inside his own system (second observer does not exist), according to the law $L = L_0 \cdot \sqrt{1-\frac{v^2}{c^2}}$. This case is not a scaling one in the sense of the LLT, but a property of the bar whose **known length** depends on its velocity through the space for a stationary observer. For velocity v , variable, L is also variable. For this case the differential dL becomes, $dL = L_0 \cdot \frac{v \cdot dv}{1-\frac{v^2}{c^2}}$.

The same consideration must be made for a mass m that crosses the space with velocity v , with known rest mass m_0 . The expressions for the variable m , and its differential, depending on its velocity v , are:

$$m = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow dm = 3 \cdot M_0 \cdot c^3 \cdot \frac{v \cdot dv}{(c^2-v^2)^{\frac{5}{2}}} = 3 \cdot \frac{M_0 \cdot c^3}{(c^2-v^2)^{\frac{3}{2}}} \cdot \frac{v \cdot dv}{(c^2-v^2)} = 3 \cdot m \cdot \frac{v \cdot dv}{(c^2-v^2)}$$

As it should be noticed, it is important to be careful with such differences.

Another aspect to be emphasized is that of the vector character of time. This vectorial character is only noticed within the relationship between times measured by two inertial observers, through coordinate transformations, when a generalized configuration is used. Only under such condition, time is mathematically forced to appear as a vector. On the contrary, time measured by one observer in his own coordinate system (second observer does not exist) can behave as we are used to know it: as a scalar, although as it is encountered in the work done by Hongbao Ma, it is possible to express time in a vectorial form [4].

5. Conclusions

If Einstein's postulates are correct (in author's opinion they are), contradictions informed in this work for Lorentz Transformations (LT) will lead to a new field of research in theoretical physics that will bring new vigor to this science. Additionally, if the Local Lorentz Transformation (LLT), presented in this work, reveals itself to be a correct approach, it will bring to the surface a branch of physics that was neglected before being developed, as a transition between Classic, and Relativistic or Quantum Physics. In author's opinion, Einstein's work was intended to go in this sense, but unsolved contradictions, introduced by LT at the very starting point of his research, probably made Einstein leave in an abrupt manner the Special Theory of Relativity to lead his investigation into a more general development: the General Theory of Relativity. In the present study, we deliberately ignored Minkowski geometry and four-dimension space-time, because VLT allows doing space-time-varying analysis in three spatial dimensions, for any movement, rectilinear or curvilinear, respecting the constancy of light speed and maintaining the consistency with Maxwell Equations. Experimental validation of this approach, for example, the accuracy of the new definition of mass rigorously obtained in equation (23), will probably require complex experiments with known rest masses accelerated at speeds close to that of light in order to establish whether the value of mass is the well-known coined by Einstein or that of the equation (23).

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Annex 1

Time As A Vector. Examples

A) In one-dimensional space, one way to interpret this could be (See Fig. A1):

$$x = c.t \quad x' = k.(x - v.t)$$

$$t' = k.(t - \frac{v}{c^2}x) \quad \Rightarrow \quad \begin{aligned} \mathbf{t}' &= k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' &= k.(\mathbf{r} - v.\mathbf{t}) \end{aligned}$$

B) For a two-dimensional space, derivation is less direct, but simple. The following equations hold (See Fig. A3):

$$\begin{aligned} x^2 + y^2 &= c^2.t^2 & x' &= k.(x - v.t.\cos \alpha) \\ x'^2 + y'^2 &= c^2.t'^2 & y' &= k.(y - v.t.\sin \alpha) \end{aligned}$$

Where, α , is the angle between trajectory of O' and X axis; By defining time components as $t_x = t.\cos \alpha$, $t_y = t.\sin \alpha$, we obtain:

$$\begin{aligned} x' &= k.(x - v.t_x) \\ y' &= k.(t - v.t_y) \end{aligned} \quad \text{Substituting and grouping:}$$

$$\begin{aligned} c^2.t'^2 &= x'^2 + y'^2 = k^2.[(x - v.t_x)^2 + (y - v.t_y)^2] \\ c^2.t'^2 &= k^2.[x^2 + y^2 + v^2.t_x^2 + v^2.t_y^2 - 2.v.x.t_x - 2.v.y.t_y] \\ c^2.t'^2 &= k^2.[c^2.t^2 + v^2.t^2 - 2.v.(x.t_x + y.t_y)] \\ c^2.t'^2 &= k^2.[c^2.(t_x^2 + t_y^2) + v^2.\frac{x^2+y^2}{c^2} - 2.v.(x.t_x + y.t_y)] \\ c^2.t'^2 &= k^2.[(c.t_x - \frac{v}{c}.x)^2 + (c.t_y - \frac{v}{c}.y)^2] \end{aligned}$$

Dividing by c^2 , a vectorial structure of time is clearly obtained and follows:

$$t'^2 = k^2.[(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2] \quad \Rightarrow \quad \begin{aligned} \mathbf{t}' &= k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' &= k.(\mathbf{r} - v.\mathbf{t}) \end{aligned}$$

C) For the three-dimensional case, see Fig. A5, the following relationships hold:

$$\begin{aligned} x^2 + y^2 + z^2 &= c^2.t^2 & x' &= k.(x - v.t.\cos \alpha.\cos \beta) & t_x &= t.\cos \alpha.\cos \beta \\ x'^2 + y'^2 + z'^2 &= c^2.t'^2 & y' &= k.(y - v.t.\sin \alpha.\cos \beta) & t_y &= t.\sin \alpha.\cos \beta \\ & & z' &= k.(z - v.t.\sin \beta) & t_z &= t.\sin \beta \end{aligned}$$

Following a similar procedure to that previously used is obtained again the familiar vector structure expression of time for three (or for any number of) dimensions:

$$t'^2 = k^2.[(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2 + (t_z - \frac{v}{c^2}.z)^2] \quad \Rightarrow \quad \begin{aligned} \mathbf{t}' &= k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' &= k.(\mathbf{r} - v.\mathbf{t}) \end{aligned}$$

All these results lead consistently to consider the behavior of time as a vector when it is referred to observers located in systems with different inertial movements.

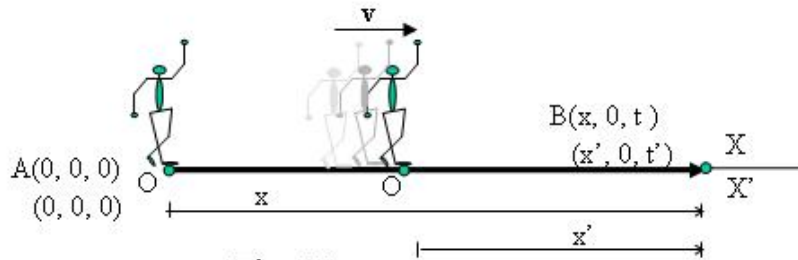


Fig. A1a

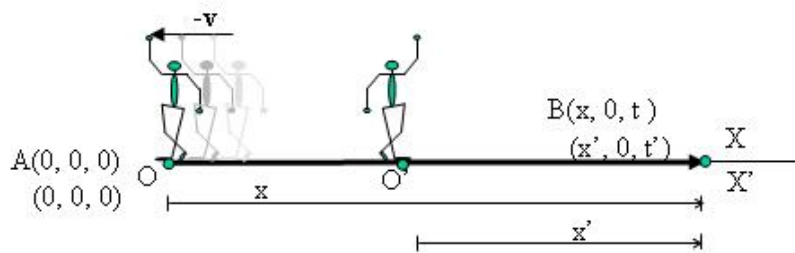


Fig. A1b

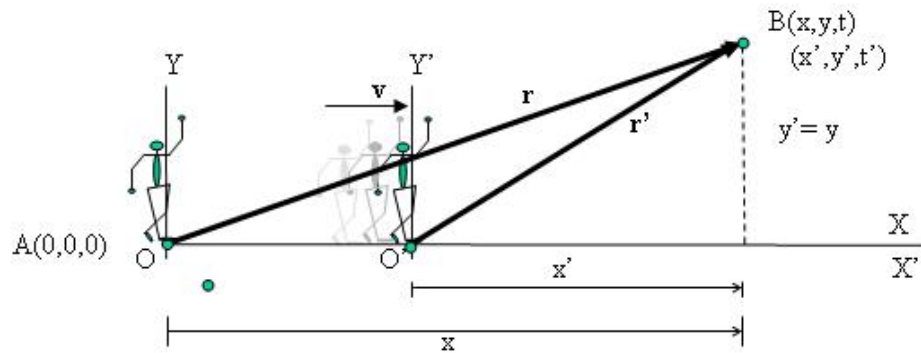


Fig. A2

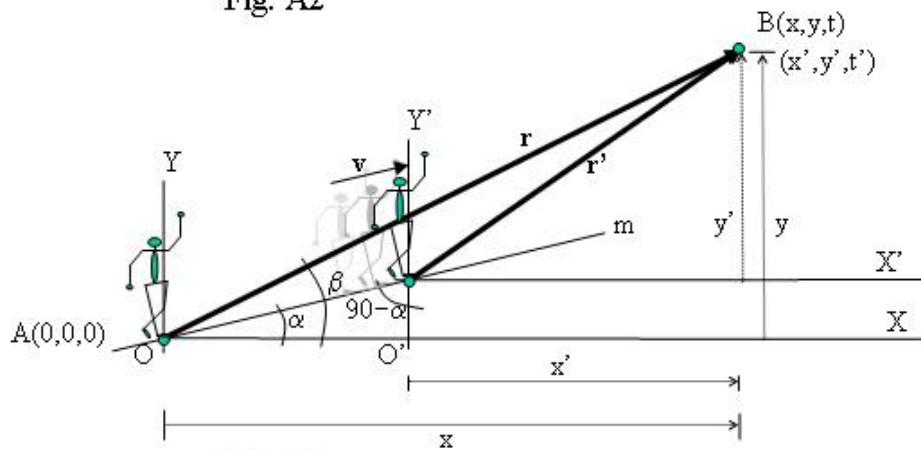


Fig. A3

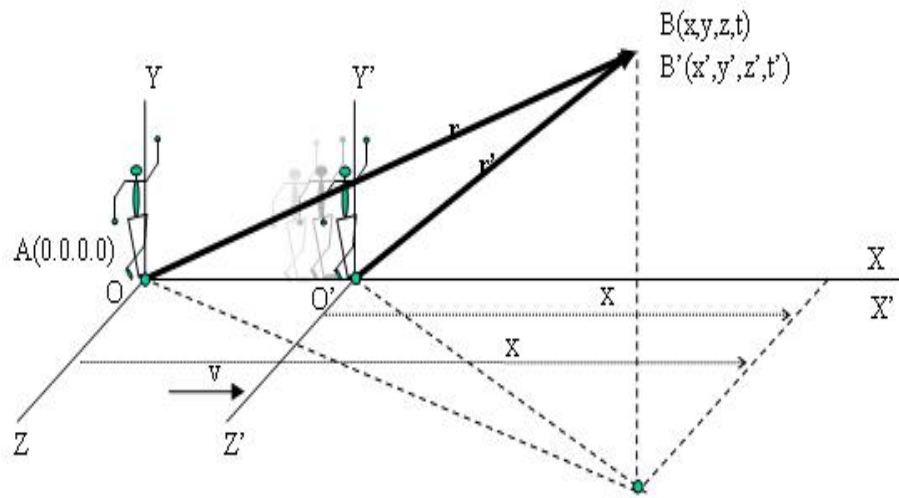


Fig. N° A4

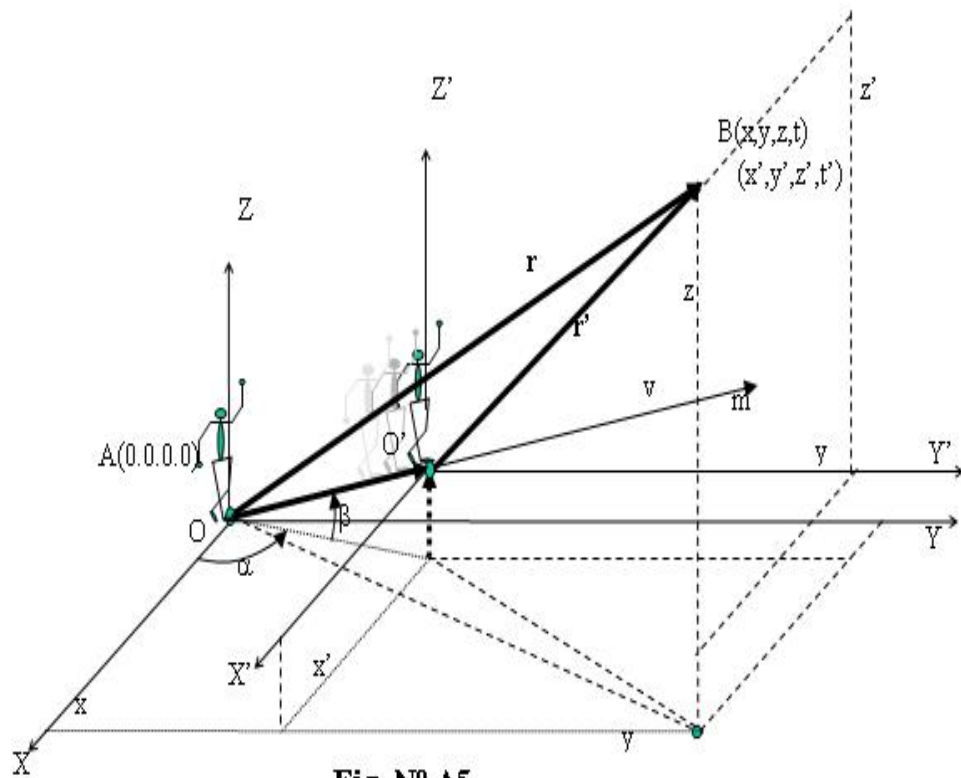


Fig. N° A5

Annex 2

Consistency Of Vectorial Lorentz Transformations And Inconsistency Of Lorentz Transformations

Given that we are working with vectors it is suitable to obtain a Wave Equation presentation in function of the light pulse radio-vector and time vector.

$$\mathbf{r} = x.\mathbf{i} + y.\mathbf{j} + z.\mathbf{k} \quad \Rightarrow \quad r = \sqrt{x^2 + y^2 + z^2} \quad \Rightarrow \quad \frac{\partial r}{\partial x} = \frac{x}{r}; \quad \frac{\partial r}{\partial y} = \frac{y}{r}; \quad \frac{\partial r}{\partial z} = \frac{z}{r};$$

In this way, each one of the components of the operator ∇ can be represented as depending on both vectors. Let's work to arrive at equations depending only on r and t . For this, we will develop the expressions of the components x , y , z :

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial}{\partial t} \frac{\partial t}{\partial x}; \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial}{\partial t} \frac{\partial t}{\partial y}; \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial r} \frac{\partial r}{\partial z} + \frac{\partial}{\partial t} \frac{\partial t}{\partial z}; \quad \text{Where, } \frac{\partial t}{\partial x} = \frac{\partial t}{\partial y} = \frac{\partial t}{\partial z} = 0$$

So, the operator $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$ can be expressed as:

$$\nabla = \frac{\partial}{\partial r} \frac{\partial r}{\partial x}\mathbf{i} + \frac{\partial}{\partial r} \frac{\partial r}{\partial y}\mathbf{j} + \frac{\partial}{\partial r} \frac{\partial r}{\partial z}\mathbf{k} = \frac{\partial}{\partial r} \frac{x}{r}\mathbf{i} + \frac{\partial}{\partial r} \frac{y}{r}\mathbf{j} + \frac{\partial}{\partial r} \frac{z}{r}\mathbf{k} = \frac{\partial}{\partial r} \frac{\mathbf{r}}{r} \quad \Rightarrow \quad \nabla \bullet \nabla = \nabla^2 = \frac{\partial^2}{\partial r^2}$$

Thus, Wave Equation can be put in a simpler manner, only as function of r and t :

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \cdot \frac{\partial^2 \varepsilon}{\partial t^2} = 0; \quad \text{For: } \left\{ \begin{array}{l} \mathbf{r}' = \frac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial r'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \frac{\partial r'}{\partial r} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial r} = \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \end{array} \right.$$

By applying Chain rule for partial derivation, respect to variables r, t :

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'} \frac{\partial r'}{\partial r} + \frac{\partial \varepsilon}{\partial t'} \frac{\partial t'}{\partial r} \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'} \frac{\partial r'}{\partial t} + \frac{\partial \varepsilon}{\partial t'} \frac{\partial t'}{\partial t}$$

Substituting values previously obtained:

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'} \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}};$$

By differentiating again, in order to form all required quadratics components of Wave Equation:

$$\frac{\partial^2 \varepsilon}{\partial r^2} = \frac{1}{(1 - \frac{v^2}{c^2})} \left(\frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right)$$

$$\frac{\partial^2 \varepsilon}{\partial t^2} = \frac{1}{(1 - \frac{v^2}{c^2})} \left(v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2.v. \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right)$$

And substituting these obtained expressions in Wave Equation, we finally have:

$$\begin{aligned} & \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \cdot \frac{\partial^2 \varepsilon}{\partial t^2} = \\ & = \frac{1}{1 - \frac{v^2}{c^2}} \left(\frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right) - \frac{1}{c^2} \cdot \frac{1}{1 - \frac{v^2}{c^2}} \left(v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2.v. \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right) \\ & \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t'^2} \end{aligned}$$

In this way, it is shown that TVL are consistent with Wave Equation (and with Maxwell Equations). So, Wave Equation under TVL has the same presentation for one and another observer, independent of the path of the moving observer and also independent of the light pulse direction, meeting in such a way Einstein relativistic postulates and being consistent with Maxwell Equations.

The problem that LT have, according to our development, is precisely the following assumptions: $y' = y$ and $z' = z$ that originates $\frac{\partial^2 \varepsilon_x}{\partial y'^2} = \frac{\partial^2 \varepsilon_x}{\partial y^2}$ and $\frac{\partial^2 \varepsilon_x}{\partial z'^2} = \frac{\partial^2 \varepsilon_x}{\partial z^2}$. They don't allow a vectorial treatment through variables r , t . For instance in two dimensions, according to Lorentz, the displacement of light measured by fixed observer at O, is $\mathbf{r} = x \cdot \mathbf{i} + y \cdot \mathbf{j}$, and the corresponding measurement done by moving one at O' is:

$$\mathbf{r}' = x' \cdot \mathbf{i} + y' \cdot \mathbf{j} = \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}} \mathbf{i} + y \cdot \mathbf{j} = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}} \mathbf{i} + y \cdot \mathbf{j} - \frac{-v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}} \mathbf{i}.$$

By observing carefully this last equation, we conclude that it is not possible to obtain an explicit expression of \mathbf{r}' as function of \mathbf{r} and \mathbf{t} . Thus, it can not be possible to obtain an expression for $\frac{\partial r'}{\partial r}$, neither for $\frac{\partial t'}{\partial t}$. In these circumstances we can not continue with the procedure of constructing the vectorial version of the original LT. It can be shown that LT really are not invariant to the Wave equation. Although in some books appears a “demonstration” of the consistency of the LT with Wave Equation, this is not quite general, this is actually a demonstration that is valid only for the particular case of one dimension: the X axis, in where the assumptions cancel out. The chosen example for such demonstration is always presented without any variation: An observer at the origin of the moving system O', which moves on the X axis and a light pulse is sent to the “space” with the usual assumptions. For instance, if this presentation is changed, by establishing that the pulse of light is going parallel to the Z axis, maintaining the moving observer on the X axis, the “demonstration” fails. For showing this, we will work out a known example taken from basic electromagnetic theory:

- (1) Let an electromagnetic plane wave move on Z axis at light speed, $z = ct$, such that the electric field on Y axis, $\varepsilon_y = \varepsilon_o \cdot \sin k \cdot (z - ct)$, depends only on the Z coordinate and time. So, field characteristics will be: $\varepsilon_x = 0$; $\varepsilon_y = \varepsilon_y(z, t)$; $\varepsilon_z = 0$; $x = y = 0$. Suppose that the system O' is moving along the X axis at a velocity v and let's assume, $z' = z$, in order to be under the same premises of LT.

The relationships that hold for this case, according to LT, are:

$$x' = \frac{-v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad x = 0; \quad y' = y = 0; \quad z' = z; \quad t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial x'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}};$$

$$\frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial x} = 0$$

With these premises, we can write: $\frac{\partial \varepsilon_y}{\partial x} = \frac{\partial \varepsilon_y}{\partial y} = 0$, and similarly, $\frac{\partial y'}{\partial t} = 0$; $\frac{\partial t'}{\partial z} = 0$; Given that time is not an explicit variable in the expression of z' , then $\frac{\partial z'}{\partial t} = 0$; and because

$z' = z$, then: $\frac{\partial z'}{\partial z} = 1$. In this way, all the equations corresponding to Wave Equation in function of the coordinate components are reduced to:

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \varepsilon_y}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0 \quad \Rightarrow \quad \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$$

Let's try to build the Wave Equation under prime variables. By using the Chain rule we will form components with the prime variables:

$$\begin{aligned} \frac{\partial \varepsilon_y}{\partial z} &= \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial z} + \frac{\partial \varepsilon_y}{\partial y'} \frac{\partial y'}{\partial z} + \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial z} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial z} = \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial z} = \frac{\partial \varepsilon_y}{\partial z'} \quad \Rightarrow \quad \frac{\partial^2 \varepsilon_y}{\partial z^2} = \frac{\partial^2 \varepsilon_y}{\partial z'^2} \\ \frac{\partial \varepsilon_y}{\partial t} &= \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial \varepsilon_y}{\partial y'} \frac{\partial y'}{\partial t} + \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial t} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial t} = \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial t} \\ \frac{\partial^2 \varepsilon_y}{\partial t^2} &= \frac{\partial^2 \varepsilon_y}{\partial x'^2} \frac{\partial x'^2}{\partial t^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2 \cdot \left[\frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \frac{\partial x'}{\partial t} \frac{\partial t'}{\partial t} \right] \end{aligned}$$

Substituting by their values we obtain:

$\frac{\partial^2 \varepsilon_y}{\partial t^2} = \frac{1}{1 - \frac{v^2}{c^2}} \left(v^2 \frac{\partial^2 \varepsilon_y}{\partial x'^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} - 2 \cdot v \cdot \frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \right)$ Introducing these results, it is obtained a different presentation of the Wave Equation for the prime variables: contrary to what is expected:

$$\frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = \frac{\partial^2 \varepsilon_y}{\partial z'^2} - \frac{1}{c^2} \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left(v^2 \frac{\partial^2 \varepsilon_y}{\partial x'^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} - 2 \cdot v \cdot \frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \right)$$

This result shows how the original LT **are not really consistent** with the Maxwell Equations, because it does not preserve the structure of Wave Equation.

(1) Let's do the same job but through the VLT. Expressing the movement of O' and that of the light pulse in a vectorial form, and remembering that components of vector time measured by O are given by the movement of O', we get:

$\alpha = \beta = x = y = t_y = t_z = 0; \Rightarrow \mathbf{t} = t \cdot \mathbf{i}; \mathbf{r} = z \cdot \mathbf{k} \Rightarrow z = r$; By applying:

$$\begin{cases} \mathbf{t}' = k \cdot \left(\mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r} \right) \\ \mathbf{r}' = k \cdot \left(\mathbf{r} - v \cdot \mathbf{t} \right) \end{cases}$$

$$\mathbf{t}' = \frac{t \cdot \mathbf{i} - \frac{v}{c^2} \cdot r \cdot \mathbf{k}}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \mathbf{r}' = \frac{r \cdot \mathbf{k} - v \cdot t \cdot \mathbf{i}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Rightarrow \quad \begin{cases} \frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; & \frac{\partial t'}{\partial r} = \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \frac{\partial r'}{\partial r} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; & \frac{\partial r'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} \end{cases}$$

Wave Equation, in function of r, t had become: $\frac{\partial^2 \varepsilon_y}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$. Operating as before, and substituting values, primed Wave Equation is consistently obtained:

$$\begin{aligned} \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} &= \left[\frac{\partial^2 \varepsilon}{\partial r'^2} \frac{\partial r'^2}{\partial r^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2 \cdot \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \frac{\partial r'}{\partial r} \frac{\partial t'}{\partial t} \right] \\ - \frac{1}{c^2} \left[\frac{\partial^2 \varepsilon}{\partial r'^2} \frac{\partial r'^2}{\partial t^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2 \cdot \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \frac{\partial r'}{\partial t} \frac{\partial t'}{\partial t} \right] & \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t'^2} \end{aligned}$$

As so it was expected. This also means that VLT are consistent with Wave Equation and in general with Maxwell Equations.

Annex 3

Vectorial Lorentz Transformations Applied To Curvilinear Movement

As we will demonstrate next, it is possible to apply the new given approach to the Lorentz Transformations, the Vectorial Lorentz Transformations (VLT), to an inertial system of coordinates with curvilinear movement, with respect to a fixed system, located in a point throughout the curvilinear trajectory of the moving system.

We can establish that, inertial systems are not only those with null acceleration, but those where “the sum of acting forces is null”. These include not only those with null acceleration in rectilinear movement, but also those in curvilinear movement with a constant Angular Momentum.

For the movement of Earth around the Sun, the summation of the gravitational force of Sun onto Earth plus the Earth’s centrifugal force gives a null result, reason why the Earth movement is inertial according to since we have defined it. In this way, earth’s movement is neither impeded nor eased by any additional external force. We will try to reproduce this movement in Fig. A6, where the first observer is on the moving system, Earth, at O' , and the second observer will be fixed on the elliptic path at the nearest point to the Sun, the perihelion.

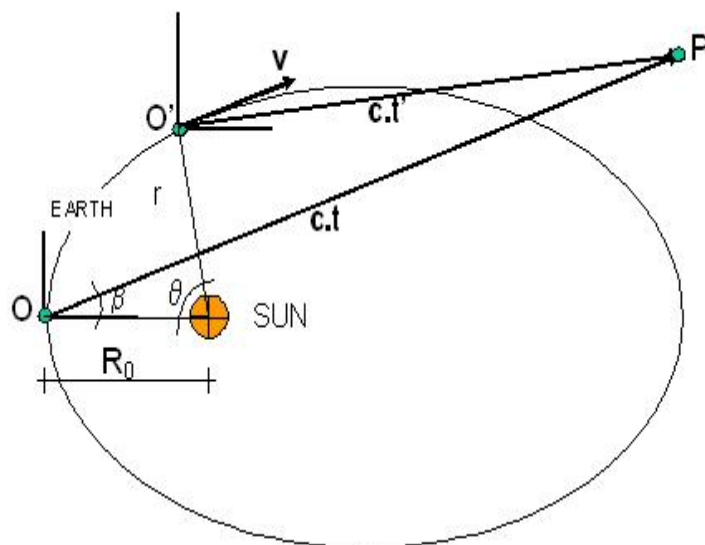


Fig. A6

Let’s denote R_0 , as the distance between Sun And Earth at the moment when observers start measuring the movement, and r , the generic position of Earth.

By taking a closer view of this movement, for two dimensions, see Fig. A7, let’s establish that, when O' and O coincide, a pulse of light is sent forming an angle β with X axis, see Fig. A6, and an angle γ between the tangential velocity v of O' with Y axis as it is shown in Fig. A7. Let’s equally define θ , as the angle swept by radius r from $r = R_0$,

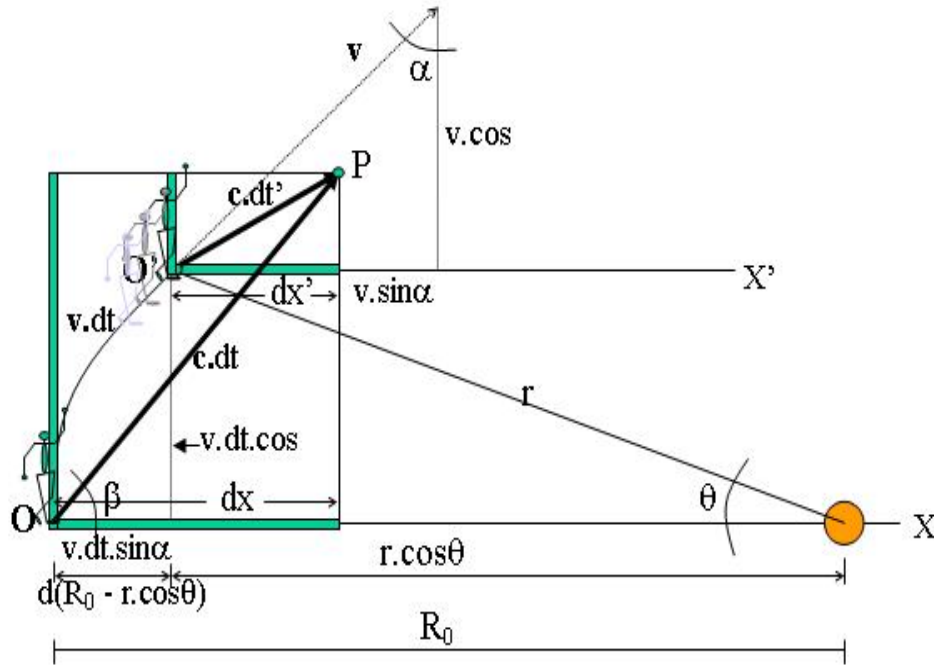


Fig. N° A7

to the new position r of the moving observer after a period of time dt . At this moment light pulse has reached point P . From figures A6 and A7, let's establish the following relationships:

$$dx' = k(dx - v.dt. \sin \gamma) \quad dy' = k(dy - v.dt. \cos \gamma) \quad (\text{A3.1})$$

From the same figure we can establish:

$$v.dt. \sin \gamma = d(R_0 - r. \cos \theta) \quad v.dt. \cos \gamma = d(R_0 - r. \sin \theta) \quad (\text{A3.2})$$

Given that the light speed is the same measured by any observer, it must be met:

$$dx^2 + dy^2 = c^2.dt^2 \quad dx'^2 + dy'^2 = c^2.dt'^2 \quad (\text{A3.3})$$

Substituting dx' , dy' , by their expressions (A3.1) and (A3.2) into (A3.3), similar expressions previously obtained for rectilinear movement are achieved:

$$dx' = \frac{dx - v.dt. \sin \gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \quad dy' = \frac{dy - v.dt. \cos \gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \quad dt' = dt. \frac{\sqrt{(\sin \gamma - \frac{v.u_x}{c^2})^2 + (\cos \gamma - \frac{v.u_y}{c^2})^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$u'_x = \frac{u_x - v. \sin \gamma}{\sqrt{(\sin \gamma - \frac{v.u_x}{c^2})^2 + (\cos \gamma - \frac{v.u_y}{c^2})^2}} \quad u'_y = \frac{u_y - v. \cos \gamma}{\sqrt{(\sin \gamma - \frac{v.u_x}{c^2})^2 + (\cos \gamma - \frac{v.u_y}{c^2})^2}} \quad (\text{A3.4})$$

Namely,

$$d\mathbf{r}' = \frac{d\mathbf{r} - v \cdot d\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad dt' = \frac{dt - \frac{v}{c^2} d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \mathbf{u}' = \frac{d\mathbf{r}'}{dt'} \quad (\text{A3.5})$$

These results show that the structure of differential VLT for curvilinear is the same previously viewed for rectilinear movement. Obviously, all this indicates that application of LLT to curvilinear movement is also valid.

Lattice Dynamics of Hydrogen Interstice in $\text{Co}_{0.92}\text{Fe}_{0.08}$

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Abstract: Lattice dynamics of hydrogen interstice in the binary alloy $\text{Co}_{0.92}\text{Fe}_{0.08}$ has been carried out to calculate the phonon dispersions along the [100], [110], [111] directions. The phonon density of states, variation of specific heat capacity and Debye's temperature with temperature are also calculated. A reasonably good agreement is found between the calculated and other theoretical and experimental results. The mean square displacement (MSD) of atoms surrounding the interstitial hydrogen atom is reported along with the defect modes.

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Keywords: Phonon dispersion, phonon density of states, specific heat capacity, Debye's temperature, mean square displacement, defect modes.

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1. Introduction

Hydrogen interstice and hence hydrogen storage in metals and alloys is a current subject of considerable experimental and theoretical investigations. A relatively small number of theoretical works have focused on diffusion in pure metals at high interstitial concentration [1-2]. The diffusion of interstitial hydrogen in metals is known to depend strongly on the host metal. Host metal properties such as lattice type, electronic structure and elastic moduli are thought to play key roles in determining the diffusion rate of hydrogen [3]. At higher hydrogen contents more complicated permeation behaviour is observed. First of all, understanding the transport of hydrogen through thin metallic films is essential for the development of coatings preventing bulk materials from hydrogen

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uptake [4]. Although some approaches in the form of Green's function technique and molecular dynamics [5-8] have been used to explain the lattice dynamics of alloys, most of the theoretical work has been done in terms of the coherent potential approximations and their modifications [9-11].

Recent research in this field shows that binary alloys are effective storage media of hydrogen [12-13]. Hydrogen in binary alloys such as $\text{Fe}_{0.5}\text{Ti}_{0.5}$ [14-15], $\text{Ni}_{0.5}\text{Fe}_{0.5}$ and $\text{Cr}_{0.7}\text{Fe}_{0.3}$ [16] have been intensively investigated by both the experimental and theoretical methods. Various theoretical models such as Born - von Karman model [17], three body electron gas phenomenological force model [18] and the transition metal model potential (TMMP) of Animalu [19] were derived to compute the phonon dispersions curves for $\text{Co}_{0.92}\text{Fe}_{0.08}$ alloy is also found to be the best media to store hydrogen. This alloy forms a substitutional face centered cubic (fcc) structure, with lattice constant 0.355nm [20]. The phonon frequencies for this alloy for the symmetry directions have been measured using the inelastic neutron scattering technique [21]. Phonon dispersion of $\text{Co}_{0.92}\text{Fe}_{0.08}$ has been calculated theoretically using the TMMP by Animalu [19]. Shyam et al [20] employed the TMMP model to study the dynamical behaviour of $\text{Co}_{0.92}\text{Fe}_{0.08}$ in which force constants are calculated up to tenth nearest neighbours and hence computed the phonon dispersion. The authors [20] also have reported the phonon frequency distribution, specific heat capacity, variation of Debye's temperature with temperature and the mean square displacement of surrounding atoms to the hydrogen interstice.

To understand the nature of hydrogen diffusion, knowledge of defect modes and displacement of the surrounding atoms to the defect is required. With this motivation in mind, this theoretical investigation has been carried out. Since the neutron diffraction experimental results are analysed on the basis of Born-von Karman model considering interactions up to sixth neighbours, it is easy to get the force constant parameters from the literature. Hence the same model has been used to work out the phonon dispersion and the phonon frequency distribution. The Green's function technique and the scattering matrix formalism have been used to calculate the MSD values of atoms surrounding the hydrogen defect and the defect modes. Our results are comparable with the existing experimental and other theoretical results.

2. Method Of Calculation

The Born-von Karman formalism has been employed to formulate the dynamical matrix for $\text{Co}_{0.92}\text{Fe}_{0.08}$ system by considering the interactions up to sixth nearest neighbours. The values of the calculated force constant parameters are given in Table 1. These parameters are procured from the literature [21], which are fitted using neutron diffraction data. The elements of the dynamical matrix are given in Appendix 1.

Diagonalisation of the dynamical matrix yields the phonon frequencies and eigen vectors. The diagonalisation is carried out using a set of 3871 wave vector points obtained by uniformly dividing the Brillouin zone. With the calculated frequency distribution,

specific heat capacity has been calculated using Debye's equation,

$$C_v = 9Nk_B \left(\frac{T}{\theta_D} \right)^3 \int_0^{\theta_D/T} \frac{x^4 e^x dx}{(e^x - 1)^2} \quad (1)$$

where $x = \frac{\hbar\omega}{k_B T}$, k_B is the Boltzmann constant, N is the number of atoms, ω is the frequency of normal mode of vibrations and θ_D is the Debye's temperature.

Hydrogen interstice in $\text{Co}_{0.92}\text{Fe}_{0.08}$ alloy occupies the minimum energy position namely the octahedral position with O_h symmetry as shown in Fig. 1. The presence of the defect can alter the frequency of normal modes of vibration and the atomic displacements due to the change in atomic force constant. Because of the defects, any one or all types of defect modes such as localised vibrational modes, gap modes and resonant modes may arise. The changes in the normal modes of vibration by the presence of defects lead to considerable changes in the thermodynamic and kinetic properties of the system. Hence it is essential to study the defect modes in detail.

The Green's function technique is an effective mathematical tool for investigating the defect modes and the amplitude of vibration of the atoms affected by defects. Following Maradudin et al. [22]. Green's function matrix can be written in terms of eigen values and eigen vectors as

$$G_{\alpha\beta} \begin{pmatrix} l & l' \\ k & k' \end{pmatrix}; \omega^2 = \frac{1}{N\sqrt{m_k m_{k'}}} \sum \frac{\vec{e}_\alpha(k/\vec{q}j) \vec{e}_\beta(k'/\vec{q}j)}{(\omega_{max}^2 - \omega_j^2(\vec{q}))} [exp(i\vec{q} \cdot (\vec{R}(l) - \vec{R}(l')))] \quad (2)$$

where ω_{max} is the maximum frequency among all normal modes of the host crystal. Green's function technique and scattering matrix formalism [22] have been used to evaluate the defect modes and the displacement of neighbours to the interstitial atom. The defect modes are obtained by solving the equation,

$$\Delta(\omega^2) = |I - g(\delta l + a\gamma a^T)| = 0 \quad (3)$$

where I is a unit matrix of order (18 x 18), g is a Green's function matrix of order (18 x 18), δl is the change in dynamical matrix due to the introduction of defect which is also of the order (18 x 18), 'a' is (18 x 3) matrix representing interaction of hydrogen with neighbours, γ is the interstitial Green's function matrix of order (3x3). The force constant parameters representing hydrogen – metal interactions in the 'a' matrix elements are calculated using the potential form discussed by Machlin [23]. The displacement of neighbouring atoms surrounding the interstitial is obtained using the equation,

$$u_1 = \{I + g(\delta l + a\gamma a^T)[I - g(\delta l + a\gamma a^T)]^{-1}\} u_\alpha \quad (4)$$

u_α is given by

$$u_\alpha \begin{pmatrix} l \\ k \end{pmatrix}; \vec{q}j = \left\{ \frac{h}{2Nm_k \omega_{qj}} \right\}^{1/2} e_\alpha(k, \vec{q}j) \exp \left[i\vec{q} \cdot r \begin{pmatrix} l \\ k \end{pmatrix} \right] \quad (5)$$

where ω_{q_j} is the frequency of normal mode of vibration and $e_\alpha(k, \vec{q}_j)$ is the eigen vector. The mean square displacement at a particular temperature T is obtained by the equation,

$$\langle u_1^2 \rangle = \frac{1}{2} \int_0^\infty \frac{u_1^2(k, \omega_{q_j})}{\omega_{q_j}} \coth\left(\frac{\hbar\omega_{q_j}}{2k_B T}\right) d\omega_{q_j} \quad (6)$$

3. Results And Discussion

The values of the force constants of $\text{Co}_{0.92}\text{Fe}_{0.08}$ alloy are fed into the dynamical matrix and then secular equation is first solved to obtain the dispersion relations along the three high symmetry directions [100], [110] and [111]. The phonon dispersion curves, so computed are plotted along with the result of Shyam et al. [20] and are shown in Fig.2. The solid line denotes the calculated result and square, triangle and circle markers denote the experimental result [21]. It has been found that the present results agree reasonably well with the experimental data.

The calculated phonon density of states is shown in Fig. 3 along with the result of Shyam et al. [20]. The agreement is found to be good. Using the calculated phonon density of states, the specific heat variation with temperature for the range 0K to 250K has been calculated and is shown in Fig. 4 along with the computed result of Shyam et al. [20]. It has been found that the present result is in good agreement with the result of Shyam et al. [20]. The phonon density based Debye's temperature variation with temperature has been calculated for the temperature range 0K to 300K and is shown in Fig. 5 along with the result of Shyam et al. [20] and both the results are found to be in good agreement.

When the hydrogen interstitial atom occupies the octahedral positions of the fcc lattice, the metal atoms close to it, relax from their equilibrium positions due to the coupling between the interstice and metal atoms. The Green's function values for this defect space have been evaluated and using these values, the MSD of the defect space atoms are calculated for temperatures 250K, 500K, 750K, and 1000K. The variation of the MSD with and without hydrogen interstice in $\text{Co}_{0.92}\text{Fe}_{0.08}$ with temperatures is shown in Fig. 6. It is clear from that figure that MSD values increase with temperature as expected. The MSD values of host atoms are greatly reduced by the inclusion of hydrogen atom. The reduction is caused by the change in coupling constants between the atoms and the defect space due to the local expansion of the lattice by the presence of interstice. This reduction is also due to the lightest mass of hydrogen atom which when makes resonant mode of vibration, the surrounding atoms tend to be at rest. Hence our model is realistic fetches reasonable results.

Using the Green's function technique and scattering matrix formalism the localised vibrational modes have been worked out for the hydrogen atom as an interstitial defect in the $\text{Co}_{0.92}\text{Fe}_{0.08}$ system. The localized mode falls at 531 cm^{-1} . Due to the lighter hydrogen the vibrational amplitude is very large in comparison with host case. Due to the cubic symmetry the localized mode is triply degenerate. Hence only one mode is

reported.

Conclusion

The present lattice dynamical investigation obviously indicates the reduction of MSD value of the $\text{Co}_{0.92}\text{Fe}_{0.08}$ alloy with hydrogen as interstitial defect. Since MSD is directly related with Debye Waller factor through the equation $W(\mathbf{k}) = (8\pi^2/3)\langle u^2 \rangle$, it can be used to find out the intensity of scattering in the X-ray diffraction studies. This result can be verified by the experimentalists for this alloy of study with hydrogen as interstice.

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Table – 1. Force constant parameters in units of 10^4 dynes cm^{-1}

Parameters	Values
A ₁	1.69
B ₁	1.78
C ₁	-0.09
A ₂	-0.30
B ₂	-0.05
A ₃	0.04
B ₃	0.007
C ₃	0.02
D ₃	0.014
A ₄	0.09
B ₄	0.12
C ₄	-0.03
A ₅	0.009
B ₅	0.022
C ₅	-0.012
D ₅	0.023
A ₆	0.052
B ₆	0.035

Appendix I

The elements of the dynamical matrix are given below:

$$D(1,1) = \frac{1}{m} [4A_1 ((\cos\pi q_x \cos\pi q_z + \cos\pi q_x \cos\pi q_y) - 2) + 4C_1 (\cos\pi q_y \cos\pi q_z - 1) + 2A_2 (\cos 2\pi q_x - 1) + 2B_2 (\cos 2\pi q_y - 1) + 2B_2 \cos 2\pi q_z - 1) + 8C_3 ((\cos\pi q_x \cos\pi q_y \cos 2\pi q_y + \cos\pi q_x \cos\pi q_y \cos\pi q_z) - 2) + 8A_3 (\cos 2\pi q_x \cos\pi q_y \cos\pi q_z - 1) + 4A_4 ((\cos 2\pi q_x \cos 2\pi q_y + \cos 2\pi q_x \cos 2\pi q_z) - 2) + 4C_4 (\cos 2\pi q_y \cos\pi q_z - 1) + 4A_5 (\cos 3\pi q_x \cos\pi q_z + \cos 3\pi q_x \cos\pi q_y) - 2) + 4D_5 ((\cos\pi q_x \cos 3\pi q_y + \cos\pi q_x \cos 3\pi q_y) - 2) + 4C_5 ((\cos\pi q_x \cos 3\pi q_y + \cos\pi q_x \cos 3\pi q_z) - 2) + 8A_6 (\cos 2\pi q_x \cos 2\pi q_y \cos 2\pi q_z - 1)]$$

$$D(2, 2) = \frac{1}{m} [4C_1 (\cos\pi q_x \cos\pi q_z - 1) + 4A_1 ((\cos\pi q_x \cos\pi q_y + \cos\pi q_y \cos\pi q_z) - 2) + 2B_2 ((\cos 2\pi q_x + \cos 2\pi q_z) - 2) + 2A_2 (\cos 2\pi q_y - 1) + 8C_3 ((\cos 2\pi q_x \cos\pi q_y \cos\pi q_z + \cos\pi q_x \cos\pi q_y \cos 2\pi q_z) - 2) + 8A_3 (\cos\pi q_x \cos 2\pi q_y \cos\pi q_z - 1) + 4A_4 (\cos 2\pi q_x \cos 2\pi q_y - 1) + 2A_4 (\cos 2\pi q_z \cos 2\pi q_y - 1) + 4C_4 (\cos 2\pi q_x \cos 2\pi q_z - 1) + 4D_5 ((\cos 3\pi q_x \cos\pi q_z + \cos\pi q_x \cos 3\pi q_z) - 2) + 4C_5 ((\cos 3\pi q_x \cos\pi q_y + \cos\pi q_y \cos 3\pi q_z) - 2) + 4A_5 ((\cos 3\pi q_y \cos\pi q_z + \cos\pi q_x \cos 3\pi q_y) - 2) + 8A_6 (\cos 2\pi q_x \cos 2\pi q_y \cos 2\pi q_z - 1)].$$

$$D(3, 3) = \frac{1}{m} [4A_1 ((\cos\pi q_x \cos\pi q_z + \cos\pi q_y \cos\pi q_z) - 2) + 4C_1 (\cos\pi q_x \cos\pi q_y - 1) + 2B_2 ((\cos 2\pi q_x + \cos 2\pi q_y) - 2) + 2A_2 (\cos 2\pi q_z - 1) + 8A_3 (\cos\pi q_x \cos\pi q_y \cos 2\pi q_z - 1) + 8C_3 ((\cos 2\pi q_x \cos\pi q_y \cos\pi q_z + \cos\pi q_x \cos 2\pi q_y \cos\pi q_z) - 2) + 4C_4 (\cos 2\pi q_y \cos 2\pi q_x - 1) + 4A_4 ((\cos 2\pi q_y \cos 2\pi q_z + \cos 2\pi q_x \cos 2\pi q_z) - 2) + 4C_5 ((\cos 3\pi q_x \cos\pi q_z + \cos 3\pi q_y \cos\pi q_z) - 2) + 4D_5 ((\cos\pi q_x \cos 3\pi q_y + \cos 3\pi q_x \cos\pi q_z) - 2) + 4A_5 ((\cos\pi q_y \cos 3\pi q_z + \cos\pi q_x \cos 3\pi q_z) - 2) + 8A_6 (\cos 2\pi q_x \cos 2\pi q_y \cos 2\pi q_z - 1)]$$

$$D(1, 2) = \frac{1}{m} [-4B_1 (\sin\pi q_x \sin\pi q_y) - 8B_3 (\sin 2\pi q_x \sin\pi q_y \cos\pi q_z + \sin\pi q_x \sin 2\pi q_y \cos\pi q_z) - 8D_3 (\sin\pi q_x \sin\pi q_y \cos 2\pi q_y) - 4B_4 (\sin 2\pi q_x \sin 2\pi q_y) - 4B_5 (\sin 3\pi q_x \cos\pi q_y + \sin\pi q_x \sin 3\pi q_y) - 8B_6 (\sin 2\pi q_x \sin 2\pi q_y \cos 2\pi q_z)]$$

$$D(2, 1) = D(1, 2)$$

$$D(1, 3) = \frac{1}{m} [-4B_1 (\sin\pi q_x \sin\pi q_z) - 8B_3 (\sin 2\pi q_x \sin\pi q_z - \sin\pi q_x \cos\pi q_y \sin 2\pi q_z) - 8D_3 (\sin\pi q_x \cos 2\pi q_y \sin\pi q_y) - 4B_4 (\sin 2\pi q_x \sin 2\pi q_z) - 4B_5 (\sin\pi q_x \sin 3\pi q_z + \sin 3\pi q_y \sin\pi q_z) - 8B_6 (\sin 2\pi q_x \cos 2\pi q_y \sin 2\pi q_z)]$$

$$D(3, 1) = D(1, 3)$$

$$D(2, 3) = \frac{1}{m} [-4B_1 (\sin\pi q_y \sin\pi q_z) - 8D_3 (\cos 2\pi q_x \sin 2\pi q_y \sin\pi q_z) - 8B_3 (\cos\pi q_x \sin 2\pi q_y \sin\pi q_z + \cos\pi q_x \sin\pi q_y \sin 2\pi q_z) - 4B_4 (\sin 2\pi q_y \sin 2\pi q_z) - 4B_5 (\sin 3\pi q_y \sin\pi q_z + \sin\pi q_y \sin 3\pi q_z) - 8B_6 (\cos 2\pi q_x \sin 2\pi q_y \sin 2\pi q_z)].$$

$$D(2, 3) = D(3, 2)$$

Where A_1, B_1 and C_1 ; A_2, B_2 ; A_3, B_3, C_3, D_3 ; A_4, B_4, C_4 ; A_5, B_5, C_5, D_5 and A_6, B_6 respectively are the force constants for the first to sixth neighbour interactions, m is statistical average mass and q_x, q_y, q_z are x, y and z components of wave vector.

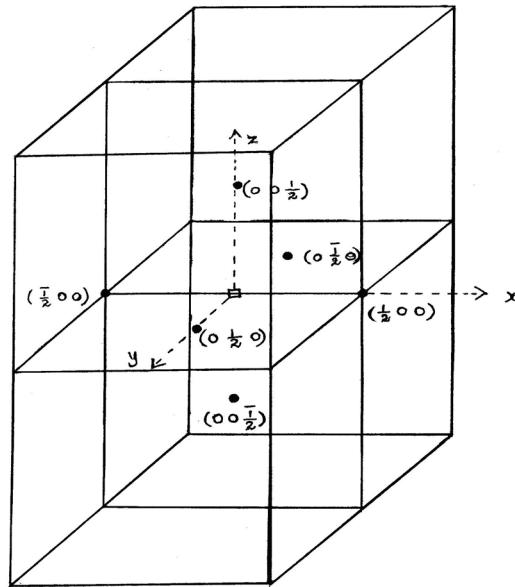
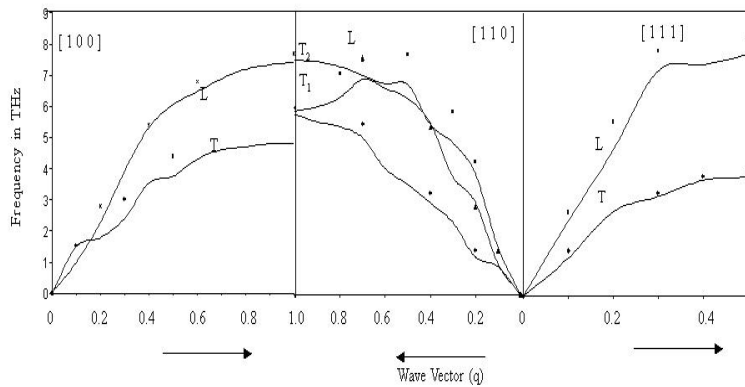


Fig. 1 Defect Space of H in $Co_{0.92}Fe_{0.08}$ (□ H atom, ● Metal atom)



●, ▲ Result of Shapiro, et al. [21] , – Calculated result

Fig. 2 Phonon Dispersion Curve for $Co_{0.2}Fe_{0.08}$ Alloy

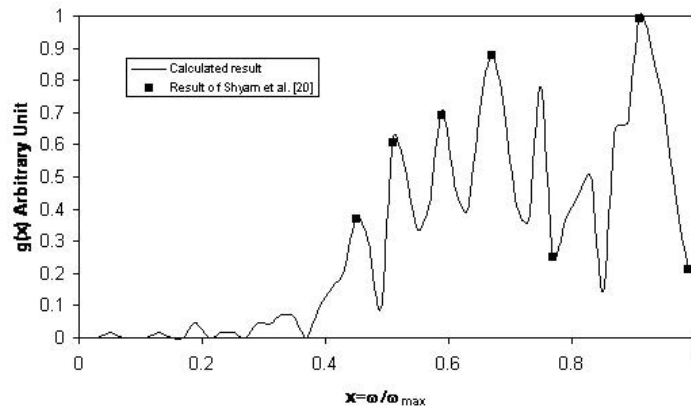


Fig. 3 Frequency distribution for $Co_{0.92}Fe_{0.08}$

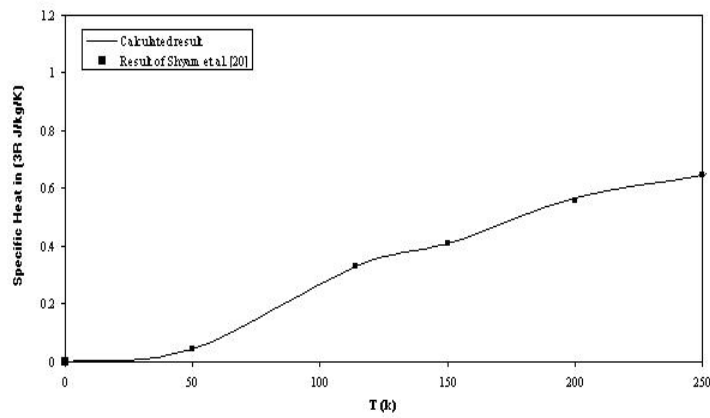


Fig. 4 Variation of the specific Heat of the $Co_{0.92}Fe_{0.08}$ Alloy with Temperature

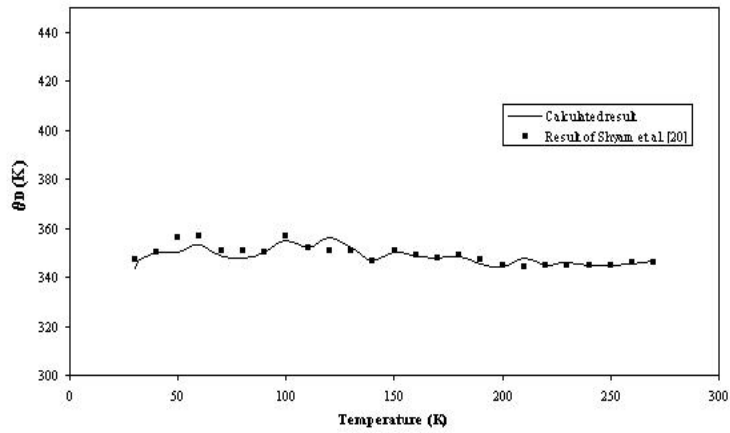


Fig. 5 Temperature dependence of the Debye's Temperature for the $Co_{0.92}Fe_{0.08}$ Alloy

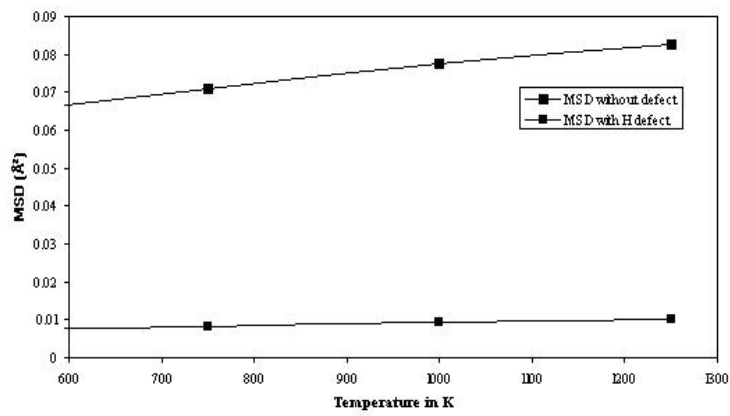


Fig. 6 MSD values for $Co_{0.92}Fe_{0.08}$

Petrov classification of the conformal tensor

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Abstract: We exhibit a flux diagram in its tensorial and Newman-Penrose representations for the Petrov classification.

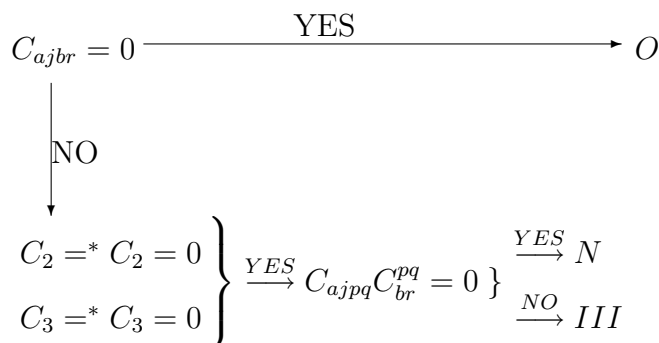
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Keywords: Conformal Tensor, NP formalism

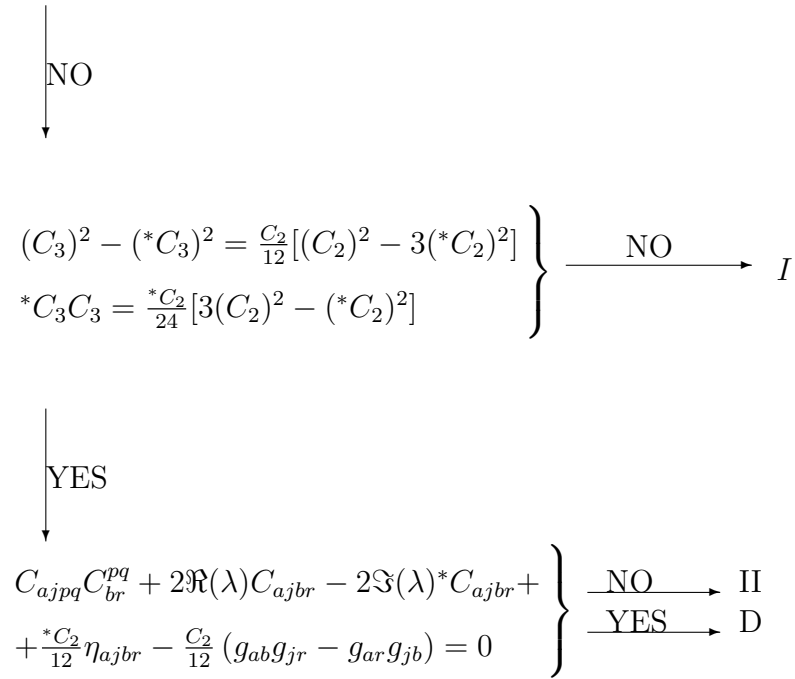
PACS (2003): 04.20.-q, 02.60.Cb, 04.90.+e

1. Introduction

We shall use the quantities and notation stated in [1]. The Petrov classification (PC) [1-8] of the Weyl tensor has a great importance in general relativity, thus it is attractive to account with an efficient procedure to determine the Petrov type of an arbitrary gravitational field. Peres [9] has worked out the tensorial version of the Petrov's matrix approach [2-4] to perform the PC. In the following figure we illustrate a flux diagram that simplifies the application of the Peres algorithm:



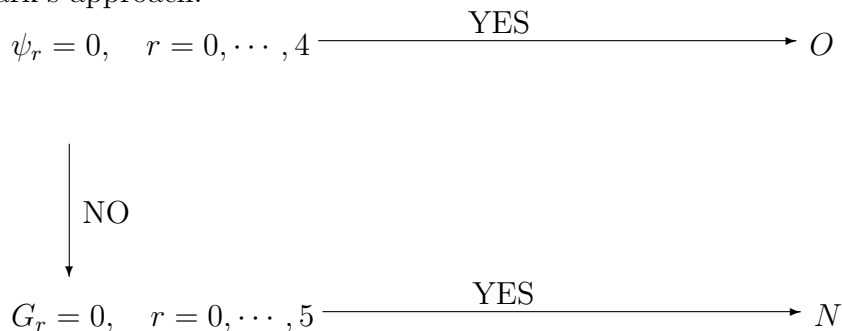
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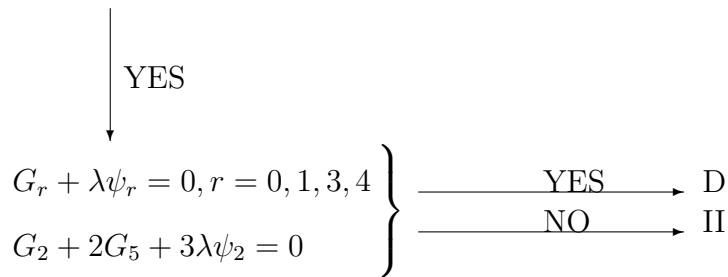
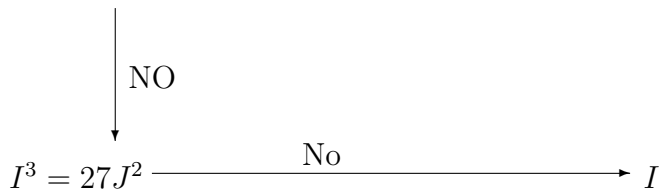
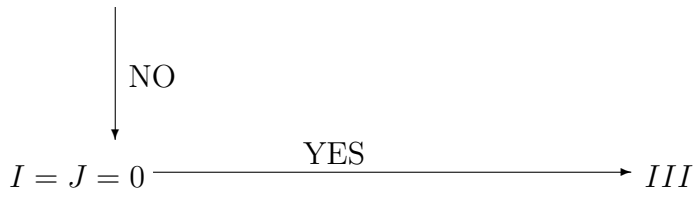


where C_{ajbr} is the Weyl tensor [1,6] associated to the metric g_{ab} , and η_{ajbr} represents the Levi-Civita tensor, besides:

$$\begin{aligned}
 *C_{ajbr} &= \frac{1}{2} \eta_{ajpq} C_{br}^{pq}, & \lambda &= \Re(\lambda) + i\Im(\lambda), \\
 C_2 &= C_{ajpq} C^{ajpq}, & C_3 &= C_{ajbr} C^{brpq} C_{pq}^{aj}, \\
 *C_2 &= *C_{ajbr} C^{ajbr}, & *C_3 &= *C_{ajbr} C^{brpq} C_{pq}^{aj}, \\
 \lambda^2 &= \frac{1}{48} (C_2 + i *C_2), & \lambda^3 &= -\frac{1}{96} (C_3 + i *C_3).
 \end{aligned}$$

D’Inverno-Russell Clark [10] employed the Newman-Penrose (NP) formalism [1,11-13], Lorentz rotations [14]-[15] and Debever-Penrose vectors [1,6,16-26] to deduce a 4th order algebraic equation [27] and the study of its roots implies a procedure for the PC. Here we show a different approach to obtain the Petrov type without the use of Debever-Penrose null principal directions and Lorentz transformations. In fact, the flux diagram for the Peres tensorial method is now expressed by using the NP technique, resulting thus the following algorithm [28] which involves fewer computations than the D’Inverno-Russell Clark’s approach.





where

$$\begin{aligned}
 G_0 &= 2(\psi_0\psi_2 - \psi_1^2), & G_1 &= \psi_0\psi_3 - \psi_1\psi_2, \\
 G_2 &= \psi_2^2 + \psi_0\psi_4 - 2\psi_1\psi_3, & G_3 &= \psi_1\psi_4 - \psi_2\psi_3, \\
 G_4 &= 2(\psi_2\psi_4 - \psi_3^2), & G_5 &= 2(\psi_1\psi_3 - \psi_2^2), \\
 I &= G_2 - G_5, & J &= -\psi_3G_1 + \frac{1}{2}(\psi_2G_5 + \psi_4G_0), \\
 \lambda^2 &= \frac{I}{3}, & \lambda^3 &= -J
 \end{aligned}$$

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On Inflation Potentials in Randall-Sundrum Braneworld Model

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Abstract: We study the inflationary dynamics of the universe in the Randall-Sundrum typeII Braneworld model. We consider both an inverse-power law and exponential potentials and apply the Slow-Roll approximation in high energy limit to derive analytical expression of relevant inflationary quantities. An upper bound for the coupling constant was also obtained and a numerical value of perturbation spectrum is calculated in good agreement with observation.

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Keywords: RS braneworld, inflation potential, perturbation spectrum.

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1. Introduction

Recently a great interest has been devoted to study cosmological inflation [1] in the framework of braneworld scenario; for a review see [2]-[6]. This issue has been motivated by observations on accelerating universe [9] and dark energy [10] as well as results on interpretations of these phenomena in terms of scalar field dynamics [11, 12]. In this regards higher dimensional cosmological models have been built and new solutions were

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obtained [4]. Of particular interest is the Randall-Sundrum (RS) braneworld picture based on considering a type IIB D3-brane embedded in AdS_5 [5], where the conventional $4d$ gravity is recovered in low energy limit despite that fifth extra dimension is non compact. As shown in [6], the RS braneworld model is described by a $4d$ effective gravity induced on the world volume of the D3-brane embedded in $5d$ Einstein gravity. The $5d$ Planck scale M_5 is assumed considerably smaller than the corresponding $4d$ effective Planck scale, $M_4 = 1.2 \times 10^{19} GeV$ opening an issue for new perspectives in cosmology at low energies. Since RS development, diverser inspired RS brane inflationary cosmological models have been constructed [7, 8] and new insights have been gained. Like in standard inflation, most of these cosmological models rest on a single scalar field rolling in a given inflaton field potential $V(\phi)$.

In this paper, we consider inflation dynamics in braneworld cosmology, first with an inverse-power low potential $V(\phi) = \mu^{\alpha+4}/\phi^\alpha$ and then with an exponential one; namely $V(\phi) = V_0 e^{-\beta\phi}$; the μ, α and β moduli will be computed later on. These two types of potential are interesting in cosmology since they are used to modeling tachyonic and quintessential matter [12, 13]. Using slow roll approximation, we compute, in the high energy limit, the inflaton time evolution $\phi = \phi(t)$ and determine the corresponding scale factor $a = a(t)$ and other cosmological quantities. We also derive an upper bound for the coupling constant and give a numerical value of perturbation spectrum, which is in good agreement with observation.

The presentation of this work is as follows: In section 2, we present the basic equations of braneworld inflation assuming the Randall-Sundrum's second model, and recall some known results, especially for chaotic inflation. In section 3, we present our results on scalar field dynamics in braneworld inflation, for both inverse power law and exponential potentials. In this context, we derive the time evolution of the scalar field and scale factor. Various spectral quantities are also calculated and an upper value of coupling constant is obtained. Our results are compared to those obtained in standard inflationary cosmology and are shown to be in good agreement with the observable quantities [14]. We end this paper by a conclusion.

2. Inflation In Randall-Sundrum Braneworld Model

2.1 Modified Einstein-Friedmann Equations

We start this section by recalling briefly some fundamentals on Randall-Sundrum type II braneworld model[6]. We first give the braneworld cosmological Einstein-Friedmann equations; then we discuss resulting modifications of Friedmann equations, slow roll parameters and perturbation spectrum.

Starting from $5d$ Einstein equation with cosmological constant Λ , and by supposing that matter fields is confined on D3-brane, Shiromizu *et al.*[15] have shown that $4d$

Einstein equation induced on the brane reads as

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left(\frac{8\pi}{M_4^2}\right) T_{\mu\nu} + \left(\frac{8\pi}{M_5^3}\right)^2 \pi_{\mu\nu} - E_{\mu\nu}. \quad (1)$$

In this relation, $T_{\mu\nu}$ is the energy-momentum tensor of matter on the brane, $\pi_{\mu\nu}$ is a tensor quadratic in $T_{\mu\nu}$ and $E_{\mu\nu}$ is a projection of the five-dimensional Weyl tensor describing the effect of bulk graviton degrees of freedom on brane dynamics. The effective cosmological constant Λ_4 on the brane is determined by the five-dimensional bulk cosmological constant Λ and the 3-brane tension λ as shown below,

$$\Lambda_4 = \frac{4\pi}{M_5^3} \left(\Lambda + \frac{4\pi}{3M_5^3} \lambda^2 \right). \quad (2)$$

Recall also that $4d$ and $5d$ Planck scales M_4 and M_5 are related as,

$$M_4 = \sqrt{\frac{3}{4\pi}} \left(\frac{M_5^2}{\sqrt{\lambda}} \right) M_5, \quad (3)$$

with λ as before. In braneworld cosmological model where the metric projected onto the brane is a spatially flat Friedmann-Robertson-Walker model with scale factor $a(t)$, Friedmann equation on the brane reads as [16],

$$H^2 = \frac{\Lambda_4}{3} + \left(\frac{8\pi}{3M_4^2}\right) \rho + \left(\frac{4\pi}{3M_5^3}\right)^2 \rho^2 + \frac{\mathcal{E}}{a^4}, \quad (4)$$

where \mathcal{E} is an integration constant arising from $E_{\mu\nu}$ and thus transmitting bulk graviton influence onto the brane. This term appears as a form of “dark radiation” and may be fixed by observation [16, 17]. However, during inflation this term is rapidly diluted, so we will neglect it. We will also assume that, in the early universe, the bulk cosmological constant is $\Lambda \sim -4\pi\lambda^2/3M_5^3$ so that Λ_4 is negligible. With above assumptions, braneworld Friedmann equation(4) reduces to,

$$H^2 = \frac{8\pi}{3M_4^2} \rho \left[1 + \frac{\rho}{2\lambda} \right]. \quad (5)$$

Note that the crucial correction to standard inflation is given by the density quadratic term ρ^2 . Brane effect is then carried here by the deviation factor $\frac{\rho}{2\lambda}$ with respect to unity. This deviation has the effect of modifying the dynamics of the universe for densities $\rho \gtrsim \lambda$. Note also that in the limit $\lambda \rightarrow \infty$, we recover standard four-dimensional general relativistic results (neglecting \mathcal{E}). Note moreover that in inflationary theory with inflaton potential $V(\phi)$, energy density $\rho = \rho(\phi)$ and pressure $p = p(\phi)$ are given by the following relations,

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi); \quad p = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad (6)$$

where ϕ is the inflaton field and the dot stands for the derivative with respect time t . The potential $V(\phi)$ is the initial vacuum energy responsible of inflation. Along with

these equations, one also has the second inflation Klein-Gordon equation governing the dynamic of the scalar field ϕ

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0. \quad (7)$$

This is a second order evolution equation which follows from conservation condition of energy-momentum tensor $T_{\mu\nu}$ on 3-brane dominated by a scalar field ϕ with a self-interaction potential $V(\phi)$. To calculate physical quantities like scale factor or perturbation spectrum, one has to solve eqs(5,7) for some specific potentials $V(\phi)$. To do so, the following approximations are needed.

2.2 Slow-roll Approximation and Perturbation Spectrum on Brane

Inflationary dynamics requires that inflaton field ϕ driving inflation moves away from the false vacuum and slowly rolls down to the minimum of its effective potential $V(\phi)$ [18]. In this scenario, the initial value $\phi_i = \phi(t_i)$ of the inflaton field and the Hubble parameter H are supposed large and the scale factor $a(t)$ of the universe growth rapidly. Using Friedman equation, the inflation condition $\ddot{a} > 0$ allows us to derive the following bound on pressure,

$$\ddot{a} > 0 \quad \Rightarrow \quad p < -\frac{\lambda + 2\rho}{3(\lambda + \rho)}\rho. \quad (8)$$

In the limit $\rho/\lambda \rightarrow \infty$, this condition reduces to $p < -\frac{2}{3}\rho$; this is a more restrictive constraint relation than the corresponding one in standard inflation relation which requires $p < -\frac{\rho}{3}$. Applying the slow roll approximation, $\dot{\phi}^2 \ll V$ and $\ddot{\phi} \ll V'$, to brane field equations (5,7), we obtain:

$$H^2 \simeq \frac{8\pi V}{3M_4^2} \left(1 + \frac{V}{2\lambda}\right), \quad \dot{\phi} \simeq -\frac{V'}{3H}. \quad (9)$$

The presence of the factor $(1 + \frac{V}{2\lambda})$ carries the brane-modification with respect to the standard slow-roll expression recovered by taking the limit $\lambda \rightarrow \infty$. Note that slow roll approximation puts a constraint on the slope and the curvature of the potential; this is clearly seen on the field expressions of ϵ and η parameters given by [7],

$$\epsilon = -\frac{\dot{H}}{H^2} \equiv \frac{M_4^2}{4\pi} \left(\frac{V'}{V}\right)^2 \left[\frac{\lambda(\lambda + V)}{(2\lambda + V)^2}\right], \quad (10)$$

$$\eta = \frac{V''}{3H^2} \equiv \frac{M_4^2}{4\pi} \left(\frac{V''}{V}\right)^2 \left[\frac{\lambda}{2\lambda + V}\right]. \quad (11)$$

Slow-roll approximation takes place if these parameters are such that $\max\{\epsilon, |\eta|\} \ll 1$ and inflationary phase ends when ϵ and $|\eta|$ are equal to one. The other important quantity related to inflation is the number N of e-folding, indicating the growing of the size of universe. Using slow roll approximation, this N number reads in present case as follows,

$$N \simeq -\frac{8\pi}{M_4^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} \left(1 + \frac{V}{2\lambda}\right) d\phi. \quad (12)$$

Before proceeding it is interesting to comment low and high energy limits of these parameters. Note that at low energies where $V \ll \lambda$, the slow-roll parameters take the standard form. At high energies $V \gg \lambda$, the extra contribution to the Hubble expansion dominates and the new factors in square brackets of eqs(10-11) become of order λ/V . The number of e-folding in the limit $V \gg \lambda$,

$$N \simeq -\frac{4\pi}{\lambda M_4^2} \int_{\phi_i}^{\phi_f} \frac{V^2}{V'} d\phi, \quad (13)$$

where ϕ_i and ϕ_f stand for initial and final value of inflaton.

To test inflation model, one must compute the spectrum of perturbations produced by quantum fluctuations of fields around their homogeneous background values. Using slow-roll equations and following [7], the scalar amplitude A_s^2 of density perturbation, evaluated by neglecting back-reaction due to metric fluctuation in fifth dimension ($E_{\mu\nu} = 0$), is given by

$$A_s^2 \simeq \left(\frac{512\pi}{75M_4^6} \right) \frac{V^3}{V'^2} \left[\frac{2\lambda + V}{2\lambda} \right]^3 \Big|_{k=aH}. \quad (14)$$

Note that for a given positive potential, the A_s^2 amplitude is increased in comparison with the standard result. Note also that in high energy limit this quantity behaves as,

$$A_s^2 \simeq \frac{64\pi}{75\lambda^3 M_4^6} \frac{V^6}{V'^2}. \quad (15)$$

On the other hand, using eqs(10-11), one can compute the perturbation scale-dependence described by the spectral index $n_s \equiv 1 + d(\ln A_s^2)/d(\ln k)$. We find,

$$n_s - 1 \simeq 2\eta - 6\epsilon, \quad (16)$$

Note that at high energies λ/V , the slow-roll parameters are both suppressed; and the spectral index is driven towards the Harrison-Zel'dovich spectrum, $n_s \rightarrow 1$ as $V/\lambda \rightarrow \infty$.

In what follows, we shall apply the braneworld formalism that we have described above by singling out two specific kinds of inflaton potentials. These are the inverse power law potential and the exponential one that have gained revival interest in recent literature in connection with dark matter and quintessence cosmology [13].

3. Scalar Field Dynamics in Braneworld Scenario

To begin, recall that chaotic inflationary model, which was first introduced by Linde [18], has been reconsidered recently by several authors in the context of braneworld scenario [7, 20]. In present work, we are interested by coupling constants for inverse power-law potential $V(\phi) = \frac{\mu^{\alpha+4}}{\phi^\alpha}$ and exponential one $V(\phi) = V_0 \exp(-\beta\phi)$.

3.1 Inverse-Power Law Potential

In the braneworld cosmology high energy limit where $V \gg \lambda$, brane effect becomes important and the Friedmann equations are simplified as,

$$H^2 \simeq \left(\frac{4\pi}{3M_5^3} \right) V^2, \quad \dot{\phi} \simeq -\frac{V'}{V} \left(\frac{M_5^3}{4\pi} \right). \quad (17)$$

and Slow-Roll parameters (10-11) reduces to:

$$\epsilon \equiv \frac{M_4^2}{16\pi} \left(\frac{V'}{V} \right)^2 \left[\frac{4\lambda}{V} \right], \quad \eta \equiv \frac{M_4^2}{8\pi} \left(\frac{V''}{V} \right) \left[\frac{2\lambda}{V} \right]. \quad (18)$$

Consider the inverse power law potential given by,

$$V = \frac{\mu^{\alpha+4}}{\phi^\alpha}, \quad (19)$$

where μ is the inflaton coupling constant and α some critical exponent. This potential has been studied in various occasions; in particular in connection with quintessence in brane inflation [13] and tachyonic inflation [21]. One of the interesting results in this matter, and which is due to Huey and Lidsey[13], is that inflation is generated for the range $\alpha > 2$. Combining results on universe observation and Slow-Roll approximation, we want to show that is possible to express the inverse power law potential coupling constant μ in terms of α and M_5 . To that purpose, consider the field expression of the Slow-Roll parameter ϵ namely,

$$\epsilon \simeq \frac{3M_5^6}{16\pi^2} \left(\frac{\alpha^2}{\mu^{\alpha+4}} \right) \frac{1}{\phi^{2-\alpha}}. \quad (20)$$

Then using the constraint relation $\epsilon \sim 1$ characterizing end of inflation, one can invert above relation as follows,

$$\phi_{end}^{2-\alpha} = \frac{3M_5^6 \alpha^2}{16\pi^2 \mu^{\alpha+4}}. \quad (21)$$

Similarly, we can compute the e-folding number by help of eq(13). We find,

$$N \simeq \frac{16\pi^2 \mu^{\alpha+4}}{3M_5^6} \frac{1}{\alpha(2-\alpha)} [\phi_f^{2-\alpha} - \phi_i^{2-\alpha}], \quad (22)$$

where ϕ_i and ϕ_f stand for initial and end inflaton field values. From these equations (21-22), we can deduce the expression of ϕ_i in term of the e-folding number N ,

$$\phi_i^{2-\alpha} = \frac{3M_5^6 \alpha}{16\pi^2 \mu^{\alpha+4}} [\alpha - N(2-\alpha)]. \quad (23)$$

Now, using observation data giving $N_{cobe} \approx 55$ [22] and identifying the initial and final inflaton field values of inflation interval as $\phi_i = \phi_{cobe}$ and $\phi_f = \phi_{end}$, we get,

$$\phi_{cobe}^{2-\alpha} = \frac{3M_5^6 \alpha}{16\pi^2 \mu^{\alpha+4}} [56\alpha - 110]. \quad (24)$$

Putting back into eq(15) after substituting equation (19), we deduce the following expression of scalar amplitude,

$$A_S \simeq \frac{64\pi(\mu^{\alpha+4})^2}{45M_5^9\alpha}\phi_{cobe}^{1-2\alpha}. \quad (25)$$

Using the observed numerical value of A_S from COBE namely $A_S \sim 2.10^{-5}$, we can invert above identity to fix the inflaton coupling constant μ in term of $5d$ Planck mass M_5 and inflaton field exponent α as shown below,

$$\mu^{\alpha+4} = \left[\frac{90 \cdot 10^{-5} \alpha M_5^9}{64\pi} \right]^{\frac{2-\alpha}{3}} \left[\frac{8\pi^2}{\alpha M_5^6 (84\alpha - 165)} \right]^{\frac{1-2\alpha}{3}}. \quad (26)$$

Note that to get sufficient inflation ($N = 55$) one can obtain from eqs(24,26) a constraint on the initial value of the field which depends on α . For $\alpha = 4$, for example, we get $\phi_i < 162.2 M_4$.

Following the same method, it is not difficult to check, by help of eqs(11,19), that the spectral index n_S reads as,

$$n_S - 1 \simeq -6\epsilon + 2\eta = \frac{1 - 2\alpha}{28\alpha - 55}, \quad (27)$$

or equivalently $n_S = \frac{26\alpha - 55}{28\alpha - 55}$ whose positivity condition requires $\alpha > 2 + \frac{3}{26}$ in agreement with Huey and Lidsey prediction [13]. Fixing a value of the inflaton field exponent α , we can compute the inflaton field coupling constant μ and the spectral index n_S . For $\alpha = 3$ for instance, we get $n_S = 0.83$ and for large α , we have 0.92 in perfect agreement with observation [22] In what follows, we consider the case of inflation exponential potential.

3.2 Exponential Inflation on Brane

Here we consider our second brane inflation model with an exponential potential [23]. This potential has been used to study tachyonic inflation [12] and quintessence [24]. In the present work, we give a quantitative study and compute physical quantities relevant to observation; in particular the perturbation spectrum. To that purpose consider the exponential potential,

$$V(\phi) = V_0 e^{-\beta\phi} \quad (28)$$

where V_0 is a constant and β is the coupling strength of the scalar field. By integrating eqs(9), one can derive the time evolution of scalar field. We find $\phi(t) = C + \frac{M_5^3}{4\pi}\beta t$, where C is an integration constant. To obtain the scale factor $a(t)$, one have to integrate the Friedmann equation (17); we get,

$$a(t) = a_i \exp \left(-\frac{V_0}{3} \frac{16\pi^2}{\beta^2 M_5^6} \exp [-\beta(C + M_5^3 \beta t)] \right). \quad (29)$$

From the study of the inflection point of this relation, one may compute t_{end} , the end time of inflation; this is given by

$$t_{end} = \frac{4\pi}{\beta^2 M_5^3} \left[-\beta C - \ln \left(\frac{3M\beta_5^6}{16\pi^2 V_0} \right) \right]. \quad (30)$$

Putting back into $\phi_{end}(t) = C + \frac{M_5^3}{4\pi}\beta t_{end}$, we determine the value of the scalar field at the end of inflation,

$$\phi_{end} = \frac{1}{\beta} \ln \left(\frac{16\pi^2 V_0}{3\beta^2 M_5^6} \right). \quad (31)$$

Using eqs(13), we can compute the number N of e-folding for the exponential potential,

$$N = -\frac{16\pi^2 V_0}{3\beta^2 M_5^6} (e^{-\beta\phi_f} - e^{-\beta\phi_i}), \quad (32)$$

where ϕ_i is the value of the scalar field at the beginning of inflation,

$$\phi_i \simeq -\frac{1}{\beta} \ln \left(\frac{21\beta^2 M_5^6}{2\pi^2 V_0} \right). \quad (33)$$

We assume now that the number of e foldings before the end of inflation at which observable perturbations are generated corresponds to $N = 55$ [22] and setting $\phi_f = \phi_{end}$, $\phi_i = \phi_{cobe}$, and $A_S^2 = 2.10^{-5}$, we can give an estimation of the coupling constant β of the scalar field. Straightforward computation leads to,

$$\beta = 1.07 \times 10^{-2} M_5^{-1}. \quad (34)$$

Since $M_5 < M_4$, one can deduce an upper bound limit of the coupling constant β as shown below,

$$\beta > 0.877 \times 10^{-21} \quad (35)$$

This will give a constraint on the initial value of the scalar field and should be compared with standard inflation result for same potential for which $\beta_0 = 708,9 \times 10^{-2} M_4^{-1}$. Note that like for chaotic inflation, we have here also a very weakly coupled scalar field on the brane compared to that of standard inflation. Using high energy limit and following same analysis as for inverse power law potential, we can compute spectral index n_s for the exponential case. We find,

$$n_s = 0.92 \quad (36)$$

which is in good agreement with observation for wich

$$0.8 < n_s < 1.05. \quad (37)$$

As far as this result is concerned, it is interesting to note that a similar value was also obtained in [25] for tachyonic inflation.

4. Conclusion

In this paper, we have studied aspects of inflationary dynamics in the framework of braneworld cosmology. It has been shown that in this scenario, the Friedmann equations governing the dynamics of scalar field in the universe are modified by an extra term which depend on the ratio of the density of matter ρ and the brane tension λ . In Slow-Roll approximation, this ratio depend only on the potential V and the tension λ .

In this short letter, we have studied brane effects on inflation for both the inverse power law and exponential potentials. In this context, we have calculated the scale factor for the potential(28) in braneworld formalism and shown that it has an exponential form, as it should be in inflation theory. We have also calculated a numerical value of the perturbation spectrum for potentials (19,28) in good agreement with observation and an upper bound for the coupling constant for exponential potential was obtained. This work may be applied to a concret physical problems such as tachyonic inflation and dark matter.

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Considerations About The Anomalous Efficiency Of Particular Thermodynamic Cycles

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Abstract: Some years ago Vignati (refs. 1, 2, 3) showed that, under some particular circumstances (*inter alia* : isobaric processes connected through internal heat exchangers), the efficiency of an Ericsson cycle involving a real gas can exceed the Carnot limit η_C , in contradiction with the second principle of thermodynamics. However, the convergence of Vignati's algorithm, giving the temperature difference between the intermediate heat exchangers, has not yet been proved. In particular, it isn't clear, if the number of intermediate heat exchangers infinitely increases, the condition of a total (perfect) heat recovery may be asymptotically approximated. This remark is relevant because the claimed anomalous efficiencies appear only in the limit of a perfect heat recovery. Considering a suitable counterexample, we prove that, in general, the residual heat discharged on the external sources does not vanish in that limit, even when the isobars exchange the same amount of heat. Therefore the violation of the second law inferred from Vignati's calculation is merely apparent, being referred to a situation which is not (in principle) physically realisable. The essentials of the Vignati's argument are then applied to an Ericsson cycle involving an ideal gas undergoing chemical reactions. Again, no contradiction arises with the second principle.

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1. Introduction

Let's consider a closed system composed of N classical identical particles; denoting with n_i the number of particles having the total energy ε_i ($i = 1, 2, \dots$) and with U the internal

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energy, the following constraints hold :

$$U = \sum_i \varepsilon_i n_i \quad (1)$$

$$N = \sum_i n_i = \text{constant} \quad (2)$$

Differentiating the eq. (1) and assuming (ref. 4) :

$$dQ = \sum_i \varepsilon_i dn_i \quad (3)$$

$$dL = -\sum_i n_i d\varepsilon_i \quad (4)$$

we easily obtain the first principle of thermodynamics :

$$dU = dQ - dL \quad (5)$$

where Q and L respectively represent the quantity of heat exchanged by the system and the work done by it.

The entropy is defined as (ref. 5) :

$$S = -k \sum_i n_i \ln(n_i) \quad (6)$$

The system is said to be in thermal equilibrium at the absolute temperature T if

$$n_i = n_0 \exp(-\varepsilon_i/kT) \quad (7)$$

During an infinitesimal equilibrium process, the entropy increase is :

$$\begin{aligned} dS &= -k \sum_i dn_i \ln(n_i) - k \sum_i n_i (dn_i/n_i) = -k \sum_i dn_i (\ln(n_0) - \varepsilon_i/kT) - kdN = \\ & -k(\ln(n_0) + 1)dN + k \sum_i dn_i \varepsilon_i/kT = (1/T) \sum_i \varepsilon_i dn_i = dQ/T \end{aligned} \quad (8)$$

In other words, the Clausius equality can be derived from microphysics¹. If the system is in thermal equilibrium with an external reservoir at the same temperature T :

$$T_{sys} = T_{res} = T \quad (9)$$

the Clausius equality can be written as :

$$dS = dQ/T_{res} \quad (10)$$

For “reservoir” we mean here a system in thermal equilibrium at a given temperature T_{res} , having so great heat capacity that it can assure the constancy of T_{res} during the

¹ This reasoning holds for free particles but, using the canonical ensemble in a consistent way, the same conclusion is obtained for the general case of interacting particles. See, for example, ref. 11.

process. If a system which is initially in equilibrium at the temperature T_{sys} interacts with the reservoir at the temperature $T_{res} \neq T_{sys}$, they exchange an amount dQ of heat in an irreversible process. The system entropy increases by an amount dS which satisfies the well known Clausius inequality :

$$dS \geq dQ/T_{res} \quad (11)$$

in which ever the reservoir temperature appears. From eq. (2) follows :

$$dS = -k \sum_i dn_i \ln(n_i)$$

and therefore, the eq. (11) becomes :

$$-k \sum_i dn_i \ln(n_i) \geq (1/T_{res}) \sum_i \varepsilon_i dn_i$$

that is :

$$\sum_i dn_i (\ln(n_i) + \varepsilon_i/kT_{res}) \leq 0 \quad (12)$$

Of course, it is not possible to derive the inequality (12) from the statistics of the equilibrium states, because it involves the time evolution of the coefficients dn_i/dt which is likely to tend to the thermal equilibrium with the reservoir at the temperature T_{res} . When the thermal interaction begins, the relation (7) holds, with $T = T_{sys}$; as a consequence:

$$\sum_i dn_i (\ln(n_0) - \varepsilon_i/kT_{sys} + \varepsilon_i/kT_{res}) \leq 0$$

that is :

$$dQ(1/T_{res} - 1/T_{sys}) \leq 0 \quad (13)$$

Defining the absolute temperature as positive, we have therefore:

$$T_{res} > T_{sys} \quad \text{if} \quad dQ > 0$$

$$T_{res} < T_{sys} \quad \text{if} \quad dQ < 0$$

In other words, the heat flows from the system into the reservoir, if the system is warmer than the reservoir, and *viceversa* if it is colder. Therefore, the essentials of the second principle of thermodynamics are given by relation (11). We arrive to the same conclusion by writing the entropy increase for the reservoir:

$$dS_{res} = (-dQ)/T_{res}$$

and for the system:

$$dS_{sys} = dQ/T_{sys}$$

According to relation (13), the total increase $dS_{tot} = dS_{res} + dS_{sys}$ is ≥ 0 .

Let us consider now a thermodynamic cycle consisting of two reversible isotherms at temperatures T_{min} , T_{max} , alternated with two other processes A , B . From eq. (5) we

have, for the amount of heat Q exchanged with external reservoirs and the work L done by the cycle, the equation $L = Q$, because the internal energy U is a function of state. Therefore :

$$L = Q = Q(T = T_{max}) - Q(T = T_{min}) + Q_A - Q_B \quad (14)$$

Now we suppose that a heat exchange occurs between the sections A and B , in such a way that $Q_A = Q_B$; therefore, the condition of a perfect heat recovery (or compensation) holds :

$$L = Q = Q(T = T_{max}) - Q(T = T_{min}) \quad (15)$$

If the global process $A + B$ is reversible (which includes a perfectly reversible thermal exchange between them), then from the Clausius equality we obtain, for the associated entropy increase, $\Delta S = \int_{AB} (\text{net amount of heat exchanged with external reservoirs at the temperature } T)/T = 0$. Therefore, the total increase of entropy along the cycle amounts to:

$$(\Delta S)_{cycle} = (\Delta S)_{T=T_{max}} + (\Delta S)_{T=T_{min}} = 0 \quad (16)$$

because S is a function of state. From the Clausius equality, eq. (10), we have:

$$Q(T = T_{max}) = T_{max}/(\Delta S)_{T=T_{max}}/,$$

$$Q(T = T_{min}) = T_{min}/(\Delta S)_{T=T_{min}}/.$$

We easily obtain the efficiency of the cycle :

$$\eta = 1 - Q(T = T_{min})/Q(T = T_{max}) = 1 - T_{min}/T_{max} = \eta_C \quad (17)$$

It is equal to the Carnot efficiency. The eq. (17) follows from the first principle [eq. (5)] and the Clausius equality [eq. (10)], which have been microphysically justified.

Assuming the thermal exchange is still compensated, but it is not reversible, then from the Clausius inequality we obtain, for the associated entropy increase, $\Delta S > \int_{AB} (\text{net amount of heat exchanged with external reservoirs at the temperature } T)/T = 0^2$. Therefore :

$$(\Delta S)_{cycle} = (\Delta S)_{T=T_{max}} + (\Delta S)_{T=T_{min}} + \Delta S = 0 \quad (18)$$

For an engine cycle,

$$(\Delta S)_{T=T_{max}} > 0 \quad , \quad \text{then} \quad (\Delta S)_{T=T_{min}} < 0 \quad \text{and} \quad |(\Delta S)_{T=T_{min}}| > |(\Delta S)_{T=T_{max}}|.$$

The equality (17) is then substituted by the inequality :

$$\eta = 1 - Q(T = T_{min})/Q(T = T_{max}) < 1 - T_{min}/T_{max} = \eta_C \quad (19)$$

In next section a particular cycle of this kind is considered in detail.

² Hereinafter we take into account only engine cycles, wherein the processes A , B involve only spontaneous thermal irreversibility without absorption of external work.

2. Heat recuperators and heat regenerators

Let us consider, on the PV plane, a thermodynamic cycle consisting of two reversible isotherms at temperatures T_{min} , T_{max} , alternated with two isobars at pressures P_{min} , P_{max} (Ericsson cycle). During the isobaric processes the gas does not exchange heat with external sources; instead, the expanding gas exchanges heat with the compressed gas. This is possible, if the cycle is accomplished by a stationary flow of gas instead of gas undergoing alternating phases.

The gas coming from the heater at temperature T_{max} (Fig. 1) is diverted in the ducting and is sent to the cooler at temperature T_{min} ; the gas coming from the cooler is diverted in another ducting and is sent to the heater. Both ductings' cross-section is shaped in such a manner as to permit to keep constant the gas pressure.

The ductings are perfectly insulated from the outside and connected through N ideal heat exchangers; the i -th heat exchanger has its definite operating temperature T_i and :

$$T_{min} < T_1 < T_2 < \dots < T_N < T_{max}$$

Each heat exchanger is assumed to be thermally insulated from the outside. The thermal conductivity of each exchanger is hypothetically infinite, so a perfect heat exchange between the ductings is guaranteed. In such a manner, we have constructed an (ideal) "heat recuperator". Both the gas being heated and the gas being cooled simultaneously pass through the recuperator; it is in this feature that it differs from a regenerator. The regenerator, in fact, is applied to the thermodynamic cycles involving alternating phases. It absorbs heat from the fluid subjected to cooling during one phase and then releases it to the fluid subjected to heating in the next phase. Coming back to consider the setup, we remark that the volume of involved gas is necessarily finite; therefore, only in extremely particular conditions the temperature of each exchanger will be stationary. These conditions are largely discussed in the next section.

Let us consider now how the recuperator influences the efficiency. The quantity of heat exchanged between two isobars is:

$$\Delta = Q(P = P_{max}) - Q(P = P_{min}) + Q_{res} = R + Q_{res}$$

where [C_p = specific heat per mole of gas at pressure p]:

$$Q(P = P_{max}) = \int_{T_{min}}^{T_{max}} C_{p_{max}} dT \quad ,$$

$$Q(P = P_{min}) = \int_{T_{min}}^{T_{max}} C_{p_{min}} dT \quad ,$$

and Q_{res} is the residual heat depending on the recuperator construction.

According to the first principle, the amount of heat R must be transferred from the external sources : if $R > 0$ the gas absorbs it from the heater at the temperature T_{max} ,

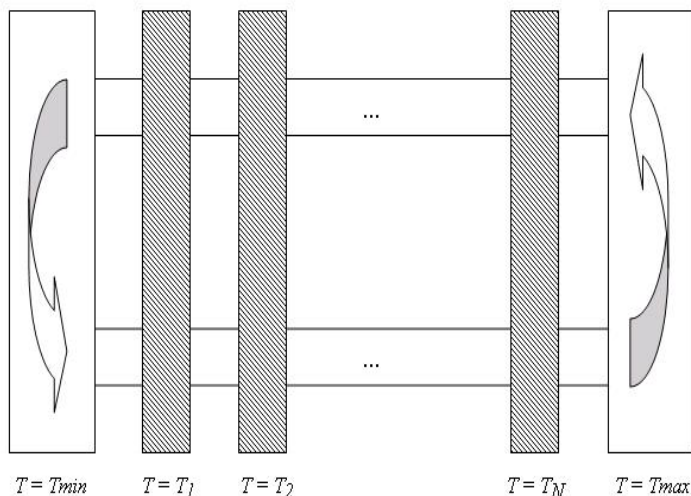


Fig. 1

if $R < 0$ the gas releases it to the cooler at the temperature T_{min} . As it is detailed in the next section, the gas absorbs the amount of heat Q_{res} from the heater and releases it to the cooler.

For example, if $R > 0$, the gas absorbs the quantity of heat :

$$Q_2 = \Delta + Q(T = T_{max})$$

from the heater at the temperature T_{max} , and it releases the quantity of heat:

$$Q_1 = Q_{res} + Q(T = T_{min})$$

to the cooler at the temperature T_{min} .

3. Some considerations about the recuperator

The heat exchange between the isobaric processes may be compensated only if:

$$R = Q(P = P_{max}) - Q(P = P_{min}) = 0 \quad (20)$$

otherwise, no matter the recuperator construction, Δ will not vanish, because a residual heat, R , will be discharged on the heater at T_{max} (if $R > 0$) or on the cooler at T_{min} (if $R < 0$). The validity of the condition (20) may be secured by selecting in a proper manner the cycle on the PV plane.

Obviously, the recuperator not influences the isothermal work. As far as the isobaric work concern, it is still equal to $p\Delta V$ provided that $R = 0$. We can conclude that under these circumstances the work L done by the cycle is not influenced by the presence of the recuperator. Since L is fixed, the efficiency of cycle in the presence of recuperator :

$$\eta = L/[Q_{res} + Q(T = T_{max})]$$

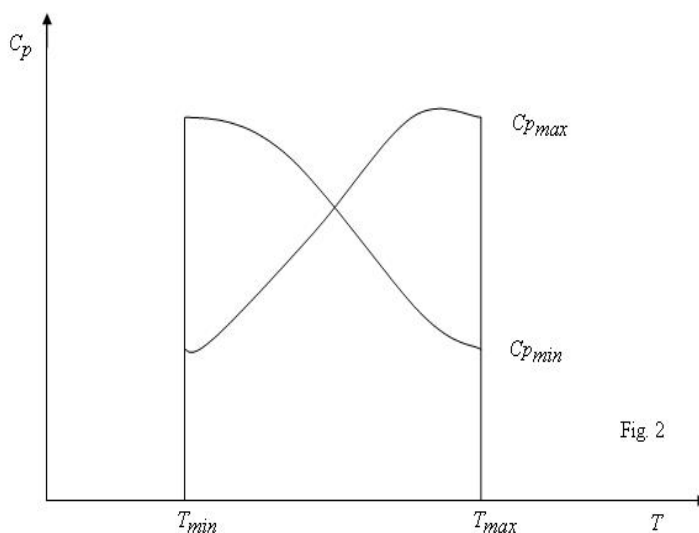
is maximum, when $Q_{res} = 0$. In other words, the maximum of efficiency is obtained when the heat exchange between the isobaric processes is exactly compensated.

The eq. (20) surely holds if

$$C_{p_{min}}(T) = C_{p_{max}}(T) \quad T_{min} \leq T \leq T_{max}$$

that is, *e.g.*, for cycles involving an ideal gas³.

There is another possibility available only for a real gas with T/T_c slightly more than 1 (where T_c is the critical temperature of gas); it is illustrated in Fig. 2. Let us consider



now the other term contributing to Δ , that is Q_{res} ; it depends on the number N of heat exchangers and their temperatures. These two factors can't be selected arbitrarily, if one needs to have a stationary temperature distribution.

The i -th heat exchanger absorbs heat from the hotter gas ($T > T_i$) entering it through one ducting, and release heat to the colder gas ($T < T_i$) entering it through another ducting. Only if these two amounts of heat are equal, the temperature, T_i , of the heat exchanger may remain constant.

Because we are considering an engine which performs a positive work, the point representing the state of a selected mass of gas must run clockwise along the cycle outline on the PV plane. So, the gas is cooled during the isobaric process at P_{min} , while it undergoes heating during the isobaric process at P_{max} .

Therefore, the i -th exchanger absorbs the amount of heat:

$$Q_{in}(i) = \int_{T_i}^{T_{i+1}} C_{p_{min}} dT \quad (21)$$

³ The cycle involving an ideal gas is a particular case of the situation presented in the last section, when $n = n'$; that is, when no dissociation/recombination occurs.

from the gas at temperature greater than T_i , flowing in it through the pipe at lower pressure. Instead, it releases the amount of heat :

$$Q_{out}(i) = \int_{T_{i-1}}^{T_i} C_{p_{max}} dT \quad (22)$$

to the gas at temperature lower than T_i , flowing in it through the pipe at higher pressure.

We may have a set of stationary temperatures $\{T_i ; i = 1, 2, \dots, N\}$ only if the condition :

$$Q_{in}(i) = Q_{out}(i) \quad (23)$$

holds for each value of i .

The temperature of each heat exchanger can be obtained, by recursively applying eqs. (21)-(23) following two distinct procedures.

According to the first procedure, $T_0 = T_{min}$ and T_1 are fixed; the eq. (22) gives $Q_{out}(1)$, and $Q_{in}(1)$ is then derived from (23). Substituting $Q_{in}(1)$ in eq. (21) we obtain an equation for the unknown T_2 ; replacing the value of T_2 in eq. (22) we obtain $Q_{out}(2)$ and so on. This iterative procedure is stopped when a temperature $T_{i+1} > T_{max}$ is finally reached. The value of i is the number N of intermediate heat exchangers which is compatible with the selected value of T_1 . The residual heat :

$$\int_{T_{max}}^{T_{N+1}} C_{p_{min}}(T) dT = \alpha$$

is not compensated; in order to reduce α it is necessary to vary T_1 . We can change T_1 continuously until we obtain $T_{N+1} = T_{max}$, so that $\alpha = 0$.

According to the second procedure, we fix $T_{N+1} = T_{max}$ and T_N ; the eq. (21) gives $Q_{in}(N)$, and $Q_{out}(N)$ is then derived from (23). Substituting $Q_{out}(N)$ in eq. (22), we obtain an equation for the unknown T_{N-1} . Replacing the value of T_{N-1} in eq. (21) as the lower extreme of integration, we obtain $Q_{in}(N-1)$ and so on. This iterative procedure is stopped when a temperature $T_{i-1} < T_{min}$ is finally reached. The value of i is then redefined as 1 and the number of iterations is the number N of heat exchangers which is compatible with the selected value of T_N . The residual heat :

$$\int_{T_0}^{T_{min}} C_{p_{max}}(T) dT = \beta$$

is not compensated; in order to reduce β it is necessary to vary T_N . We can change T_N continuously until we obtain $T_0 = T_{min}$, so that $\beta = 0$.

Anyhow, the same set of stationary temperatures $\{T_i ; i = 1, 2, \dots, N\}$ is obtained for the same value of N ; it means that we have obtained a recuperator which is in thermal equilibrium, but not a perfect heat recovery between two isobaric processes. As we can

see in Fig. 1, the gas flowing through the pipe at lower pressure into the cooler at temperature $T = T_{min}$ releases to it an amount of heat :

$$\int_{T_{min}}^{T_1} C_{p_{min}}(T) dT = Q_{res} \quad , \quad (24)$$

which is not compensated; it is discharged on the cooler. Analogously, the gas flowing through the pipe at higher pressure into the heater at $T = T_{max}$ releases to it an amount of heat :

$$\int_{T_{max}}^{T_N} C_{p_{max}}(T) dT \quad ,$$

which is not compensated. This integral amounts to $-Q_{res}$ as a consequence of eqs. (21)-(23). Therefore, the gas absorbs from the heater at $T = T_{max}$ an amount of heat :

$$Q_{res} = \int_{T_N}^{T_{max}} C_{p_{max}}(T) dT \quad ; \quad (25)$$

which contributes to Δ , as it has been exposed before.

If the eq. (20) holds, one has $\Delta = Q_{res}$; in such a case, the efficiency η is the highest when $Q_{res} = 0$. However, the condition $Q_{res} = 0$ (corresponding to a complete or “perfect” heat recovery) is unattainable, because N is a finite number. The question arises if

$$\lim_{N \rightarrow \infty} Q_{res} = 0 \quad , \quad (26)$$

that is, if it is possible to approximate indefinitely the condition of perfect (and reversible) heat recovery by increasing the number of intermediate heat exchangers.

It is easy to find a situation which violates the eq. (26). Let us consider a particular case of the situation represented in Fig. 2, for which an analytical calculation is possible. We represents it in Fig. 3. The specific heat per mole of gas along the isobars is expressed as ($C, A > 0$):

$$C_{P_{max}}(T) = C + \frac{2A}{(T_{max} - T_{min})} (T_x - T)$$

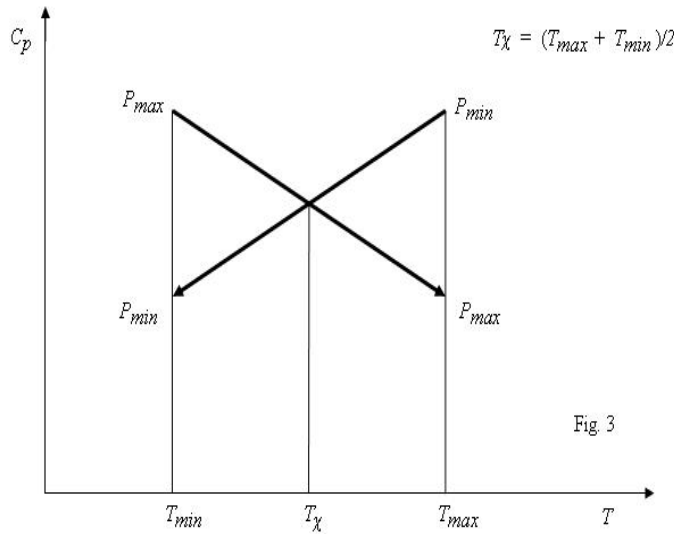
$$C_{P_{min}}(T) = C - \frac{2A}{(T_{max} - T_{min})} (T_x - T) \quad .$$

Therefore: $Q(P = P_{max}) = Q(P = P_{min}) = C(T_{max} - T_{min})$

so that $\Delta = Q_{res}$.

Let us assume that $T_0 = T_{min}$ and $T_1 < T_x$. The integral (22), evaluated for $i = 1$, is the area under the curve $C_{P_{max}}(T)$ in the interval (T_0, T_1) , that is :

$$S = C(T_1 - T_0) + \frac{2A}{(T_{max} - T_{min})} \left[T_x (T_1 - T_0) - \frac{1}{2} (T_1^2 - T_0^2) \right] \quad .$$



The integral (21), evaluated for $i = 1$, is the area under the curve $C_{P_{min}}$ in the interval (T_1, T_2) ; this area is:

$$S = C(T_2 - T_1) - \frac{2A}{(T_{max} - T_{min})} \left[T_{\chi} (T_2 - T_1) - \frac{1}{2} (T_2^2 - T_1^2) \right] .$$

From the equality of these areas [eq. (23)] one obtains

$$\frac{T_2 - T_1}{T_1 - T_0} = \frac{C + \alpha [T_{\chi} - ((T_1 + T_0)/2)]}{C - \alpha [T_{\chi} - ((T_1 + T_2)/2)]} , \text{ where } \alpha = 2A / (T_{max} - T_{min}) .$$

And, in general :

$$\frac{T_{i+1} - T_i}{T_i - T_{i-1}} = \frac{C + \alpha [T_{\chi} - ((T_i + T_{i-1})/2)]}{C - \alpha [T_{\chi} - ((T_i + T_{i+1})/2)]} .$$

Consider now the i_0 -th heat exchanger, with $T_{i_0+1} > T_{i_0} > T_{i_0-1} \geq T_{\chi}$. For $i > i_0$ the expressions in brackets are negative, so that :

$$\frac{T_{i+1} - T_i}{T_i - T_{i-1}} \leq \frac{C}{C - \alpha [T_{\chi} - ((T_i + T_{i+1})/2)]} \leq \frac{C}{C - \alpha (T_{\chi} - T_{i_0-1})} = q < 1 .$$

That is :

$$T_{i+1} - T_i \leq q^{i-i_0+1} (T_{i_0} - T_{i_0-1}) .$$

Let's denote with T_{init} the temperature of $(i_0 - 1)$ -th heat exchanger; n is the number of heat exchangers between T_{init} and T_{max} and $\Delta T_i = T_i - T_{i-1}$ is the temperature difference between two adjacent heat exchangers. Summing up the $n + 1$ temperature differences (we assume that $T_{n+1} = T_{max}$) one has :

$$\sum_i \Delta T_i = T_{max} - T_{init} ,$$

so that :

$$\Delta T_{i_0} \sum_{i \geq i_0} q^{i-i_0} = \Delta T_{i_0} \frac{1 - q^{n+1}}{1 - q} \geq T_{max} - T_{init} .$$

For $n \rightarrow \infty$, this expression becomes :

$$\Delta T_{i_0} \geq (1 - q) (T_{\max} - T_{\text{init}}) \quad ,$$

where $q = C/[C - \alpha (T_\chi - T_{\text{init}})]$. Therefore, the first segment (T_{i_0-1}, T_{i_0}) of the sequence does not tend to vanish in the limit $n \rightarrow \infty$, but it tends to a positive value dependent on the arbitrarily selected temperature T_{init} , except perhaps for $T_\chi = T_{\text{init}}$. As a consequence, the integrals (21), (22) in this interval, in general, do not vanish, and a finite residual heat is discharged on the external heater.

In each situation similar to one illustrated in Fig. 3 the curve at P_{\min} is above the curve at P_{\max} for $T > T_\chi$. Since the area subtended by the lower curve on a given segment of temperatures must be equal to the area subtended by the upper curve on the subsequent segment [eq. (23)], this segment must be shorter than the previous one. The area subtended by the lower curve on the new segment will be less than the area subtended by the same curve on the previous segment. Therefore, the segment following this one will be shorter than preceding one and so on.

This result is reasonable, because (see Fig. 3) it is $C_{P_{\min}}(T) > C_{P_{\max}}(T)$ for $T > T_\chi$; therefore from eq. (23) immediately follows that $\Delta T_i > \Delta T_{i+1}$, so we have an infinite sequence of shorter and shorter segments ΔT_i which forms the interval (T_χ, T_{\max}) . Just in this case, indeed, the temperature difference between the “last” heat exchanger and T_{\max} is infinitesimal, so that no residual heat is discharged on the heater. But if the first segment was too short, the decrease of the following segments would be too sharp to guarantee the complete covering of the temperature axis until T_{\max} . A finite temperature difference would remain with respect to the external source which would have to discharge a finite amount of heat. In general, if the first interval is too short, relation (26) is not satisfied and the proposed algorithm does not converge; the sequence converges towards a temperature $T_{\text{limit}} < T_{\max}$; the integral of the specific heat on $[T_{\text{limit}}, T_{\max}]$ is a residual heat discharged on the external heater. Referring to Fig. 3, this happens when :

$$\Delta T_{i_0} \leq (1 - q) (T_{\max} - T_{\text{init}}) \quad .$$

On the other hand, the first interval must not be too long, otherwise it remains finite in the limit $n \rightarrow \infty$. According to our algorithm [eqs. (21),(22),(23)], a finite residual heat is discharged on the external heater so that, in conclusion, the eq.(26) can hold only for $T_\chi = T_{\text{init}}$ (that is, $q = 1$). In other words, a heat exchanger must operate at temperature T_χ .

A specular reasoning is valid for the segment (T_{\min}, T_χ) : starting from the heat exchanger, in which the reaction takes place, and approaching to T_{\min} , the temperature interval between two adjacent heat exchangers has to become shorter and shorter and the segment has to be completely covered, otherwise there is a finite temperature difference between the heat exchangers and the cold source and the latter has to discharge a finite amount of heat.

4. The Vignati's argument

In his interesting work, Vignati studied the situation illustrated in Fig. 2. According to his heuristic argument, the eq. (26) was *ever* true and then was *ever* possible to approximate at least ideally, the condition $Q_{res} = 0$. Therefore, in order to have $\Delta = \theta$ it was enough to individuate the Ericsson cycles on the PV plane in such a way that $Q(P = P_{max}) \approx Q(P = P_{min})$. The cycles, involving real gas, had to be near the critical temperature of gas ($T_{min}/T_c \approx 1$).

Vignati paid a particular attention to argon, because for this gas extensive tabulations and accurate modelizations were available from NBS-27 report (ref. 6). He was able to find several cycles, whose efficiency η , in the presence of a recuperator, (given that $Q_{res} = 0$) was, in fact, more than η_C .

The original ERICSSON code written by Vignati, in GWBASIC language, to perform the calculation of $\eta(Q_{res} = 0)$ according to NBS-27, is published in appendix to ref. 1. We have rechecked each line of this code, even making comparison with NBS-27 equations and tables, searching for a mistake, without results. We also reproduced the original Vignati's calculations and obtained the same results. For example, at

$$\begin{aligned} P_{max} &= 62atm, & P_{min} &= 55atm \\ T_{max} &= 180^0K, & T_{min} &= 152^0K \end{aligned} \quad (27)$$

(we remark here that for argon $T_c = 150.86$ °K) we have $R = 5.68$ J/mol, a very small value. The cycle efficiency, without recuperator, is 0.025; the cycle efficiency with recuperator and $Q_{res} = 0$ is $\eta(Q_{res} = 0) = 0.270$. The Carnot efficiency between the temperatures T_{max} and T_{min} amounts to $\eta_C = 0.155$. Therefore, for this cycle: $\eta(Q_{res} = 0) > \eta_C$.

The report NBS-27 gives analytic expressions both for the internal energy of gas argon, U , as a function of P , T and its equation of state. In calculating the efficiency, the heat exchanged during each process is evaluated by applying the first principle in the form of $Q = \Delta U + \int pdV$.

Therefore, according to the Vignati judgement, the result $\eta(Q_{res} = 0) > \eta_C$ should reveal a contradiction between the first and the second principle, if it was possible to obtain the perfect (and reversible) heat compensation.

However, numerical computations performed independently by Vignati himself and by us shown, without any doubt, that even for a great number of intermediate heat exchangers ($N = 20$ or more), the effective residual heat Q_{res} is high enough to decrease the efficiency η to values below the Carnot limit η_C (ref. 7).

Of course, by numerical methods only, it is impossible to prove that for argon $Q_{res} \rightarrow 0$ with $N \rightarrow \infty$, because starting from a certain value of N , the increase in temperature between the adjacent heat exchangers becomes less than the processor rounding error, as a consequence the computer program fails to enter a loop. In any case, the physical realization of a heat recuperator, having heat exchangers so finely graduated in

temperature, is not likely. The original Vignati's argument would be taken into account merely as a *gedanken*⁴.

However, a critical re-examination of Vignati's analysis, based upon the results presented in previous sections, is now possible. At first we remark that, for $Q_{res} = 0$, the cycle is completely reversible, therefore [according to the eq. (17)] the efficiency must be equal to η_C . Otherwise we have a violation of eqs. (5) and (10) [which have been microphysically justified !] , rather than a violation of the second principle.

The key of the puzzle consists in the violation of eq. (26). In the limit $N \rightarrow \infty$ the condition $Q_{res} = 0$ is not reached, because at least one of the following conditions occurs :

- (1) The temperature of some heat exchanger is not stationary and a heat exchange between the isobaric processes and external heat sources is requested in order to guarantee the thermal equilibrium.
- (2) The heat recovery does not cover the whole range of temperature (T_{min}, T_{max}); in other words, the cycle does not close itself. So, the cycle really does not exist.

In the first case the efficiency calculated, according to Vignati's prescription, as

$$\eta(Q_{res} = 0) = 1 - Q(T = T_{min})/Q(T = T_{max}) \quad (28)$$

is not the correct efficiency; in the second case, it is not the efficiency of a physically possible cycle. Obviously, no contradiction arises if this fictitious efficiency exceeds the Carnot limit.

The argument can be restated as follows. Let us consider a particular cycle with a heat input $Q(T = T_{max})$ and a heat output $Q(T = T_{min})$; we assume these amounts of

⁴ A chapter of Vignati's book (1) is dedicated to the Xu Yelin experiment (8). We do not believe this interesting experiment, recently updated (9) and independently reproposed by other research workers [it coincides with the first setup investigated in ref. (12)], really violates the second principle. In fact, let us consider a closed circuit (electromagnetically and optically insulated) wherein a non biased diode, in thermal equilibrium with the environment, is inserted. It is possible to arrange the work function of cathode and anode and their surfaces in such a manner that a net current of thermoelectrons flows from the cathode towards the anode. Therefore an electric current, maintained by the thermal interaction between the diode and the environment, flows along the circuit. However, a single thermoelectron has the same energy when it leaves the cathode and when it comes back to it; so the amount of heat exchanged between the environment and the apparatus is really null. In absence of electric resistance, the work done by the circuit is also null. In the presence of a resistor, the current flowing in it generates a voltage between cathode and anode. The single thermoelectron gains an amount of energy when it interacts with the electric field generated by the presence of the resistor, and then, it releases that energy to the resistor (eq. 1e of ref. 8). As a consequence, we have here a mere transformation of electric work in heat or in other kinds of work. Thermal fluctuations serve here just to maintain the energy distribution of thermoelectrons, without any net transfer of energy to the apparatus; there is not a conversion of environmental heat in work. In other terms, the apparatus is not a heat engine. The rather interesting fact is that the spontaneous self-organization of a disordered motion (thermal fluctuations) leading to an ordered motion (the electric current) occurs *in thermal equilibrium*. Ideally, the environment may be substituted with an insulated thermostat operating at the same temperature, so obtaining self-organization in an insulated apparatus in thermal equilibrium !

heat to be independent on the number of heat exchangers N . The cycle efficiency is then :

$$\eta = [Q(T = T_{max}) - Q(T = T_{min})] / [Q(T = T_{max}) + Q_{res}(N)] = \{ [Q(T = T_{max}) - Q(T = T_{min})] / Q(T = T_{max}) \} \{ Q(T = T_{max}) / [Q(T = T_{max}) + Q_{res}(N)] \} = \eta_0 / [1 + Q_{res}(N) / Q(T = T_{max})], \text{ (for } N > 0)$$

where $\eta_0 = [Q(T = T_{max}) - Q(T = T_{min})] / Q(T = T_{max})$. If $Q_{res}(N) \rightarrow 0$ for $N \rightarrow \infty$, then $\eta \rightarrow \eta_0$ in the same limit. But in this limit, due to the complete reversibility, the eq. (17) holds, so that $\eta_0 = \eta_C$. Then, in general, the following relation holds :

$$\eta = \eta_C / [1 + Q_{res}(N) / Q(T = T_{max})] < \eta_C$$

in agreement with the second principle. Therefore, no contradiction exists between the first and the second principle at this level.

Turning the argument on his head, we can say that a reversible cycle with an efficiency, calculated according to eq. (28), exceeding the Carnot limit, is not physically possible because the reversible limit $Q_{res}(N) \rightarrow 0$ for $N \rightarrow \infty$ does not exist for it.

For example, the cycle (27) may be realised only without recuperator or with a finite number N of heat exchangers. The numerical calculations performed with Vignati's code itself give an efficiency without recuperator which is less than η_C (in agreement with the first principle and the Clausius equality, see Appendix). For several finite values of N , the numerical calculations performed with Vignati's RECUPERF code or, in alternative, with our RECUPE code, give an efficiency below η_C (in agreement with the second principle and the Clausius inequality).

The reader is referred to Table n. 1 for a synopsis; the reported efficiency is estimated, following Vignati's procedure, as $\eta = \eta_0 / [1 + Q_{res} / Q(T = T_{max})]$, where η_0 is given by eq. (28). We have here $\eta_0 \neq \eta_C$, because the reversible limit does not exist for this cycle. No violation of the second principle [eq. (19)], even mathematically possible, is evidenced.

Table n. 1; The efficiency of the cycle (27), estimated with RECUPERF and RECUPE codes;

N	η/η_C
0	0.161
1	0.260
2	0.320
5	0.424
10	0.207
20	0.194

5. A cycle with chemical reactions

In order to eliminate the difficulties related to numerical calculations, we consider in this section a modified setup for which an analytical treatment is possible. The system model is an Ericsson engine involving an ideal gas, a recuperator being situated between two isobaric processes. One of the recuperator's heat exchangers operates at temperature T_χ between T_{min} and T_{max} .

Let us introduce a modification in the original Vignati's setup, supposing that in this heat exchanger at temperature T_χ (with $T_{min} \leq T_\chi \leq T_{max}$), and only in it, due to a suitable catalyser installed there, the inflowing gas undergoes such chemical reactions as:

- a dissociation, when it moves from T_{min} towards T_{max} ;
- a recombination, when it moves from T_{max} towards T_{min} .

The reaction is assumed to occur at constant pressure, and the reaction heat generated at $T = T_\chi$ during the dissociation process is assumed to be integrally absorbed during the recombination process occurring in the same heat exchanger at the same temperature.

Since the number of freedom degrees of the gas molecule changes during the reaction, the gas specific heat $C_p(T)$ changes as well. Hereinafter we limit our attention to a given mass of gas small enough to keep uniform its temperature and pressure during the whole cycle. This mass will amounts to n and n' moles respectively before and after the reaction occurring in the pipe at higher pressure.

We redefine $C_p(T)$ to be the heat capacity of this mass. The $C_p(T)$ curve, which is identical for both isobaric processes, is represented in Fig. 4. This curve is covered

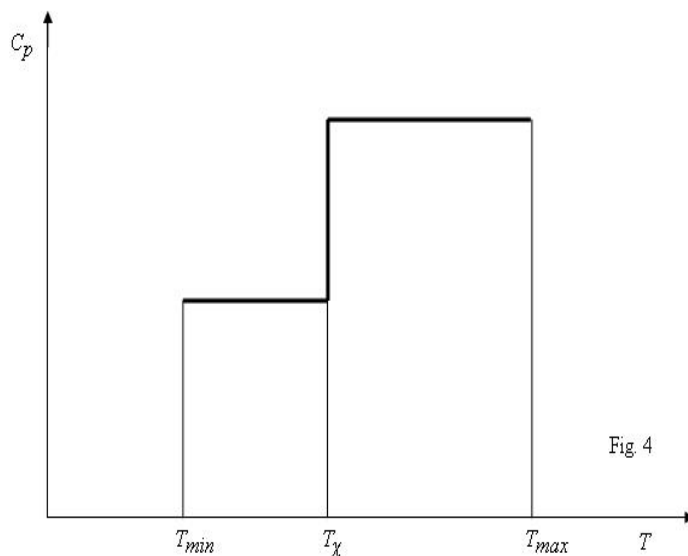


Fig. 4

from the left to the right during the isobaric process at higher pressure; while it is covered from the right to the left during the isobaric process at lower pressure. Therefore, $Q(P = P_{max}) = Q(P = P_{min})$, so that $\Delta = Q_{res}$.

The work done by the isobaric process at P_{max} may be expressed as a sum of the work

done by expanding gas , at temperature range from T_{min} to T_χ [= $P_{max} (V\chi^- - -V_1')$], the work of expansion done by the reaction [= $P_{max} (V\chi^+ - -V\chi^-)$] and the work done of expanding gas at temperature range from T_χ to T_{max} [= $P_{max} (V_1 - -V\chi^+)$]. Since :

$$P_{max}V_1 = n'RT_{max}, \quad P_{max}V_1' = nRT_{min}$$

this work is $P_{max}(V_1 - V_1') = n'RT_{max} - nRT_{min}$.

The work done by the isobaric process at P_{min} may be express as a sum of the compression work at temperature range from T_{max} to T_χ [= $P_{min} (V\chi^+ - -V_2)$], the compression work done by the reaction [= $P_{min} (V\chi^- - V\chi^+)$] and the compression work at temperature range from T_χ to T_{min} [= $P_{min} (V_2' - V\chi^-)$]. Since :

$$P_{min}V_2' = nRT_{min}, \quad P_{min}V_2 = n'RT_{max}$$

this work is $P_{min}(V_2' - V_2) = nRT_{min} - n'RT_{max}$.

The total work done by two isobaric processes of the cycle is then zero. The work produced by the cycle is expressed as the sum of the isothermal contributes only :

$$L = n' RT_{max} \ln (V_2/V_1) + n RT_{min} \ln (V_1'/V_2') .$$

The Vignati's efficiency $\eta(Q_{res}=0)$ can be expressed as⁵ :

$$\frac{L}{Q_2} = 1 + \frac{n}{n'} \frac{T_{min}}{T_{max}} \frac{\ln (V_1'/V_2')}{\ln (V_2/V_1)} = 1 + \frac{n}{n'}(\eta_C - 1).$$

Therefore $\eta(Q_{res}=0) > \eta_C$ for

$$1 - \frac{n}{n'} + \frac{n}{n'}\eta_C \geq \eta_C$$

$$1 - \frac{n}{n'} \geq \eta_C \left(1 - \frac{n}{n'} \right) .$$

The condition $n' > n$ is satisfied for a dissociation; in such a case we have $0 < 1 - n/n' < 1$, that is $\eta_C \leq 1$; this is an ever true statement. Therefore, if $n' > n$ is ever $\eta(Q_{res}=0) > \eta_C$.

Let us assume that $T_0 = T_{min}$ and let us fix a value of $T_1 \in (T_{min}, T_\chi)$. The value of integral (22) is determined and equal to the value of integral (21). The equality between two integrals [eq. (23)] is expressed as:

$$(T_1 - T_0)Cp_{max} = (T_2 - T_1)Cp_{min}$$

For temperatures in the interval (T_{min}, T_χ) we have $Cp_{max} = Cp_{min}$, so that $T_1 - T_0 = T_2 - T_1$ and, in general :

$$T_{i+1} - T_i = T_i - T_{i-1} \quad \text{for} \quad T_{min} \leq T \leq T_\chi$$

In others words, the temperature interval between two adjacent heat exchangers is constant on the segment (T_{min}, T_χ) . Denoting with N_1 the number of exchangers operating at temperature intermediate between T_{min} and T_χ one has

⁵ We pose $Q_2 = Q(T = T_{max})$, $Q_1 = Q(T = T_{min})$, $P_2 = P_{max}$, $P_1 = P_{min}$.

$$\Delta T_1 = T_1 - T_0 = (T_\chi - T_{min})/(N_1 + 1)$$

a condition which assure that the amounts α and β of the residual heat on this segment are null.

At the temperature T_χ there is the $(N_1 + 1)$ -th heat exchanger, that is $T_{N_1+1} = T_\chi$. Applying the same line of reasoning to $T > T_\chi$, we can obtain :

$$T_{N_1+2} - T_{N_1+1} = T_{N_1+3} - T_{N_1+2}$$

and in general :

$$T_{i+1} - T_i = T_i - T_{i-1} \quad \text{for} \quad T_\chi \leq T \leq T_{max}$$

By denoting with N_2 the number of heat exchangers operating in the temperature interval (T_χ, T_{max}) one has

$$T_{N_1+2} - T_{N_1+1} = (T_{max} - T_\chi)/(N_2 + 1)$$

a condition which assure that the amounts α and β of the residual heat on this segment are null.

We have, therefore, N_1 heat exchangers operating in the interval (T_{min}, T_χ) , one heat exchanger operates at $T = T_\chi = T_{N_1+1}$ wherein the reactions occur and N_2 heat exchangers operate in the interval (T_χ, T_{max}) ; the total number of heat exchangers is

$$N = N_1 + N_2 + 1$$

The heat exchanger wherein the reactions occur absorbs a heat amount $(T_{N_1+2} - T_{N_1+1})C_p(T > T_\chi)$ from the cooling gas; it releases a heat amount $\Delta T_1 C_p(T < T_\chi)$ to the heating gas. In order to make T_χ value stationary, these amounts must be equal, so that :

$$T_{N_1+2} - T_{N_1+1} = T_{N_1+2} - T_\chi = \frac{C_p(T < T_\chi)}{C_p(T > T_\chi)} \Delta T_1$$

The uncompensated residual heat that the gas absorbs from the heater at T_{max} therefore is:

$$Q_{res} = C_p(T > T_\chi)(T_{N_1+2} - T_\chi) = C_p(T < T_\chi) \Delta T_1$$

It is easy to verify that $Q_{res} \rightarrow 0$ for $\Delta T_1 \rightarrow 0$, that is for $N_1 \rightarrow \infty$ or $N_2 \rightarrow \infty$. The situation of perfect heat recovery may be indefinitely approximated. The eq. (26) is then ever satisfied.

Therefore, the condition $Q_{res} \rightarrow 0$ involving a perfect (and reversible) heat recovery is ever, at least ideally, achievable. If it is actually achieved, we have a cycle with an efficiency $\eta = \eta(Q_{res} = 0) > \eta_C$.

However, this efficiency takes in account only the pV work. The proposed cycle can not do mechanical pV work only; indeed, the total increase of entropy along it is :

$$Q_2/T_{max} - Q_1/T_{min} + (\Delta S)_{dissociation} + (\Delta S)_{recombination} = 0$$

where :

$$Q_2 = n'RT_{max}\ln(P_1/P_2), \quad Q_1 = nRT_{min}|\ln(P_1/P_2)|$$

The isobaric sections do not contribute in the reversible limit, because their thermal exchange is compensated. Therefore :

$$Q_2/T_{max} - Q_1/T_{min} = (n' - n)R|\ln(P_1/P_2)| > 0 \quad , \text{ because } n' > n$$

Of consequence :

$$(\Delta S)_{dissociation} + (\Delta S)_{recombination} < 0$$

But :

$$(\Delta G)_{dissociation} = (\Delta H)_{dissociation} - T_\chi(\Delta S)_{dissociation}$$

$$(\Delta G)_{recombination} = (\Delta H)_{recombination} - T_\chi(\Delta S)_{recombination}$$

$$(\Delta H)_{dissociation} = -(\Delta H)_{recombination}$$

so that :

$$(\Delta G)_{dissociation} + (\Delta G)_{recombination} = -T_\chi[(\Delta S)_{dissociation} + (\Delta S)_{recombination}] > 0$$

Therefore, the chemical processes of gas dissociation and recombination request an external (not pV) work. This work must be supplied from the cycle. The maximum work requested amounts to $T_\chi [(\Delta S)_{dissociation} + (\Delta S)_{recombination}]$.

The (minimum) useful work done by the cycle is then :

$$L' = L + T_\chi[(\Delta S)_{dissociation} + (\Delta S)_{recombination}] = L + T_\chi[Q_1/T_{min} - Q_2/T_{max}]$$

and the effective efficiency is :

$$\eta' = L'/Q_2 = (Q_2 - Q_1)/Q_2 + (T_\chi/Q_2)[Q_1/T_{min} - Q_2/T_{max}] =$$

$$1 - (T_\chi/T_{max}) + (Q_1/Q_2)(T_\chi/T_{min} - 1) =$$

$$1 - (T_\chi/T_{max}) - (1 - \eta)(1 - T_\chi/T_{min}) =$$

$$T_x[(1 - \eta)/T_{min} - 1/T_{max}] + \eta = (T_x/T_{min})[1 - \eta - T_{min}/T_{max}] + \eta =$$

$$(T_x/T_{min})(\eta_C - \eta) + \eta = \eta - |\eta - \eta_C|/T_{min}T_x$$

We can see that η' decreases when T_x increases. It is $\eta' > \eta_C$ when $T_x < T_{min}$, a condition which is never satisfied. Therefore, it is $\eta' < \eta_C$, a result which is still in agreement with the second principle of thermodynamics.

Conclusions

The efficiency of a perfectly reversible cycle can be inferred from the first principle and the Clausius equality, without relation with the second principle. The anomalous efficiencies, calculated by Vignati assuming the reversible limit $Q_{res} = 0$ for particular cycles with gas argon, may be explained as referred to actually forbidden cycles.

For finite values of N and $Q_{res} \neq 0$, the efficiencies calculated according to the Vignati's prescription (Table n. 1) give results in agreement with the second principle, so that no contradiction arises with the usual laws of thermodynamics. This conclusion is strengthened by an analytical calculation on cycles involving chemical reactions.

Appendix

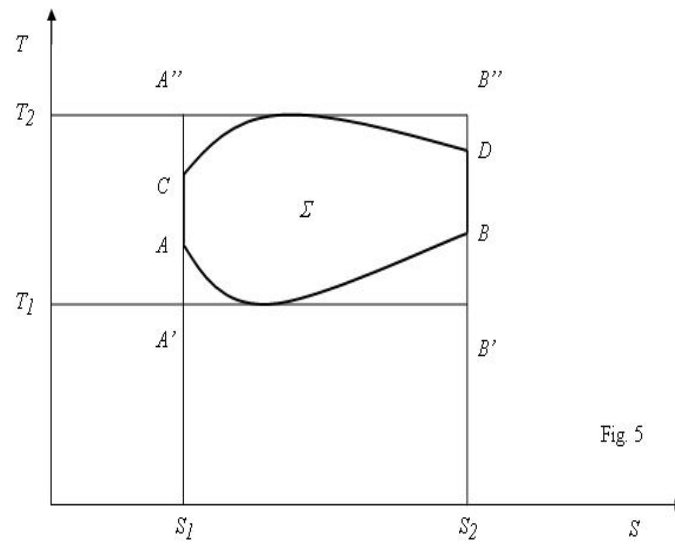
Let us consider a cycle represented by a closed curve on the plane TS (Fig. 5). Each point of this curve represents a state in equilibrium with an external reservoir depending on the point, because the temperature $T = T_{sys} = T_{res}$ is a function of cycle point. Therefore, the system exchanges heat with a sequence of external sources which repeats by itself at each cycle.

Under these conditions, the eq. (10) holds and dQ is the infinitesimal element of area of the plane TS . So, if no internal heat recovery occurs, the area Σ enclosed in the cycle is the total amount of heat, Q , exchanged with the external reservoirs:

$$Q = \text{area } \Sigma, \text{ if no internal heat recovery occurs.}$$

Therefore, referring to Fig.5, denote with Q_2 , the area under the upper branch CD and with Q_1 the area under the lower branch AB , then the quantity of exchanged heat is $Q_2 - Q_1$. According to the first principle, this heat is equal to the work L done by the cycle; the efficiency $\eta = L/Q_2$ is then expressed as $(\text{area } \Sigma)/Q_2$. As one may verify from Fig. 5, abscissa and ordinate extremes, T_1, T_2, S_1, S_2 , have once been fixed, Q_1 is minimum for $A \equiv A', B \equiv B'$. In this case, Q_2 is maximum for $C \equiv A'', D \equiv B''$. The maximum of the efficiency $\eta = (Q_2 - Q_1)/Q_2 = 1 - Q_1/Q_2$ is then obtained, when the cycle coincides with the Carnot rectangle $A'B'B''A''$. Its value is :

$$\eta_{\max} = \frac{(S_2 - S_1)(T_2 - T_1)}{(S_2 - S_1)T_2} = 1 - \frac{T_1}{T_2} = \eta_C$$



Therefore, if no internal heat recovery occurs, the Carnot theorem may be inferred, for a perfectly reversible cycle, from eqs. (5), (10).

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