EJTP **3, No. 13** (2006) 71–97 *Electronic Journal of Theoretical Physics*

Some Important Features of Ultra-Light Particles, Induced Cosmological Constant and Massive Gravitons in Modern Cosmology Theories

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Received 16 June 2006, Accepted 17 October 2006, Published 20 December 2006

Abstract: Some important features of ultra-light masses and induced cosmological constant implemented in Einstein gravity theory from supergravities arguments and non-minimal coupling effects are presented and discussed in some details in modern cosmology where massive gravitons are taken into account.

Keywords: Ultra-Light Particles, Induced Cosmological Constant, Gravitons, Accelerated Expansion, Relativistic Theory of Gravity, Brane, Cyclic Universe
PACS (2006): 04.50.+h, 98.80.Es, 95.36.+x

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1. Introduction

The recent discovery of dark energy as a theoretical explanation of the accelerated expansion of the Universe is a big unattended surprise in cosmology. In fact, the recent observations of the Type Ia supernovae have led to the idea that our universe undergoes accelerated expansion at the present epoch tending to a flat de-Sitter space-time as predicted by inflation theory [1,2,3]. Theoretical physics explain this fact by the presence of a non-luminous or dark matter known as "quintessence or Λ-field" with negative pressure obeying the state equation $w = p/\rho < -1/3$ and accounting for the missing energy if one really believes inflation theory in all its aspects predicting $\Omega = 1$ [4,5,6]. In reality, this strange matter present in the early phase of the Universe adds to the cosmological constant problem (the lambda problem) another ones. Several theoretical models, theories and possible solutions have been proposed, including a cosmological constant, a time-varying energy density, real and complex scalar fields, dilaton from string theory, supersymmetric exotic particles, massive neutrinos, holographic dark energy, $N = 8$ and $N = 2$ supergravity, M/string theory, etc [7,8,9,10,11]. In most of these theories, we still find difficulties to solve the cosmological constant problem, to find a suitable dynamical theoretical scenario generating a small "lambda" and explain at the same time the recent astrophysical observed parameters. In reality, there exists observational evidence where $w < -1$ (tachyonic or phantom dark energy) corresponding to a very fast acceleration or superacceleration [12,13]. It is highly important to know precisely which principles and observations restrictions we need to follow and obey. We believe that minimal coupling theories between the scalar field the scalar Ricci curvature cannot achieve such a superacceleration regime, especially with time-dependent state equation, but can be achieved in models containing only a real (or complex scalar field), non-minimally coupled to the Ricci curvature and with positive definite kinetic energy density [14,15,16]. Note that non-minimal couplings can in general be constrained by classical tests of gravity theories [17]. Recently, we have investigated a particular cosmological model with complex scalar self-interacting inflation field non-minimally coupled to gravity, based on supergravities argument [18]. It was shown that in the case of non-minimal coupling between the scalar curvature and the density of the scalar field such as $L = -(\xi/2)\sqrt{-g}R\phi\phi^*$ ($R$is the scalar curvature or the Ricci scalar,ξ is the non-minimal coupling constant and g is the scalar metric) and for a particular scalar complex potential field $V(\phi\phi^*) = (3m^2/4)(\zeta\phi^2\phi^{*2} - 1)$,($\zeta = O(1)$), inspired from supergravity inflation theories, ultra-light masses m ($|m^2| \approx H^2$) are implemented naturally in the Einstein field equations (EFE), leading to a cosmological constant Λ in accord with observations. The metric tensor of the spacetime is treated as a background and the Ricci scalar in the non-minimal coupling term, regarded as an external parameter, was found to be $R = 4\Lambda - 3m^2 = 4\bar{\Lambda}$where $\bar{\Lambda} = \Lambda - (3m^2/4)$ is the effective cosmological constant. In fact, the property $|m^2| \sim H^2$ is required in many theoretical quintessence models based on extended supergravity. These ultra-light masses occur in supergravity quintessence model with de-Sitter minimum and have important additional features in astrophysics, cosmology, the early universe and the standard electroweak model [19,20,21,22,23,24]. Adopting this result, we explored in a recent paper some of the consequences concerning their presence in the field equations. Briefly talking, when we ignore all gauge coupling, the Klein-Gordon equation (KGE) with no mass term associated with this choice reads $\Box\phi - (R/6)\,\phi - \partial V/\partial\phi^* = 0$. In fact, a possible mass term for the scalar field and the cosmological constant are embodied in our quartic complex potential $V(\phi\phi^*)$ and our scalar curvature$R = 4\Lambda - 3m^2$. By substituting into the last equation, we obtain $\Box\phi - M^2\phi = 0$ where $M^2 = \tilde{M}^2 + 2h\phi\phi^*$, $\tilde{M}^2 = (2\Lambda/3) - (m^2/2) = \xi R$ and $h = (3/4)\,\zeta m^2 > 0$. If we treat the Ricci scalar as a parameter, the complex potential takes the following form $\tilde{V}(\phi\phi^*) = (2\Lambda\xi - (3m^2/2)\,\xi)\,\phi\phi^* + (h/2)\,(\phi\phi^*)^2$ which can be written in a real-form as $V(\varphi) = (1/2)\,\mu^2\varphi^2 + (\lambda/4)\,\varphi^4 \equiv (\lambda/2)\,(\varphi^2 - v^2)^2 + V_0$ assuming $\phi(t) = \varphi(t)\exp(i\theta(t))$ where $\mu^2 = (4\Lambda - 3m^2)\,\xi$, $\lambda = (3/2)\,\zeta m^2 > 0$, $\mu^2 = -\lambda v^2$ and $V_0 = -\mu^4/4\lambda \equiv -(4\Lambda - 3m^2)^2\,\xi^2\Big/4\lambda$. The point $\varphi = \mathrm{v}$ corresponds to the minimum of the negative potential with of course a spontaneous symmetry breaking (SSB). In our model, the presence of ultra-light particles is responsible for inducing the SSB provided that $4\Lambda < 3m^2$ and $\xi > 0$. Note that both constraints have no effects on the sign of V_0. For the particular case $4\Lambda = 9m^2$ and $\zeta = 0$ we have$R = 6m^2 > 0$, and as a result we can obtain $(\Box - R/6)\,\phi = 0$. The Higgs field is then supplanted by a massless KGE conformally coupled to the scalar curvature with tiny real rest mass. Consequently, for $h = \Lambda = 0$, the effective cosmological constant is negative, SSB still occurs and is controlled by the presence of the term m^2. The major feature of our Mexican potential is its dependence on the scalar curvature and on the non-minimal coupling parameter. When the scalar curvature is zero or $4\Lambda = 3m^2$, the potential vanishes whatever is the sign of ξ. That is the universe is flat without inflation. But in reality, SSB occurs if $4\Lambda < 3m^2$, that is we are dealing with a negative potential unless $\xi < 0$. We are in fact interested in the following two cases: ($\xi > 0, 4\Lambda < 3m^2$) and ($\xi < 0, 4\Lambda < 3m^2$) corresponding for SSB in the first evolution stage of the universe. This would imply that we live in an open Friedmann-Robertson-Walker (FRW) accelerating universe with negative spatial curvature and positive cosmological constant[25]. The resulting field equations for a matter free universe but dominated with dark energy scalar field give the Ricci scalar for zero index curvature as $R = 6\left(\dot{H} + H^2\right) = 4\Lambda - 3m^2 = 4\bar{\Lambda}_-$ or in the following form $\dot{H} + H^2 = 2\bar{\Lambda}_-\big/3 \equiv \tilde{V}_-$. This is a Riccatti equation indicating the possibility of a time or Hubble-dependent effective cosmological constant[26-32]. Another interesting case corresponds to $\tilde{V} > 0$ and positive scalar curvature. This is achieved when we deal with negative potential and negative cosmological constant, e.g. $V(\phi\phi^*) = -(3m^2/4)\,(\zeta\phi^2\phi^{*2} - 1)$ and $\Lambda < 0$. This case is favoured by string theorists. In this way $R = 6\left(\dot{H} + H^2\right) = -4\hat{\Lambda} + 3m^2, \hat{\Lambda} = -\Lambda$ and $\dot{H} + H^2 \equiv \tilde{V}_+$. ($X_\pm$ corresponds to whether X is positive or negative respectively). In fact, letting $r^2 = \tilde{V}_+$, the solution is given by $H(t) = r(e^{rt} - ke^{-rt})/(e^{rt} + ke^{rt})$, $k$is a constant[33]. Let us observe that as time becomes infinite, the Hubble parameter approaches the limiting value r, that is $H = \sqrt{\tilde{V}_+}$ implying $3H_\infty^2 = 2\left(3m^2 - 4\hat{\Lambda}\right), \Lambda < 0, 3m^2 > -4\Lambda$. In this way, $m \approx H_\infty$

as expected from sueprgravities theories[5]. The new potential $\tilde{V}$is negative and this will lead to a contraction[34]. Therefore, we can write $H\,(t) = H_\infty(e^{rt} - ke^{-rt})/(e^{rt} + ke^{rt})$. If at the origin of time, $H\,(t) \equiv H_0$, then $k = (1-\theta)/(1+\theta)$, $\theta \equiv H_0/H_\infty$. In this case, the solution can be written in the following form $H\,(t) = H_\infty(\theta + k\tanh rt)/(1+\theta\tanh rt)$. The corresponding scale factor $a\,(t)$ evolves as $a\,(t) \propto (\cosh rt + \theta\sinh rt)^{H_\infty/r}$assuming that at the origin of time $a = a_0$, e.g. a non-singular Universe.[2*] A negative cosmological constant may be consistent with recent astronomical observations if the present accelerating is generated by another dark energy component [35].

Motivated by these results, we present in this paper some additional important cosmological and astrophysical features and implications of the ultra-light masses and the induced cosmological constant. The paper is organized in independent sections as follows: in section **2**, we will explore the evolution of a homogeneous flat universe but with positive scalar curvature dominated by ultra-light matter and the cosmological constant and we investigate about the necessary condition for eternal accelerated expansion with no need of any form of tachyonic matter. In section **3**, some important aspects and features of a cosmological model with phenomenological decay of the ultra-light masses, the vacuum cosmological constant and the matter density are discussed. In section **4**, we use extended supergravities theories to show the important role playing by the decaying gravitons masses and decaying quantized ultra-light masses in the evolution Universe within the framework of the Relativistic Theory of Gravitation (RTG). In section **5**, we proposed a modification of the Standard Hot Big Bang Cosmology (*SHBBC*), in which the Universe is flat with a total energy density taken to be the sum of the contributions from vacuum and ultra-light mass term responsible of the dominant driver of expansion at a late epoch of the Universe. In section **6** we illustrate the modification of the gravitational wave propagation through a special vacuum medium with a total density being the sum of the true vacuum density and the ultra-light particle supergravity density and finally conclusions are given in section **7**.

2. Supergravity, Ultra-light Masses and New Condition for Eternal Accelerated Expansion

As we mentioned in the previous paragraph, it is a matter of fact that the discovery of the accelerated expansion of the universe plays an important and leading role in modern cosmological theories [1,2]. After a lot of investigations, the dark energy seems for the majorities the responsible. Of course, one can deal with dark energy in a polite and soft way including the celebrated cosmological constant and quintessence [35]. Within the same context and within the framework of standard Friedmann-Roberston-Walker (**FRW**) cosmology, the resulting dynamical equations for an homogeneous flat universe dominated

[2] **The possibility that we are living in a non-singular accelerating Universe with positive scalar Ricci curvature generated from negative extended supergravity potential is an interesting idea, in particular for string theorists. It may have additional interesting features that are under progress.*

by the cosmological constant and the ultra-light masses are[18,19,20,21,23,24,36,37]:

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda}{3}\left(1 + \frac{3m^2}{\Lambda}\right) \tag{1}$$

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3}\left(1 - \frac{3m^2}{2\Lambda}\right) \tag{2}$$

We know from inflation scalar theory that $m^2/\Lambda \approx \dot{\phi}$, $\dot{\phi} \equiv d\phi/dt$ being the derivative with respect to time of the inflaton scalar field [34]. For this, we feel motivated to define $\dot{F}^2 = \alpha m^2/\Lambda$,$F$ represents our new field and α is a positive constant. That is

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda}{3}\left(1 + 3\frac{\dot{F}^2}{\alpha}\right) \tag{3}$$

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3}\left(1 - \frac{3}{2}\frac{\dot{F}^2}{\alpha}\right) \tag{4}$$

The expansion is accelerated if $\ddot{a}/a > 0$, that is $m^2/\Lambda \equiv \dot{F}^2\big/\alpha < 2/3$. As a result, the universe is flat but with positive scalar curvature. In fact, one can recover equations (3) and (4) from another alternative by assuming that the cosmos is dominated by a special fluid with density and pressure behaving like:

$$\rho = \frac{\Lambda}{8\pi G}\left(1 + 3\frac{\dot{F}^2}{\alpha}\right) \tag{5}$$

$$\rho + 3p = -\frac{\Lambda}{4\pi G}\left(1 - \frac{3}{2}\frac{\dot{F}^2}{\alpha}\right) \tag{6}$$

The negative fluid pressure is easily found to be independent of $\dot{F}$, that is $p = -\Lambda/8\pi G$ ($\Lambda > 0$) and from equation (5) $p = -\rho\Big/\left(1 + 3\dot{F}^2\right)$ or:

$$p = -\frac{\rho}{1 + 3\frac{m^2}{\Lambda}} \equiv \omega\rho \tag{7}$$

where $\omega = -1/(1 + 3m^2/\Lambda) = -1/(5 - R/\Lambda) \approx -1/(5 - R/\gamma H^2)$ (*where* $R = 4\Lambda - 3m^2$ *and* $\Lambda \equiv \gamma H^2$, $0 < \gamma \leq 3$). The dependence of ω on the scalar curvature is a remarkable feature. Equation (7) tells us that for $m^2 = 0$ or $\dot{F} = 0$, $p = -\rho$ or $\omega = -1$. From equation (6), $\dot{F}$ reaches its limiting value at $\dot{F}^2 = 2\alpha/3$. In this case, $2\Lambda = 3m^2$ and $p = -\rho/3$ or $w = -1/3$. That is $-1 \leq w \leq -1/3$ or in other words, we have eternal accelerated expansion. That is the flat universe with positive scalar curvature filled with fluid having negative pressure, positive density and positive kinetic energy is accelerating with time with no need of tachyon matter. If for our massive field $m^2 << \Lambda \sim H^2$, than $p \approx -\rho\,(1 - 3m^2/\Lambda)$. In this way, $w \approx 3m^2/\Lambda - 1 > -1$. A remarkable feature of equations (3) and (4) is the presence of some similarity with Sen tachyon matter cosmology [36,37]. In fact, letting

$$V_{\text{effective}}(F) \equiv \tilde{V}(F) = \frac{\Lambda_{\text{effective}}}{8\pi G} \tag{8}$$

$$\Lambda_{\text{effective}} = \Lambda \left(1 + 3\frac{\dot{F}^2}{\alpha}\right)\sqrt{1 - \frac{\dot{F}^2}{\alpha}} \tag{9}$$

$$= \Lambda \left(1 + 3\frac{m^2}{\Lambda}\right)\sqrt{1 - \frac{m^2}{\Lambda}} \tag{10}$$

$$= \Lambda \left(1 + 3\mu H^2\right)\sqrt{1 - \mu H^2} \tag{11}$$

$$= \left(3H^2 + 2\dot{H}\right)\left(1 + 3\mu H^2\right)\sqrt{1 - \mu H^2} \tag{12}$$

where $\tilde{V}(F)$ is the effective potential of the field F and μ is assumed to be a positive constant, equations (3) and (4) takes the form:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\frac{\tilde{V}(F)}{\sqrt{1 - \frac{\dot{F}^2}{\alpha}}} \tag{13}$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\frac{\tilde{V}(F)}{\left(1 + 3\frac{\dot{F}^2}{\alpha}\right)\sqrt{1 - \frac{\dot{F}^2}{\alpha}}}\left(1 - \frac{3}{2}\frac{\dot{F}^2}{\alpha}\right) \tag{14}$$

The cosmological constant in contrast to Sen's model is not set equal to zero. Equation (13) is the Friedman equation in the standard form within the framework of Sen tachyon field (α=1) and equation (14) is the Raychaudhuri equation within the same framework but with the presence of the term $1 + \left(3\dot{F}^2\big/\alpha\right)$. In this way, the accelerated cosmos expansion ($m^2/\Lambda \equiv \dot{F}^2\big/\alpha < 2/3$, $\Lambda > 0$) is dominated by a special fluid with density and pressure as:

$$\rho = \frac{\tilde{V}(F)}{\sqrt{1 - \frac{\dot{F}^2}{\alpha}}} \tag{15}$$

$$p = -\tilde{V}(F)\frac{1}{\left(1 + 3\frac{\dot{F}^2}{\alpha}\right)\sqrt{1 - \frac{\dot{F}^2}{\alpha}}} = -\frac{\rho}{1 + 3\frac{\dot{F}^2}{\alpha}} \equiv w\rho \tag{16}$$

with

$$\rho + p = \frac{\tilde{V}(F)}{\sqrt{1 - \frac{\dot{F}^2}{\alpha}}}\left(\frac{3\frac{\dot{F}^2}{\alpha}}{1 + 3\frac{\dot{F}^2}{\alpha}}\right) \tag{17}$$

where again $w = -1/(1 + 3m^2/\Lambda)$. The entropy conservation of our special fluid will be:

$$\frac{d}{dt}\left(\frac{\tilde{V}(F)}{\sqrt{1 - \frac{\dot{F}^2}{\alpha}}}\right) + 3\frac{\dot{a}}{a}\left(\frac{\tilde{V}(F)}{\sqrt{1 - \frac{\dot{F}^2}{\alpha}}}\left(\frac{3\frac{\dot{F}^2}{\alpha}}{1 + 3\frac{\dot{F}^2}{\alpha}}\right)\right) = 0 \tag{18}$$

and the field energy decreases with time as it is expected due to the universe accelerated expansion. In summary, we have investigated the possibility of having accelerated eternal expansion without implementing any form of tachyonic matter as long as the following condition $m^2/\Lambda < 2/3$ holds. As a result, the Ricci scalar curvature is positive. The cosmos is assumed to be filled with a special fluid with positive cosmological constant, positive energy density and negative energy pressure obeying the particular state equation $p = w\rho$, where $-1 \leq w = -1/(1 + (3m^2/\Lambda)) \leq -1/3$. This indicates how importance is the presence of the ultra-light masses implemented in our theory from supergravity and non-minimal coupling arguments as necessary ingredients in modern cosmology.[3*]

3. Cosmic Acceleration with Two Decaying Cosmological Constants

The unified modern gauge field theories inform us that it is impossible to neglect the Einstein cosmological constant "Λ" introduced by Albert Einstein in the search for static cosmological solutions to his General Relativity Theory. This famous Lambda is so tiny but in the context of high energy physics, it is not really the case. When quantum corrections are used, that is, within the context of quantum gravity, we get at Planck scale, a cosmological constant cancelled at the electroweak scale energy scale and below to one part in 10^{119} [38-44]. As a result, the problem still persists and it seems that the "*lambda*" problem is a difficult physical problem at very "*low energies scales*". There exist in literature several attempts to reduce the vacuum density to a very small value over cosmological timescales, at least from dynamical point of view. It has also been noted that spontaneous symmetry breaking and phase transitions can be induced by curvature via the non-minimal coupling with the external gravitational field [45-49]. It was shown for the simple case of a scalar field with a potential depending on the Ricci scalar or spacetime curvature that if the field starts near the origin, it will slows down and the vacuum density will decrease as inverse of the square of time ($\rho_{vac} \propto t^{-2}$). These suggest the beautiful possibility that the universe evolves to a state with time decaying effective cosmological constant. Such scenarios are interesting because *a red-shifting vacuum energy can have effects over many expansion times, while a constant vacuum density could only have become dynamically important at very recent epochs....and dynamical models of decaying vacuum energy of a rather general variety are consistent with observational cosmology; however, the deviation from the standard model must be small*[44]. Particle creation of light nonminimally coupled scalar fields due to the changing geometry of a spacetime at an early inflationary phase was also treated and discussed in details by several authors leading to a total density parameter $\Omega_T \approx 1$ [50-55]. Inspired from the relation between the cosmological constant, the ultra-light masses and the Ricci scalar, we assume that in

[3] * *An important issue to treat in a future work is the impact of all these in string theory, the connection to tachyon field theory as well as the implication of the effective cosmological constant represented by equation (12) in standard* ***FRW*** *cosmology.*

addition to the ordinary matter and radiation, there is a time-dependent energy density associated with the ultra-light masses, say $\rho_m(t) \equiv 3m^2(t)/8\pi G$ and $\Lambda = \Lambda(t)$ where $m(t)$ represents the ultra-light masses, assuming at first that the gravitational constant "G" is constant and that the ultra-light pressure is zero. This additional or extra energy density must certainly appear in the Friedman dynamical equation (*solution of the Einstein's field equations*) describing the "*flat*" FRW standard cosmology as follows:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\rho + \frac{3m^2}{8\pi G}\right) + \frac{\Lambda}{3} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} + m^2 \tag{19}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3m^2}{8\pi G} + 3p\right) + \frac{\Lambda}{3} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} - \frac{m^2}{2} \tag{20}$$

"p" and "ρ" are the pressure and density of the perfect fluid with equation of state assuming through this work to be $p = (\tilde{\gamma} - 1)\rho$, "$\tilde{\gamma} \equiv w + 1$" being a constant. Equations (19) and (20) give:

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p + \Lambda \tag{21}$$

$$\frac{d(\rho a^3)}{dt} + p\frac{d}{dt}\left(a^3\right) + \frac{a^3}{8\pi G}\frac{d}{dt}\left(\Lambda + 3m^2\right) = 0 \tag{22}$$

The goal of this work is to study the effect of the presence of phenomenological decaying ultra-light masses on cosmic acceleration. For this we follow Arbab, Ozer, Lima, etc., and we consider the following three phenomenological laws: $m^2(t) = \varepsilon\dot{a}^2/a^2$, $\Lambda(t) = \beta\ddot{a}/a$ and $\rho(t) = (3\delta/8\pi G)\,\dot{a}^2/a^2$, ε,β and δ are constants[56-66]. Remember that the FRW Ricci scalar contains both terms $\dot{a}^2/a^2$ and $\ddot{a}/a$[67]. Equation (21) yields:

$$(2-\beta)\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\left(1 + 3\delta(\tilde{\gamma}-1)\right) = 0 \tag{23}$$

with the general solution:

$$a(t) = \left(C\frac{\beta - 3 - 3\delta(\tilde{\gamma}-1)}{\beta - 2}t\right)^{\frac{\beta-2}{\beta-3-3\delta(\tilde{\gamma}-1)}} \tag{24}$$

C is a constant, $\beta \neq 2$. Note that in this scenario, the scale factor is independent of the ultra-light masses parameter ε. It follows that:

$$\Lambda(t) = \frac{(\beta-2)(\beta+6\delta(\tilde{\gamma}-1))}{(\beta-3-3\delta(\tilde{\gamma}-1))^2}\frac{1}{t^2} \tag{25}$$

$$m^2(t) = \frac{\varepsilon(\beta-2)^2}{(\beta-3-3\delta(\tilde{\gamma}-1))^2}\frac{1}{t^2} \tag{26}$$

$$\rho(t) = \frac{3\delta}{8\pi G}\frac{(\beta-2)^2}{(\beta-3-3\delta(\tilde{\gamma}-1))^2}\frac{1}{t^2} \tag{27}$$

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1+3\delta(\tilde{\gamma}-1)}{2-\beta}, \beta \neq 2 \tag{28}$$

$$H \equiv \frac{\dot{R}}{R} = \frac{\beta - 2}{\beta - 3 - 3\delta\left(\tilde{\gamma} - 1\right)}\frac{1}{t} \tag{29}$$

Equation (28) represents the deceleration parameter.

- For the matter dominated universe, $\tilde{\gamma} = 1$, than for $\beta > 2$, $q < 0$, $\Lambda > 0$, $m^2 = (\varepsilon\left(\beta - 2\right)/\beta)\,\Lambda > 0$, the density parameter of the universe $\Omega_M = \rho/\rho_c = \delta$ ($\rho_c = 3H^2/8\pi G, H \equiv \dot{a}/a$ *are respectively the critical density and the Hubble parameter*), the density parameter of the Λ-vacuum $\Omega_\Lambda = \Lambda/3H^2 = (\beta/3\left(\beta - 2\right))$, the density parameter of the m^2-vacuum defined as $\Omega_m = m^2/H^2 = \varepsilon$. The total density parameter defined as $\Omega_T = \Omega_M + \Omega_\Lambda + \Omega_m$. In our scenario, the ultra-light masses vary as $m^2\left(t\right) = \varepsilon(\beta - 2)/(\beta - 3)t^2$. In order to have $m^2 > 0$, we need to have $\varepsilon > 0$ and $\beta > 3$. As a result for:

$\beta = 3.5, q = -2/3, \Omega_M^0 = \delta, \Omega_\Lambda^0 = 7/9, \Omega_m^0 = \alpha, \Omega_T^0 = \delta + 7/9 + \varepsilon \Rightarrow \delta + \varepsilon \approx 2/9$so that $\Omega_T^0 \approx 1$
$\beta = 4, q = -0.5, \quad \Omega_M^0 = \delta, \Omega_\Lambda^0 = 2/3, \Omega_m^0 = \alpha, \Omega_T^0 = \delta + 2/3 + \varepsilon \Rightarrow \delta + \varepsilon \approx 1/3$for $\Omega_T^0 \approx 1$
$\beta = 5, q = -0.33, \quad \Omega_M^0 = \delta, \Omega_\Lambda^0 = 5/9, \quad \Omega_m^0 = \alpha, \Omega_T^0 = \delta + 5/9 + \varepsilon \Rightarrow \delta + \varepsilon \approx 4/9$ for $\Omega_T^0 \approx 1$
$\beta = 6, q = -0.25, \Omega_M^0 = \delta, \Omega_\Lambda^0 = 1/2, \quad \Omega_m^0 = \alpha, \Omega_T^0 = \delta + 1/2 + \varepsilon \Rightarrow \delta + \varepsilon \approx 1/2$ for $\Omega_T^0 \approx 1$

(*The subscript* 0 *denotes the present value of the quantity*). Arbab suggests the value $\beta = 5$ as the best fit-value with the age of the universe [57]. In our scenario, for this value of "β", the universe is accelerating with time with a positive decaying cosmological constant, positive decaying ultra-light masses and positive decaying matter density accompanied with $\Omega_T^0 \approx 1$, $\delta \in (0.3 \pm 0.1)$ and $\varepsilon \approx 4/9 - \delta > 0$. The age of the universe is found to be from equation (29) to be larger than the standard model. If $\beta = 3$, we fall into the inflationary regime.

- For the radiation dominated universe, $\tilde{\gamma} = 4/3$ yielding from the above equations and requirements:

$$\Lambda\left(t\right) = \frac{\left(\beta - 2\right)\left(\beta + 2\delta\right)}{\left(\beta - 3 - \delta\right)^2}\frac{1}{t^2} \tag{30}$$

$$m^2\left(t\right) = \frac{\varepsilon\left(\beta - 2\right)^2}{\left(\beta - 3 - \delta\right)^2}\frac{1}{t^2} \tag{31}$$

$$\rho\left(t\right) = \frac{3\delta}{8\pi G}\frac{\left(\beta - 2\right)^2}{\left(\beta - 3 - \delta\right)^2}\frac{1}{t^2} \tag{32}$$

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1 + \delta}{2 - \beta}, \beta \neq 2 \tag{33}$$

As a result, the density parameter of the universe at the radiation epoch $\Omega_M = \rho/\rho_c = \delta$, the density parameter of the Λ-vacuum $\Omega_\Lambda = \Lambda/3H^2 = ((\beta + 2\delta)/3\left(\beta - 2\right))$, the density parameter of the m^2-vacuum defined as $\Omega_m = m^2/H^2 = \varepsilon$. The total density parameter defined as $\Omega_T = \Omega_M + \Omega_\Lambda + \Omega_m > 1$.

If the value of $\beta = 5$ still holds in radiation dominated era, $\Omega_\Lambda^0 = (5 + 2\delta)/9$, $\Omega_T^0\left(radiation\right) = \varepsilon + \delta + (5 + 2\delta)/9 \Rightarrow \varepsilon + 11\delta/9 \approx 4/9$so that $\Omega_T^0\left(radiation\right) \approx 1$,

$q < 0$, $\Lambda > 0$ and $m^2 > 0$. Again this implies $\varepsilon \approx (4 - 11\delta)/9 > 0$ if $\delta < 0.36$. For $\beta = 3$, $a(t) \propto t^{-1/\delta}$. For the same value of β, but for $\tilde{\gamma} = 0$ ($p = -\rho$), we find $a(t) \propto t^{1/3\delta}$, $q = 3\delta - 1 < 0$ for $\delta < 1/3$. For this particular case, the vacuum universe is accelerating with time. Astronomical observations predict $\delta = 0.3 \pm 0.1$, that is to conclude that for the particular value $\beta = 3$, the universe will not belong to an inflationary regime as predicted by inflation cosmology. For $\beta = 5$ and $\tilde{\gamma} = 0$, $a(t) \propto t^{3/(2+3\delta)}$ with the deceleration parameter $q = (3\delta - 1)/(3) < 0$ for $\delta < 1/3$. In summary for $\beta = 5$ and $\delta < 1/3$ with $(\Lambda(t), m^2(t), \rho(t) \propto t^{-2})$:

Vacuum state: $p = -\rho$, $a(t) \propto t^{3/(2+3\delta)}$, $q < 0$
Radiation dominated epoch: $p = \rho/3$, $a(t) \propto t^{3/(2-\delta)}$, $q < 0$
Matter dominated epoch: $p = 0, a(t) \propto t^{3/2}$, $q < 0$

For the particular case $\delta = 0.25$ with $(\Lambda(t), m^2(t), \rho(t) \propto t^{-2})$:

Vacuum state: $p = -\rho, a(t) \propto t^{1.09}, q < 0$
Radiation dominated epoch:$p = \rho/3, a(t) \propto t^{1.7}, q < 0$
Matter dominated epoch: $p = 0, a(t) \propto t^{1.5}, q < 0$

For this particular case, the universe will undergo an accelerating regime when passing from the vacuum state to the radiation dominated epoch, continue its acceleration with time but with a tiny decrease of its accelerating expansion. If the gravitational constant is assumed to vary with time, than for the case of matter dominated era, one can easily prove that $G(t) \propto t^{\beta/(\beta-3)}$ while in the radiation dominated era, $G(t) \propto t^{(\beta+2\delta)/(\beta-3-\delta)}$ and in the vacuum state, $G(t) \propto t^{(\beta-6\delta)/(\beta-3+3\delta)}$[57]. Again, for $\beta = 5$ and $\delta = 0.25$ with $(\Lambda(t), m^2(t), \rho(t) \propto t^{-2})$:

Vacuum state: $p = -\rho, a(t) \propto t^{1.09}, q < 0, G(t) \propto t^{2.5}$
Radiation dominated epoch: $p = \rho/3, a(t) \propto t^{1.7}, q < 0, G(t) \propto t^{3.14}$
Matter dominated epoch: $p = 0, a(t) \propto t^{1.5}, q < 0, G(t) \propto t^{1.27}$

In this case, the gravitational constant passes through different passes, it increases when passing from the vacuum state to the radiation dominated epoch, continue decreasing but with a reducing factor when passing to the matter dominated era[57,67,68,69,70]. In conclusion, for the special parameters $\beta = 5$ and $\delta = 0.25$, the universe will passes through different phases. The transition from the vacuum state to the radiation one is followed by a increasing in the gravitational constant, an increasing of the cosmic acceleration while the ultra-light masses, the cosmological constant and the density decreases as $1/t^2$. The transition from the radiation dominated epoch to the matter dominated one is also followed by increasing of the gravitational constant but with a reduced term and this reducing follows also the scale factor as well as $m^2(t), \Lambda(t)$ and $\rho(t)$. Certainly the exact choice and exact values of the parameters within the theory described need more astrophysical data.

4. Extended Supergravities and Accelerated Universe from Relativistic Theory of Gravitation with Decaying Gravitons and Quantized Ultra-Light Masses

In **RTG**, Minkowski space is the fundamental space for all physical fields. The Special Theory of Relativity (**STR**) was assumed to play the major role in the construction of **RTG** [71,72]. The Faraday-Maxwell physical gravitational field in Minkowski space is introduced and allows the notion of an energy-momentum tensor of the gravitational field to be used. This means that the gravitational field could be localize. The gravitational field in **RTG** is characterized by the curvature tensor alone. While in Einstein General Relativity (**EGR**), it is characterized by both the curvature and the 4-vector of force. The **RTG** denies the total geometrization. The gravitational field is described by a real physical field with energy-momentum density and the rest mass m and a polarization states corresponding to spin two and zero, that is a symmetric second rank tensor $\phi^{\mu\nu}$constraints to the condition $D_\mu \phi^{\mu\nu} = 0$ where D_μ is the covariant derivative in the Minkowski space. The field $\phi^{\mu\nu}$is related to the Minkowski space metric $\gamma^{\mu\nu}$via the Lagrangian density of matter according to the following rule

$$L_M\left(\tilde{\gamma}^{\mu\nu}, \phi_{matter}\right) \to L_M\left(\tilde{g}^{\mu\nu}, \phi_{matter}\right) \tag{34}$$

where $\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\phi}^{\mu\nu}$ and $\tilde{a}^{\mu\nu} = \sqrt{-g}a^{\mu\nu}, a = \{g, \gamma, \phi\}$.The effective Riemann space is produced by $\phi^{\mu\nu}$ in the Minkowski space. This means that complicated topology is excluded from RTG. The total Lagrangian density of matter and gravitational field is then given by

$$L = \frac{1}{16\pi}\tilde{g}^{\mu\nu}\left(G^\lambda_{\mu\nu}G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\rho}G^\sigma_{\nu\lambda}\right) - \frac{m^{*2}}{16\pi}\left(\frac{1}{2}\gamma_{\mu\nu}\tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma}\right) + L_{matter}\left(\tilde{g}^{\mu\nu}, \phi_{matter}\right) \tag{35}$$

where $G^\lambda_{\mu\nu} = 1/2\tilde{g}^{\mu\nu}\left(D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}\right)$ and $m^*$is the graviton mass. Using the last action principle $\delta L/\delta\tilde{g}_{\mu\nu} = \delta L_{matter}/\delta\phi_{\mu\nu}$ =0 and the fact that $\nabla_\lambda T^{\lambda\mu} = 0$ where ∇_λ is the covariant derivative, the complete system of equations is ($G = \hbar = c = 1$):

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{m^{*2}}{2}\left[g^{\mu\nu} + \left(g^{\mu\nu}g^{\nu\beta} - \frac{1}{2}g^{\nu\beta}g^{\alpha\beta}\right)\gamma^{\alpha\beta}\right] = \frac{8\pi}{\sqrt{-g}}T^{\mu\nu} \tag{36}$$

with $D_\mu \tilde{g}^{\mu\nu} = 0$. In **RTG**, the insertion of the cosmological term into the field equations (36) kills its physical and mathematical logic and structure, because the additional repulsing physical field is not affected by the presence of matter. So, we will reject the possibility of adding a cosmological constant to equation (36). It was proved that the evolution of the isotropic and homogeneous **FRW** Universe in the context of the **RTG** could be a flat $D = 4$ manifold with density greater than the critical density $\rho_c = 3H_0^2/8\pi G$

where H_0 is Hubble constant and as a result, some missing dark energy must exist. One important feature of the **RTG** is $m^* \neq 0$ and due to this, the Universe is free from singularity, evolves cyclically and with deceleration parameter greater than unity, e.g. in contrast with recent observations of the Type Ia supernovae suggesting that our universe undergoes accelerated expansion at the present epoch tending to a flat de-Sitter space-time as predicted by inflation theory. In our ultra-light masses theory, the total density can be interpreted as the sum of the ordinary matter and an ultra-light one (ρ_m) where$\rho_m = \pm 3m^2/8\pi G$, $m \approx H$ (+ for positive potential and - for negative potential). In fact, in the absence of matter, $\rho_m = 3m^2/8\pi G \equiv \rho_c = 3H_0^2/8\pi G$ for $m = H_0$. In this way, ρ_m can be interpreted as dark energy density. In de-Sitter regime where $\rho_{matter} = 0$, the time-dependent Hubble constant is $H(t) = \rho_m/3$. In this way, we have an inflation or deflation regime corresponding to whether ρ_m is positive or negative. Note that the positive sign corresponds to a positive scalar curvature unless $m^2 < 0$(tachyons; we note that negative energy density occurs also in wormholes physics). In reality a large class of supersymmetric Calabi-Yau string compactifications, have classical configurations with negative energy density, e.g. from a four dimensional perspective, there can be constrained finite regions of space-time manifolds with negative energy and positive scalar curvature on a compact Ricci flat manifold admitting a covariant constant spinor leads to negative energy density [73]. The role of positive Ricci scalar curvature is negligible at late times, but can played an important and crucial role in the early stages of the Universe [74-82]. In order to reconcile the **RTG** with recent observations, we would like to use another alternative:$D = 4$ *extended supergravities having a de Sitter solution corresponding to the extrema of the negative effective potentials* $V(\phi)$ *for some scalar fields* ϕ. An interesting results of these solutions is that the squared mass of these scalars in all theories with $N = 2$ (*extended supergravities with unstable de-Sitter (dS) vacua*) is quantized in units of the Hubble constant H_0. That is $m^2 = nH_0^2$, n are integers (in units of unity Planck Mass). It was also proved that the accelerated expansion of the Universe in the framework of **RTG** can be achieved by the introduction of weak-coupling light quintessence in the energy momentum tensor generating gauge coupling corrections. Motivated by these results, we will restudy the **RTG** with negative effective supergravity power-law potentials with maximum $V_0 = 3m^2/4$ at $\phi = 0$ and zero cosmological constant and where the field equations are:

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{m^{*2}}{2}\left[g^{\mu\nu} + \left(g^{\mu\nu}g^{\nu\beta} - \frac{1}{2}g^{\nu\beta}g^{\alpha\beta}\right)\gamma^{\alpha\beta}\right] = \frac{8\pi}{\sqrt{-g}}\left[\left(p + \rho - \frac{3m^2}{8\pi G}\right)u_\mu u_\nu + pg_{\mu\nu}\right] \tag{37}$$

ρ being the density and p the pressure. In other words, we will take into considerations the contribution of $\rho_m = -3m^2/8\pi G$.As we mentioned below, this density can be viewed positive if $m^2 < 0$ and negative if $m^2 > 0$. By applying this field equation to the flat FRW flat metric $ds^2 = -dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j =, i, j = 1, 2, 3$, the dynamical equations in matter dominated epoch are found to be($P = 0$):[70,71]

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\tilde{\rho}}{3} - \frac{m^{*2}}{6}\left(1 - \frac{1}{a^2}\right)^2\left(1 + \frac{1}{2a^2}\right) \tag{38}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\tilde{\rho}}{3} - \frac{m^{*2}}{6}\left(1 - \frac{1}{a^6}\right) \tag{39}$$

where $\tilde{\rho} = \rho - 3m^2/8\pi G$. As a result, it is easy to show that the density of the gravitational matter is

$$\rho_{m^*} = \rho - \rho_c = \frac{m^{*2}}{16\pi G}\left(1 + 6\frac{m^2}{m^{*2}}\right) \approx 2.5\rho_c\left(1 + 4\frac{nH_0^2}{\pi^2}\right) \tag{40}$$

where $\rho_c = 3H_0^2/8\pi G$, $H_0 = \dot{a}/a$ and $m^{*2} \approx (3/2)\,\pi^2 H_0^2$as deduced from the cyclic **RTG**. In reality, for the dust dominant epoch of the Universe ($\rho_{total} \propto a^{-3}(t)\,, t >> t_0$), $a(t) \propto \sin^{2/3}(\lambda t/2)$, $\lambda = 3m^*/\sqrt{6}$ with a good accuracy. Making use of equation (38), one can deduce the actual value of the deceleration parameter:

$$q_0 = -\left(\frac{\ddot{a}}{a}\right)_0 \frac{1}{H_0^2} \approx 4.2 - \frac{n}{2} \tag{41}$$

In order to have negative deceleration parameter, we need to have $n > 9$ with n integer. In this way, we have a cyclic expansion of the Universe but with $q_0 < 0$. If instead we use a positive effective supergravity potential then $q_0 = 4.2 + n/2$, not in agreement with recent astrophysical observations unless $n < 0$. In the absence of macroscopic matter, $q_0 = 2.4 - n/2$ and accelerated expansion occurs also for $n > 9$. Note that in $N = 2$supergravity having de-Sitter vacua without tachyons, near **dS** extremum all masses of scalar particles are quantized in terms of the Hubble parameter with $n = 12, -2, -6$[83]. This new cyclic cosmology provides a surprisingly new picture of the Universe history in which inflation exists. The Universe undergoes a periodic eternal sequence of Big Bangs and Big Crunches and where each bang is followed by a stage with $q_0 < 0$. This is a mysterious feature of the model explanation has to be found and need more detailed investigations. From phenomenological point of view, one may be inspired from the interesting property of scalar masses quantized in terms of cosmological constant in **dS** vacuum and explore the **RTG** with our supergravity potential (positive/negative) assuming that the ultra-light masses decays as $m^2 = \pm\omega\dot{a}^2/a^2, \omega$ is a positive/negative parameter respectively (the negative (positive) sign corresponds to the positive (negative) potential). That is:
Negative Supergravity Potential: $\tilde{\rho} = \rho - 3m^2/8\pi G, m^2 = \omega\dot{a}^2/a^2, \omega < 0$
Positive Supergravity Potential: $\tilde{\rho} = \rho + 3m^2/8\pi G, m^2 = \omega\dot{a}^2/a^2, \omega > 0$

In reality, the case where $\omega < 0$ corresponds to negative $(\text{mass})^2$ and in gauged supergravities it corresponds to AdS_4 and plays an important role in string theory, string black hole and branes [84,85]. In this way we have two possible scenarios and equations (38) and (39) are modified as:

(1) ***Negative Supergravity Potential*:**

$$(1+\beta)\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{m^{*2}}{6}\left(1 - \frac{1}{a^2}\right)^2\left(1 + \frac{1}{2a^2}\right) \tag{42}$$

$$\frac{\ddot{a}}{a} - \frac{\beta}{2}\frac{\dot{a}^2}{a^2} = -\frac{4\pi G\rho}{3} - \frac{m^{*2}}{6}\left(1 - \frac{1}{a^6}\right) \tag{43}$$

Equation (42) can be written as:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi\bar{G}\rho}{3} - \frac{\bar{m}^{*2}}{6}\left(1-\frac{1}{a^2}\right)^2\left(1+\frac{1}{2a^2}\right) \tag{44}$$

where $\bar{G} = G/(1+\omega)$ and $\bar{m}^{*2} = m^{*2}/(1+\omega)$with $-1 < \omega < 0$are the modified gravitational constant and gravitons mass. Assuming again $\rho(t) = \chi(\gamma)/a^3(t)$ with the state equation$p(t) = (\gamma-1)\rho(t)$, $\chi(\gamma)$ being integration constant depending on the evolution state of the cosmic fluid, the solution is given by:

$$a(t) \approx \sqrt[3]{\frac{32\pi G\chi(1)}{3m^{*2}}}\sin^{2/3}\left(\frac{m^* t\sqrt{3}}{2\sqrt{2(1+\omega)}}\right) \tag{45}$$

The resulting critical mass of the gravitons is then $m^{*2} \approx (3(1+\omega)/2)\pi^2 H_0^2$ and the density of the gravitational field is $\rho_{m^*} \approx \bar{m}^{*2}\big/16\pi\bar{G} \approx 2.5\rho_c$. The second Friedmann equation (43) gives:

$$\begin{aligned}\frac{\ddot{a}}{a} &= \frac{\omega}{2}\left(\frac{8\pi\bar{G}\rho}{3} - \frac{\bar{m}^{*2}}{6}\left(1-\frac{1}{a^2}\right)^2\left(1+\frac{1}{2a^2}\right)\right) - \frac{4\pi G\rho}{3} - \frac{m^{*2}}{6}\left(1-\frac{1}{a^6}\right)\\ &\approx \frac{\omega}{2}\left(\frac{8\pi\bar{G}\rho}{3} - \frac{\bar{m}^{*2}}{6}\right) - \frac{4\pi G\rho}{3} - \frac{m^{*2}}{6} \quad = -\frac{4\pi G}{3}\left(\frac{3(1+\omega)\rho_{m^*}+\rho_c}{1\pm\omega}\right)\\ &\qquad = -\frac{4\pi G\rho_c(8.5+7.5\omega)}{3(1+\omega)}\end{aligned} \tag{46}$$

and the resulting deceleration parameter is $q_0 \approx 4.2 + (7.5\omega/2)$. For $-1 < \omega < 0$, $0.45 < q_0 < 4.5$.

(2) ***Positive Supergravity Potential*:**

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi\bar{G}\rho}{3} - \frac{\bar{m}^{*2}}{6}\left(1-\frac{1}{a^2}\right)^2\left(1+\frac{1}{2a^2}\right) \tag{47}$$

$$\frac{\ddot{a}}{a} + \frac{\omega}{2}\frac{\dot{a}^2}{a^2} = -\frac{4\pi G\rho}{3} - \frac{m^{*2}}{6}\left(1-\frac{1}{a^6}\right) \tag{48}$$

where $\bar{G} = G/(1-\omega)$ and $\bar{m}^{*2} = m^{*2}/(1-\omega)$are the modified gravitational constant and gravitons mass corresponding to this special case with $\omega < 1$ and the resulting deceleration parameter is $q_0 \approx 4.2 - (7.5\omega/2)$. For $0 < \omega < 1$, $0.45 < q_0 < 4.5$ as in the previous case but with some amelioration. One may in both cases generates negative deceleration parameter if we require the presence of tachyon mass and negative gravitational constant, a special case not favored by observations but favored with cyclic string and brane theorists [84-92]. One can also interpret this scenario by stating that the negative gravity force generated in the presence of tachyons prevent the Universe from being completely crushed into a singularity [93,94]. One can reconcile these situations by assuming a time-decreasing gravitons mass as $m^{*2} \equiv \gamma\dot{a}^2/a^2 + \eta\ddot{a}/a \propto 1/t^2 \to 0$ as $t \to \infty$, γ and η are free parameters of the model. In this way, for negative and positive supergravity potentials, we find a power-law evolution of the scale factor as $a \propto t^p$

where $p = (4 + \eta)/(6 + \eta + \gamma)$independent of ω. In this way, for $\alpha < -2$and$\delta > 0$, $p > 0$ and accelerated expansion may occur. The gravitons mass evolves as $m^{*2} = ((\gamma + \eta)\, p^2 - \eta p)/t^2 > 0$for $(\gamma + \eta)\, p > \eta, p > 1, \gamma < -2$.That is there is a critical scale factor where the gravitons mass tends to zero, e.g. $a\,(t) = (C\,((\gamma + \eta)/\eta)\, t)^{(\eta+\gamma)/\eta}$. For $\gamma < -2$ and$\eta > 0$, $(\eta + \gamma)/\gamma < 1$ and the universe is not yet in the stage of accelerated expansion. From the other sight, one can have accelerated expansion if during the decaying of the gravitons masses $\gamma, \eta > 0$. In this way, the decaying of the graviton masses is responsible of the cosmic acceleration. At very later times, when all the gravitons disappear, the Universe enters a slow dying epoch where he scale factor evolves as $a \propto t^p$ with $p < 1$.

5. Accelerating Flat-Brane cosmology in the presence of Ultra-Light Masses

As we mentioned in the previous sections, the first evidence for the accelerating universe came from observations of distant supernovae. However, the data were also consistent with an open universe because the total energy density was less than the so-called critical density with a low mass density and no cosmological constant. The energy density of the universe is composed of matter (*both ordinary visible matter and invisible or dark matter*) and the energy density of the vacuum. The size of the latter, which is sometimes called quintessence or "*dark energy*" defines the cosmological constant. This new accelerating energy has a larger energy density than the mass density of the Universe. Many cosmologists and high energy physicists have explored a cosmological constant, a decaying vacuum energy and quintessence as possible explanations for such an acceleration [91,92]. In the context of quantum field theories the notion of empty space has been replaced with that of a vacuum state, defined to be the ground (*lowest energy density*) state of a collection of quantum fields. A peculiar and truly quantum mechanical feature of the quantum fields is that they exhibit zero-point fluctuations everywhere in space, even in regions, which are otherwise "empty". These zero-point fluctuations of the quantum fields, as well as other "vacuum phenomena" of quantum field theory, give rise to an enormous vacuum energy density [95]. Zero-point energies of particle physics theories cannot be ignored when gravitation is taken into account and densities are given by$\rho_{vac} \approx m^4 c^3/\hbar^3$, where "$m$" is the ultra-violet cut-off. In this section, we would like to study a radiative flat universe consisting only of vacuum energy and ultra-light densities. In others words, we take the total energy density to be the sum of two terms: the contributions from vacuum with density$\rho_{vac} \approx m^4 c^3/\hbar^3$ and the contribution from ultra-light masses with densities$\rho_m = 3m^2 c^4/8\pi G\hbar^2, m \leq \hbar H/c^2$, "$H$" is the Hubble constant. The ultra-light term may arise from self-interactions between light particles. The nature of this force is unclear, but could be interpreted as a long-range quantum fifth force. From equilibrium and statistical mechanics considerations based on the scaling of the partition function, one finds the equation of state $p = -a\rho/3$ where$F\,(r) \propto r^{a-1}$("a" is a positive number). The new model has then the attractive characteristic that quantum matter alone is sufficient to provide a flat geometry. As a first approximation, we will suppose that is of the same nature in both densities ρ_{vac} and ρ_m. The Friedman-Robertson-Walker (FRW) equation is modified by the addition of this new term and takes one of the forms:

$$H^2 = A\rho_{vac} + B\rho_{vac}^{1/2} \tag{49}$$

$$H^2 = A\rho_m + B\rho_m^2 \tag{50}$$

$A \equiv 8\pi G/3$ and B is a constant. Equation (50) appears in the general in the context of *brane cosmology* [96.97,98]. In the brane world, all matter fields and forces except gravity are localized on the 3−brane in a higher dimensional spacetime. In models where gravity is confined on the brane the Einstein's gravitational equations on the 3 brane are given by:

$$G_{\mu\nu}^{(4)}+\Lambda^{(4)}g_{\mu\nu}+C^a_{bcd}n_a n^c q^b_\mu q^d_\nu = \kappa_4^2 T_{\mu\nu}+\kappa_5^4-\frac{1}{4}\tau_{\mu\alpha}\tau^\alpha_\nu+\frac{1}{12}\tau\tau_{\mu\nu}+\frac{1}{8}q_{\mu\nu}\left(\tau_{\alpha\beta}\tau^{\alpha\beta}-\frac{\tau^2}{3}\right) \tag{51}$$

where $G_{\mu\nu}^{(4)}$ is the Einstein tensor with respect to the intrinsic metric $g_{\mu\nu}$, $\Lambda^{(4)}$ is the 4-dimensional cosmological constant given by $\Lambda^{(4)} = \kappa_5^2/2(\Lambda^{(5)} + \kappa_5^2\sigma^2/6)$, $T_{\mu\nu}$ is the energy-momentum tensor of matter fields confined on the brane defined by $T_{\mu\nu} = -\sigma q_{\mu\nu} + \tau_{\mu\nu}$, σ being the brane tension and $\tau_{\mu\nu}$ is the matter-energy momentum, $E_{\mu\nu}$ is part of the 5-dimensional Weyl tensor and carries some information about bulk geometry with $E^\mu_\mu = 0, n_a = (1,0,0,0,0)$, $\kappa_4^2 = 8\pi G_4 = \kappa_5^4\sigma/6$ and $\kappa_5^2 = 8\pi G_5$ are 4-dimensional and 5-dimensional gravitational constants. Assuming the Friedmann-Robertson-Walker metric, equation (51) yields the Friedmann equation was found to be:

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3m_4^2} + \frac{\rho^2}{36m_5^6} + \frac{C}{R^4} \tag{52}$$

where $m_4 = \kappa_4^{-1} = 2.4\times 10^{18}GeV$, $m_5 = \kappa_5^{-2/3}$,"k"is a curvature constant, "ρ" is the total energy density of matter, C is a constant coming from $E_{\mu\nu}$. $\Lambda^{(4)}$ is set equal to zero. The important future of equation (4) is the presence of the last two terms. When $C = k = 0$, equation (52) looks like equation (50). In principle, equation (52) can be written in the following form:

$$H^2 + \frac{k}{R^2} = \frac{8\pi G_4\rho}{3} + \frac{4\pi G_4\rho^2}{3\sigma} - \frac{E_{00}}{3} \tag{53}$$

In fact, one can consider the vacuum state of a particle field as containing itself virtual pairs of particles with mass assuming here to be of the same order of ultra-light particles (m)and with an effective density $n \propto 1/\lambda_c$ where $\lambda_c = \hbar/mc$ is the Compton wavelength. Then one can considers the gravitational interaction energy of these virtual pairs, which is $G\lambda_c/m^2$¸ for one pair, thus leading to a contribution to an effective energy density in the vacuum of order of magnitude$\rho_c = Gm^6c^4/\hbar^4$. If we take now the energy density to be the sum of three terms$\rho_m, \rho_{vac}, \rho_c$, then the following equations holds:

$$H^2 = A\rho_{vac}^{1/2} + B\rho_{vac} + C\rho_{vac}^{3/2} \tag{54}$$

$$H^2 = A\rho_m + B\rho_m^2 + C\rho_m^3 \tag{55}$$

$$H^2 = A\rho_c^{1/3} + B\rho_c^{2/3} + C\rho_c \tag{56}$$

In fact, such equations may arise from fundamental theories of gravity in higher dimensions or from an extra contribution to the energy-momentum tensor on the right hand side of (*ordinary four dimensional*) Einstein's equations as our cosmological model [99,100,101]. Equation (54) (or (55)) is identical to the one obtained not only in brane but also in Cardassian models with ρ replaced byρ_m [102,103,104]. Only when $m = \hbar H/c^2$, the two densities are equal, then we fall into the classical and standard model. From equation (50), equation (55) could be viewed as brane perturbations. Replacing the density in equation (5) by$\rho_m = 3m^2/8\pi G_4$, we get:

$$H^2 + \frac{k}{R^2} = \frac{8\pi G_4 \rho_m}{3} + \frac{4\pi G_4 \rho_m^2}{3\sigma} - \frac{E_{00}}{3} \tag{57}$$

or

$$H^2 + \frac{k}{R^2} = A\rho_m + B\rho_m^2 - \frac{E_{00}}{3} \tag{58}$$

This equation is identical to equation (55) if $E_{00} \propto \rho_m^3 \approx \rho_c \approx \rho_{vac}^{3/2} \approx m^6 \approx H^6 \quad (\hbar = c = 1)$, that is$H^6 \equiv \dot{a}^6/a^6 \propto 1/a^4$, or $a\,(t) \propto t^{3/2}$ (*accelerating expansion*) and $k = 0$ (*flat geometry*). In this way, the radiation energy varies with time as $E_{00} \propto t^{-6}$which is a too fast decreasing function. Finally, we study and discuss the phenomenology of the ansatz in equation (49). We first mention that the ghost contribution ρ_m has a negative pressure (*simple calculations gives* $p_m = -\rho_m/2$), which is responsible for the Universe's acceleration. In this way, the quantum fifth force varies as $F\,(r) \propto a^{0.5}$. From equation (49), it is clear that this kind of behavior is qualitatively very different from the standard braneworld cosmology (*as equation* (50)) because it implies a modification of gravity at very low energy scales rather than very high ones. We suppose that the new density in equation (48) is considered to be initially negligible. It only comes to dominate recently, at the redshift $z_{eq} \approx O\,(1)$ indicated by the supernovae observations. Once the second term dominates, it causes the universe to accelerate even if there is no macroscopic matter contribution. We take the vacuum density to scale with the redshift as$\rho_{vac} \propto a^{-3}$. That is, when the second term in equation (1) dominates, it causes the Universe to accelerate as the scale factor grows as $a \propto t^{4/3}$so that$\ddot{a} > 0$. The second term starts to dominate at a redshift z_{eq} such that $A\rho\,(z_{eq}) = B\rho^{0.5}\,(z_{eq})$. In this way[105,106]:

$$A = \frac{H_0^2}{\rho_o^{vac}} - \frac{B}{\sqrt{\rho_0^{vac}}} \tag{59}$$

$$B = \frac{H_0^2\,(1+z_{eq})^{3/2}}{\rho_0^{vac}\left[1 + (1+z_{eq})^{3/2}\right]} \tag{60}$$

where ρ_0^{vac} is the present vacuum density. We have one parameter in this model, z_{eq} or B. Observations of the cosmic background radiation show that we live in a flat Universe.

We defines the critical quantum density as

$$\rho_c = \frac{3m_0^2 c^4}{8\pi G\hbar^2} \times \frac{1}{\left[1 + (1 + z_{eq})^{3/2}\right]} \tag{61}$$

where $m_0 = \hbar H_0/c^2$. As a result:

$$\begin{aligned} \rho_c^{vac} &\approx 0.35 \frac{3H_0^2}{8\pi G} \\ &\approx 0.35 \times 1.88 \times 10^{-29} h_0^2 gm \times cm^{-2} \end{aligned} \tag{62}$$

where h_0 is the Hubble constant today in unit of 100km/s/Mpc. The critical density is much lower than previously estimated and satisfy most of the observational constraints. It became much lower if we reformulate the problem with equation (53), but than the acceleration is constant. Several characteristics of this model are in progress.

6. Massive Gravitons, Massive Ultra-Light Particles and the Cosmological Constant in General Relativity

In the previous sections, we mention that one can describe dark energy in some $D = 4$ extended supergravities that have a de Sitter solutions. These **dS** solutions correspond to the extrema of the effective potentials $V(\phi)$ for some scalar fields "ϕ". An interesting result of these solutions is that the squared mass of these scalars in all theories with $N = 2$ (*extended supergravities with unstable dS vacua*) is quantized in units of the Hubble constant "H_0". That is $m^2 = nH_0^2$ where "n"are integers of order of unity (*in units* $M_P = 1, M_P$*being the Planck mass*). In extended supergravities with a positive cosmological constant, one always has $3m^2 = n\Lambda$, "Λ" is the cosmological constant having dimension two in energy unit. Along this, reducing the vacuum density (*using the context of non-minimal coupling of scalar fields to gravity*) to a very small value over cosmological timescales was discussed by several authors [18,19,20,21,22,107-111]. In this work, we will be interested on extended supergravities with positive cosmological constant and positive "m^2". For that we adopt the chaotic inflaton complex potential-like $V(\phi\phi^*) = am^2(\zeta\phi^2\phi^{*2} + 1)$, $a \approx 3/4$. In this particular case and for $\zeta << 1$, the Einstein field equations $G_{\mu\nu} - \Lambda g_{\mu\nu} + \sum T_{\mu\nu} = 0$ ($G_{\mu\nu}$ *is the Einstein curvature tensor and* $\sum T_{\mu\nu}$ *is the sum of all the stress-energy tensor implemented in the theory*) yields the following Ricci scalar $R = -4\Lambda + 3m^2$ [18]. As a result, a possible candidate static field equations corresponding to this scalar curvature are:

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\left(\frac{3m^2}{4} - \Lambda\right) g_{\alpha\beta} \tag{63}$$

"m" is the ultra-light masses and "$g_{\alpha\beta}$" is material interior metric tensor. In general, this equation is identical to the general Einstein field equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -8\pi G \left[(p + \rho) u_\alpha u_\beta - p g_{\alpha\beta}\right] \tag{64}$$

if we assume $p = -\rho$ and $32\pi G\rho = 3m^2 - 4\Lambda$, "$G$" being the gravitational constant. As long as $3m^2 > 4\Lambda$, the density remains positive. It follows that if the mass of the ultra-light particle vanishes, the cosmological constant must be assumed negative in order to get positive vacuum density and a corresponding negative pressure. That is to say also that the total vacuum density is the sum of the true vacuum energy represented by the cosmological constant and the false vacuum energy represented by the ultra-light masses. At the same time, there are some theoretical arguments that the graviton mass can have a non-zero rest mass as a result of the interaction with matter [112,113,114]. The gravitational waves were showed to be absorbed by the false vacuum having the equation of state $p = -\rho$. Thus we expect that there must exit a special relation between the graviton mass, the ultra-light mass and the cosmological constant. In fact, conformal relativity is a theory of mass having zero Weyl conformal curvature tensor and a special metric tensor$g_{\alpha\beta} = e^{\psi}\eta_{\alpha\beta}$, $\eta_{\alpha\beta}$ is the Minkowski metric and "ψ" is some function of the coordinates in the theory described. The resulting Einstein field equations[115]:

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = -8\pi G\left[(p+\rho)\,\delta^0_\alpha\delta^0_\beta - pg_{\alpha\beta}\right] \tag{65}$$

becomes adopting that the density of the background cosmic fluid is $\rho = (3m^2 - 4\Lambda)/32\pi G$:

$$\frac{\partial^2\psi}{\partial x^\alpha\partial x^\beta} + \frac{1}{2}\eta_{\alpha\beta}\eta^{\mu\nu}\frac{\partial^2\psi}{\partial x^\mu\partial x^\nu} - \frac{3}{2}g_{\alpha\beta}\psi = 8\pi G\rho g_{\alpha\beta} \tag{66}$$

or in the following form and in natural units:

$$\left(\Box - \frac{64\pi G}{3c^2}\left(\frac{3m^2c^4}{32\pi G\hbar^2} - \frac{\Lambda c^2}{8\pi G}\right)\right)\psi = \frac{32\pi G}{3c^2}\left(\frac{3m^2c^4}{32\pi G\hbar^2} - \frac{\Lambda c^2}{8\pi G}\right) \tag{67}$$

Here $\Box \equiv \partial^2/\partial x^\mu\partial x_\mu$. Comparing to the Proca-Yukawa massive gravitoelectomagnetics static theory describing a universe as a mode collective behavior of dust and graviton particles [116,117]:

$$\left(\Box - \frac{m_g^2c^2}{\hbar^2}\right)\psi = \frac{1}{2}\frac{m_g^2c^2}{\hbar^2} \tag{68}$$

where "m_g" is the graviton mass, we get from equations (68) and (66):

$$\Lambda = \frac{3}{8}\frac{c^2}{\hbar^2}\left(2m^2 - m_g^2\right) \equiv \frac{3}{8}\frac{m^2c^2}{\hbar^2}\left(2 - \frac{m_g^2}{m^2}\right) \equiv \frac{3}{8}\frac{m^2c^2}{\hbar^2}\left(2 - f^2\right) \tag{69}$$

where $f \equiv m_g/m$. This equation is of interest: *it relates for the first time the cosmological constant to both the ultra-light mass and the graviton mass.* As long as $f < \sqrt{2}$ or $m_g < \sqrt{2}m$, the cosmological constant is positive. If "m" is set equal to the "*Hubble mass*", that is $m \equiv m_H^0 = \hbar H_0/c^2 \approx 3.8\cdot 10^{-66}h\,(g)\,, h = 0.71\pm 0.07$, then we get $m_g < \sqrt{2}\hbar H_0/c^2$ [118]. For $m_g = \sqrt{2}\hbar H_0/c^2 \approx 1.4m_H^0$, the cosmological constant vanishes. Equation (69) takes then the simple form:

$$\Lambda = \frac{3}{8}H_0^2\left(2 - f^2\right) \tag{70}$$

If $m_g = m_{H_0}$, than $\Lambda = 3m_g^2c^2/8\hbar^2 = 3/8H_0^2 = 0.375H_0^2$. This could explain the weakness of the cosmological constant. It is worth noting that equations (69) and (70) are in agreement with MAXIMA-1 recent constraints on cosmological parameters suggest that $0 < \Lambda < 2.28H_0^2$ [119]. Within the framework of Relativistic Theory of Gravity, some authors claimed that $0.24 \leq f^2 \leq 1.08$ [71,72,120,121]. As a result, the Ricci scalar as well as the cosmic fluid densities $\rho = 3m^2c^4f^2/64\pi G\hbar^2$ determined from the above requirements are both positive. For $m = m_g = m_{H_0}$, $\rho = 3H_0^2f^2/64\pi G < \rho_c \equiv 3H_0^2/8\pi G$ in agreement with observations. In conclusion, a gravitational wave propagating through a special medium with a total density which is the sum of the true vacuum density and the ultra-light particle false density is influenced by these latter. This is manifested by the production of massive gravitons, by reducing the value of the cosmological constant and the density of matter $\rho < \rho_{critical} = 3H_0^2/8\pi G$, relating the ultra-light masse, the graviton mass and the cosmological constant in one simple relation and finally by generating a positive curvature scalar $R = 3m^2 - 4\Lambda > 0$.

7. Conclusions

In this work, we illustrated some important aspects, implications and features of ultralight masses in modern cosmology. We showed the importance of the presence of the ultra-light masses as necessary ingredients in modern cosmology. The evolution of a homogeneous flat universe with positive scalar curvature dominated by ultra-light matter and the cosmological constant are discussed and we investigated about the necessary condition for eternal accelerated expansion with no need of any form of tachyonic matter. We discussed some important aspects and features of a cosmological model with phenomenological decay of the ultra-light masses, the vacuum cosmological constant and the matter density as $m^2(t) = \varepsilon\dot{a}^2/a^2$, $\Lambda(t) = \beta\ddot{a}/a$ and $\rho(t) = (3\delta/8\pi G)\dot{a}^2/a^2$ respectively, α,β and δ are constants. We showed that, for the special parameters $\beta = 5$ and $\delta = 0.25$, the universe will passes through different phases. The transition from the vacuum state to the radiation one is followed in our scenario by a increasing in the gravitational constant, an increasing of the cosmic acceleration while the ultra-light masses, the cosmological constant and the density decreases as $1/t^2$. Within the same context, the transition from the radiation dominated epoch to the matter dominated one is also followed by increasing of the gravitational constant but with a reduced term and this reducing follows also the scale factor as well as $m^2(t), \Lambda(t)$ and $\rho(t)$. We discussed also the role of decaying ultralight masses and gravitons masses within the framework of RTG. We showed the important role playing by the decaying gravitons masses and decaying quantized ultra-light masses from extended supergravities in the evolution of the (RTG) Universe for positive and negative supergravities potentials. The decaying of the graviton masses is responsible of the cosmic acceleration. At very later times, when all the gravitons disappear, the Universe enters a slow dying epoch where he scale factor evolves as $a \propto t^p$ with $p < 1$. We proposed also a simple modification of the Standard Hot Big Bang Cosmology (*SHBBC*), in which the Universe is flat with a total energy density taken to be the sum of the contributions from vacuum and ultra-light mass term responsible of the dominant driver of expansion at a late epoch of the Universe. When the new term dominates, the universe was found to be accelerated with time. The ultra-light quantum energy density required to close the Universe was found to be much smaller than in *SHBBC*, so that quantum matter can be sufficient to provide a flat geometry. It was also found that quantum ultra-light matter particles interactions are interpreted as a quantum fifth force and it is found to vary as$F(r) \propto r^{0.5}$. Finally, we illustrated the modification of the gravitational wave propagation through a special vacuum medium with a total density being the sum of the true vacuum density and the ultra-light particle supergravity density by reducing the value of the cosmological constant up to $\Lambda = 3H_0^2(2-f^2)/8$ ($f \equiv m_g/m_{H_0}$) and the density of matter $\rho < \rho_{critical} = 3H_0^2/8\pi G$, by relating the ultra-light masse, the graviton mass and the cosmological constant in one simple relation and finally by generating a positive Ricci scalar curvature. The gravitational wave propagating through a special medium with a total density which is the sum of the true vacuum density and the ultra-light particle false density was shown to be influenced by these latter. This is manifested by the production of massive gravitons, by reducing the value of the cosmological constant and the density of matter $\rho < \rho_{critical} = 3H_0^2/8\pi G$, relating the ultra-light masse, the graviton mass and the cosmological constant in one simple relation and finally by generating a positive curvature scalar $R = 3m^2 - 4\Lambda > 0$. We hope that this work will lead to some important cosmological, astrophysical and theoretical consequences (*the cosmological constant problem, the quantization of General Relativity, the dark matter problem, the effects of the presence of ultra-light masses and gravitons on black holes physics, etc*) [122-131]. For this, further consequences and details are in process.

Appendix

The parameters used in this paper are summarized as fellows:

(1) ξ is the non-minimal coupling constant appearing in our supergravity inflationary potential $V(\phi\phi^*) = (3m^2/4)(\zeta\phi^2\phi^{*2} - 1)$ (Section **1**)

(2) $\zeta = O(1)$ is a supergravity inflaton scalar field parameter (Section **1**)

(3) $\theta \equiv H_0/H_\infty$ introduced in attempt to describe non-singular universe accelerating Universe with positive scalar Ricci curvature generated from negative extended supergravity potential (Section **1**)

(4) $\dot{F}^2 = \alpha m^2/\Lambda, F$ represents the new inflaton field with α is a positive constant introduced in attempt to investigate about the necessary condition for eternal accelerated expansion with no need of any form of tachyonic matter (Section **2**)

(5) $\omega = -1/(1+3m^2/\Lambda) \equiv p/\rho$ is the state equation coefficient (Section **2**)

(6) $w \equiv \omega = p/\rho$(Section 3)

(7) $\tilde{\gamma} \equiv w + 1$(Section 3)

(8) $m^2(t) = \varepsilon\dot{a}^2/a^2$, $\Lambda(t) = \beta\ddot{a}/a$ and $\rho(t) = (3\delta/8\pi G)\dot{a}^2/a^2$ are the three phenomenological laws for time-varying ultra-light masses, cosmological constant and matter density (Section **4**)

(9) $m^2 = \pm\omega\dot{a}^2/a^2$is the phenomenological time-varying law for the ultra-light masses within the framework of **RTG** (Section **4**)
(10) $m^{*2} \equiv \gamma\dot{a}^2/a^2 + \eta\ddot{a}/a$ is the phenomenological time-varying law for the gravitons masses within the framework of **RTG** (Section **4**)
(11) $a \propto t^p, \quad p = (4+\eta)/(6+\eta+\gamma)$ is the corresponding scale factor parameter (notes 9 and 10) (Section **4**)
(12) $p = -a\rho/3, a$ is a positive number (Section **5**)
(13) $A \equiv 8\pi G/3, B$ is a constant used in the brane world scenario (Section **5**)
(14) $m^2 = nH_0^2$ where "n" are integers of order of unity (Mostly in all sections)
(15) $a \approx 3/4$ used in the positive supergravity inflationary potential $V(\phi\phi^*) = am^2(\zeta\phi^2\phi^{*2} + 1)$ (Section **6**)

Acknowledgments

The author would like to thank the anonymous referees for their useful comments and suggestions.

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