

Particle Interference without Waves

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Received 25 November 2006, Accepted 3 December 2006, Published 20 December 2006

Abstract: After eighty years of Quantum Mechanics (QM) we have learned to live with wave functions without worrying about their physical nature. This attitude is certainly justified by the extraordinary success of the theory in predicting and explaining not only all the phenomena encountered in the domain of microphysics, but also some spectacular nonclassical macroscopic behaviors of matter. Nevertheless one cannot ignore that the *wave-particle duality* of quantum objects not only still raises conceptual problems among the members of the small community of physicists who are still interested in the foundations of our basic theory of matter, but also induces thousands and thousands of physics students all around the world to ask each year, at their first impact with Quantum Mechanics, embarrassing questions to their teachers without receiving really convincing answers. Remember that Feynman once said “It is fair to say that nobody understands Quantum Mechanics”. My purpose is to show that these difficulties can only be faced by pursuing a line of research which takes for granted the irreducible nature of randomness in the quantum world. This can be done by eliminating *from the beginning* the unphysical concept of wave function. I believe that this elimination is conceptually similar to the elimination of the aether, together with its paradoxical properties, from classical electrodynamics, accomplished by relativity theory. In our case the lesson sounds: No wave functions, no problems about their physical nature. Furthermore, the adoption of a statistical approach from the beginning for the description of the physical properties of quantum systems sounds methodologically better founded than the conventional *ad hoc* hybrid procedure of starting with the determination of a system’s wave function of unspecified nature followed by a “hand made” construction of the probability distributions of its physical variables.

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Keywords: Wigner-Feynman approach of Quantum Mechanics, Quantum Interpretations and Representations, Classical Ensemble Theory, Foundations of Quantum Mechanics

PACS (2006): 03.65.w, 05.20.Gg, 03.65.Ta

1. Introduction

After eighty years of Quantum Mechanics (QM) we have learned to live with wave functions without worrying about their physical nature. This attitude is certainly justified by the extraordinary success of the theory in predicting and explaining not only all the phenomena encountered in the domain of microphysics, but also some spectac-

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ular nonclassical macroscopic behaviors of matter. Nevertheless one cannot ignore that the *wave-particle duality* of quantum objects not only still raises conceptual problems among the members of the small community of physicists who are still interested in the foundations of our basic theory of matter, but also induces thousands and thousands of physics students all around the world to ask each year, at their first impact with Quantum Mechanics, embarrassing questions to their teachers without receiving really convincing answers. Remember that Feynman once said “It is fair to say that nobody understands Quantum Mechanics” [1].

Typical examples of this dissatisfaction are the nonseparable (or nonlocal) character of long distance correlated two-particle systems [2] and the dubious meaning of the superposition of state vectors of measuring instruments [3], and in general of all macroscopic objects (Schrödinger’s cat).

In the former case experiments have definitely established that Einstein was wrong in claiming that QM has to be completed by introducing extra “hidden” variables in order to specify the “objective” physical state of each particle, but have shed no light on the nature of the entangled two-particle state vector responsible for the peculiar quantum correlation between them, a correlation which exceeds the classical one expected from the constraints of conservation laws.

In the latter case, generations of theoretical physicists in neoplatonist mood have insisted in claiming that the realistic aspect of macroscopic objects is only an illusion valid For All Practical Purposes (in jargon FAPP). The common core of their views is the belief that the only entity existing behind any object, be it small or large, is its wave function, which rules the random occurrence of the object’s potential physical properties.

The most extravagant and bold version of this approach is undoubtedly the one known as the Many Worlds Interpretation of QM [4], which goes a step further by eliminating the very founding stone on which QM has been built, namely the essential randomness of quantum events. Chance disappears: the evolution of the whole Universe is written – a curious revival of Laplace - in the deterministic evolution of its wave function.

“The Many-Worlds Interpretation (MWI) – in the words of Lev Vaidman, one of its most eminent supporters [5] - is an approach to quantum mechanics according to which, in addition to the world we are aware of directly, there are many other similar worlds which exist in parallel at the same time and in the same space. The existence of the other worlds makes it possible to *remove randomness* and action at a distance from quantum theory and thus from all physics.”

I believe that it is grossly misleading to attribute the epistemological status of “consistent physical theory” to this sort of science fiction, which postulates the existence of myriads and myriads of *physical objects* (indeed entire worlds!) which are *in principle undetectable*. My purpose is to show that these difficulties can only be faced by pursuing a line of research which goes in the opposite direction, namely which takes for granted the irreducible nature of randomness in the quantum world.

This can be done by eliminating *from the beginning* the unphysical concept of wave function. I believe that this elimination is conceptually similar to the elimination of

the aether, together with its paradoxical properties, from classical electrodynamics, accomplished by relativity theory. In our case the lesson sounds: No wave functions, no problems about their physical nature.

Furthermore, the adoption of a statistical approach from the beginning for the description of the physical properties of quantum systems sounds methodologically better founded than the conventional *ad hoc* hybrid procedure of starting with the determination of a system's wave function of unspecified nature followed by a "hand made" construction of the probability distributions of its physical variables.

2. The Phase Space Approach to the Formulation of Quantum Mechanics

This statistical approach goes back to Wigner's formulation [6] of QM in phase space. Wigner functions replace the classical probability distributions as a perfectly valid tool for calculating all the statistical properties of any ensemble of quantum particles, in spite of the fact that they lack the fundamental property of being non-negative. This is indeed the price one has to pay in order to introduce in the classical expressions the constraint of the uncertainty principle.

Almost fifty years later, Feynman [7] reconsidered the possibility of giving up the assumption that the "probability" for an event must always be a positive number. This extension of the probability concept does not lead to absurd consequences. On the contrary, Feynman argued that, once the "strong mental block against negative probabilities" has been overcome, "they are entirely rational, and their use simplifies calculations and thought in a number of applications in physics." One should add also that the validity of Feynman's approach has been strikingly supported by the recent direct experimental determination of the Wigner functions for the states of quantum harmonic oscillators [8]

This line of thought has been developed by myself in the last few years [9]. I have in fact shown that the Wigner representation of quantum mechanics in phase space can be derived from two basic physical principles (the Heisenberg uncertainty principle for canonically conjugated variables and the Planck quantization of variables such as energy or angular momentum) *without need of introducing Schrödinger waves*. An important consequence of these postulates is that the random variables obeying these constraints *must* be represented by non commuting mathematical entities whose explicit expression, however, never enters in the theory, which deals only with their ensemble averages.¹

The aim of the present paper, however, is not to argue further on general grounds in favor of this new approach. In principle, in fact, the new formalism allows the solution of any quantum mechanical problem in terms of Wigner functions without ever having

¹ A generalization of this approach to field quantization (¹⁰) has further shown that this approach leads also to the clarification of the notion of *wave/particle duality*, which still holds but does not involve Schrödinger waves. It refers instead to the dual nature of the quantum field as a unique physical entity *objectively existing in ordinary three dimensional space*. I will not be concerned with this question in this paper.

to introduce Schrödinger waves. However, since the most elementary evidence of the existence of Schrödinger waves is usually considered to be the “interference pattern” of a beam of particles passing through a Mach Zender interferometer, it may be interesting to show that the notion of irreducible randomness allows a straightforward explanation of this phenomenon without ever having to introduce this puzzling and elusive entity.

3. The Statistical Properties of a Classical Ensemble of Particles with Random Dichotomic Variables

To achieve this goal it is necessary to start by considering a beam of classical particles (fig.1) impinging on a beam splitter A which randomly may be either reflected to proceed along a path L_1 or transmitted to proceed along a path L_2 . In fact one should at this point stress that, classically, *randomness is a purely epistemic phenomenon*, namely that it is a consequence of the impossibility of reconstructing by means of a suitable detection apparatus the ‘real’ deterministic path of each particle which, depending on the initial conditions, will lead it to be either reflected or transmitted in the interaction with the beam splitter’s atoms. The actual physical state of each particle of a classical ensemble *must* be different from that of the other ones: if they all had exactly the same state they could not behave differently.

In order to describe the statistical aspects of the phenomenon we define a random variable \mathbf{A} with the value $a=1$ in the case of reflection and the value $a=-1$ in the case of transmission. At the end of L_1 the particles impinge on the upper side of a second beam splitter B and may be either reflected (value $b=1$ of the random variable \mathbf{B}) and detected by a detector D_1 or transmitted ($b=-1$) and detected by a detector D_2 . The particles arriving from L_2 impinge on the opposite side of B to be either transmitted ($b=-1$) to reach the counter D_1 , or reflected ($b=1$) to reach the counter D_2 . To summarize: the counter D_1 receives both the particles with $a=-1, b=-1$ and those ones with $a=+1, b=+1$, and the counter D_2 receives the particles with $a=+1, b=-1$ and those with $a=-1$ and $b=+1$.

We can define therefore this classical statistical ensemble of particles by means of three numbers: x, y, z representing respectively the ensemble average $\langle \mathbf{A} \rangle$ of \mathbf{A} , the ensemble average $\langle \mathbf{B} \rangle$ of \mathbf{B} , and the ensemble average $\langle \mathbf{AB} \rangle$ of their product (correlation coefficient). It is useful for later reference to introduce an independent notation \mathbf{C} for the product \mathbf{AB} because, in spite of the fact that for each particle the value of \mathbf{C} is determined by the product of the values of \mathbf{A} and \mathbf{B} , the ensemble average of the values of \mathbf{C} is statistically independent from the ensemble averages of the other two variables, because it depends on physical factors other than those of \mathbf{A} and \mathbf{B} taken separately: x and y represent the average values of the random events of reflection or transmission of each particle in the interaction with the splitters, while z depends on the physical features of the two paths. No direct connection exists however in this description between these features (length difference, physical nature of the phase shifter S or relative time delay) and the value of z . In a classical particle ensemble there are no waves, of course, and in a

purely probabilistic description, the connection can only show up in the relation between the ensemble average values of the random events. We will come back to this point when we will discuss the difference between the classical and the quantum ensemble.

In a given physical ensemble, given the values of x, y, z , the probabilities for the corresponding four alternatives ($a = \pm 1, b = \pm 1$) are given by:

$$p_{ab} = \left(\frac{1}{4}\right)[1 + ax + by + abz] \quad (1)$$

where

$$x = \langle \mathbf{A} \rangle = \sum a p_{ab}; \quad y = \langle \mathbf{B} \rangle = \sum b p_{ab}; \quad z = \langle \mathbf{C} \rangle = \sum ab p_{ab} \quad (2)$$

If we impose that $0 \leq p_{ab} \leq 1$, as it must be for classical probabilities, then one finds that the values of x, y, z should satisfy the condition of being the coordinates of a point inside the equilateral octahedron with vertices

$$x = \pm 1, z = 0, y = 0; \quad y = \pm 1, z = 0, x = 0; \quad z = \pm 1, x = 0, y = 0 \quad (3)$$

These limiting values correspond respectively to: (i) pure reflection (or pure transmission) by A followed by equally probable reflection *and* transmission by B, and zero correlation; (ii) equally probable reflection *and* transmission by A followed by pure reflection (or pure transmission) by B, again with zero correlation; (iii) complete correlation or autocorrelation between the two beam splitters with equiprobable transmission and reflection by both A and B. *Apart from these limiting cases, all other possible ensembles have average values of the random variables with values less than one.* This means that always particles with both values ± 1 of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are present.

The above constraint leads to the relation between x, y, z

$$-1 \leq x + y + z \leq +1 \quad (4)$$

The counting rates of the counters D_1 and D_2 are therefore given by

$$P_1 = p_{++} + p_{--} = \left(\frac{1}{2}\right)(1 + z); \quad P_2 = p_{+-} + p_{-+} = \left(\frac{1}{2}\right)(1 - z) \quad (5)$$

This procedure of defining an ensemble by giving the three values of x, y, z with the constraint (4), is however too general for our purpose of preparing the ground to the formulation of the constraints which define the corresponding quantum ensemble.

To pursue this aim it is convenient to select a class of ensembles, characterized by the values of three new random variables, $\mathbf{U}, \mathbf{V}, \mathbf{W}$, which satisfy a sort of classical *uncertainty principle* - a property that will become the actual uncertainty principle of the corresponding quantum ensemble - defined by the properties

$$\langle \mathbf{U} \rangle = u; \quad \langle \mathbf{V} \rangle = 0; \quad \langle \mathbf{W} \rangle = 0 \quad (6)$$

\mathbf{U} is the variable whose average value u labels the ensemble while the other two variables satisfy the constraint of being completely undetermined, namely that all their possible

values are equally probable. This program is fulfilled by choosing an arbitrary unit vector of components α, β, γ in the x, y, z space and defining \mathbf{U} in terms of the variables $\mathbf{A}, \mathbf{B}, \mathbf{C}$ as a linear combination

$$\mathbf{U} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}; \quad \alpha^2 + \beta^2 + \gamma^2 = 1 \quad (7)$$

In order to satisfy the two constraints (6) for \mathbf{V} and \mathbf{W} we have to define, in addition to (α, β, γ) two other mutually orthogonal unit vectors (λ, μ, ν) and $((\beta\nu - \gamma\mu), (\gamma\lambda - \alpha\nu), (\alpha\mu - \beta\lambda))$ such that

$$\mathbf{V} = \lambda\mathbf{A} + \mu\mathbf{B} + \nu\mathbf{C}; \quad \lambda^2 + \mu^2 + \nu^2 = 1; \quad \alpha\lambda + \beta\mu + \gamma\nu = 0 \quad (8)$$

$$\mathbf{W} = (\beta\nu - \gamma\mu)\mathbf{A} + (\gamma\lambda - \alpha\nu)\mathbf{B} + (\alpha\mu - \beta\lambda)\mathbf{C} \quad (9a)$$

The definitions (7) (8) (9) can be transformed into equations for x, y, z by taking the average values of the variables $\mathbf{A}, \mathbf{B}, \mathbf{C}$, involved and making use of the relations between α, β, γ and λ, μ, ν . The result is

$$x = \alpha u; \quad y = \beta u; \quad z = \gamma u \quad (9b)$$

The constraint (4) leads therefore to the constraint for u

$$-1 \leq u(\alpha + \beta + \gamma) \leq +1 \quad (10)$$

which implies that the **absolute value of u is always smaller than one**, because $\alpha + \beta + \gamma$ is always greater than one (except in the limiting cases of the vertices of the octahedron). This confirms the above statement that *particles with both values ± 1 of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are always present in any ensemble*.

In terms of the new parameters defining the ensemble the probabilities p_{ab} become

$$p_{ab} = \left(\frac{1}{4}\right)[1 + u(a\alpha + b\beta + ab\gamma)] \quad (11)$$

and the counting rates of the counters

$$P_1 = p_{++} + p_{--} = \left(\frac{1}{2}\right)(1 + u\gamma); \quad P_2 = p_{+-} + p_{-+} = \left(\frac{1}{2}\right)(1 - u\gamma) \quad (12)$$

4. The Quantum Ensemble

We now try to construct the quantum mechanical theory of our experimental setup by introducing the necessary changes to the definitions of the classical ensemble. The first step is to assume as a **postulate** that *randomness is an irreducible property of quantum events*. After all, this is exactly what all physics students learn in their introductory courses when they are taught that the actual decay of a single unstable radioactive nucleus is a purely random event which may occur at any time within the limit of a typical lifetime.

This postulate means that the different behavior of each particle going through the interferometer is not due to *hidden variables* having a priori different values, but the effect of individual intrinsically random events triggered by chance occurring in a dispersion-free quantum ensemble in which **all the particles have the same physical state**. This common state should be either one or the other of the two states corresponding to the values ± 1 of a suitable dichotomic random variable \mathbf{U} .

If we assume the **uncertainty principle** as the founding principle of quantum theory we must further assume that **for each particle** the value of the two other statistically independent variables \mathbf{V} , \mathbf{W} is completely undetermined, namely that their values ± 1 are equally probable.

This principle justifies our choice of the variables \mathbf{U} , \mathbf{V} , \mathbf{W} with their definitions (7) (8) (9) and their properties (6), as the proper classical framework to start with for the construction of the corresponding quantum ensemble

The quantum constraints imply therefore that the absolute value of the ensemble average value u of \mathbf{U} (which, accordingly to eq. (10) was in the classical case **always** smaller than one) must be now be **always** equal to one (for all values of α, β, γ), because **all the particles have the value** $+1$ (or -1) of \mathbf{U} . Eqs. (9) are therefore now replaced by

$$x = \alpha; \quad y = \beta; \quad z = \gamma \quad (13)$$

and the constraints (4) and (10) by

$$x^2 + y^2 + z^2 = 1 \quad (14)$$

The immediate consequence of the replacement of (4) with (14) (*which implies that x, y, z are always outside of the octahedron*) is clearly that the *probabilities* p_{ab} cannot be any more constrained within the interval $0 \leq p_{ab} \leq 1$. This is not unexpected, in the light of the considerations exposed in the introductory sections.

There is however an even more striking consequence of our quantum postulate. It follows from the observation that, if all the particles have the same value $+1$ (or alternatively -1) of \mathbf{U} then *the ensemble average* of \mathbf{U}^2 must necessarily be

$$\langle \mathbf{U}^2 \rangle = 1 \quad (15)$$

Until now we have implicitly assumed that \mathbf{A} , \mathbf{B} , \mathbf{C} are the same random variables of the classical ensemble, satisfying the commutative property of multiplication of ordinary numbers. However, under this assumption, if one introduces for \mathbf{U} the expression (7) and takes into account eqs. (13), the ensemble average of \mathbf{U}^2 turns out to be

$$\langle \mathbf{U}^2 \rangle = 1 + 6\alpha\beta\gamma \quad (16)$$

This means that the quantum random variables \mathbf{A} , \mathbf{B} , \mathbf{C} cannot satisfy the commutative law of multiplication of ordinary numbers. Also this result, of course is not unexpected.

Therefore, the only way to fulfill the physical requirement of the uncertainty principle is in fact to assume that the variables \mathbf{A} , \mathbf{B} and their correlation \mathbf{C} are represented by mathematical objects which do not satisfy the commutative law of multiplication. Typical objects of this kind are Hermitian matrices with eigenvalues ± 1 . It should be kept in mind that In order to ensure that the correlation variable \mathbf{C} be an Hermitian matrix (with real eigenvalues), it should be related to the product of the two Hermitian matrices \mathbf{A} and \mathbf{B} by (i is the imaginary unit);

$$\mathbf{AB} = i\mathbf{C} \quad (17)$$

With this assumption the ensemble averages of the variables coincide with the expectation values (traces) of the corresponding matrices. It is immediate to see that if one assumes for them the commutation and anticommutation relations of the three Pauli spin matrices

$$\mathbf{AB} - \mathbf{BA} = 2i\mathbf{C}; \quad \mathbf{AB} + \mathbf{BA} = 0 \quad \text{and cyclic permutations} \quad (18)$$

one obtains

$$\mathbf{UV} - \mathbf{VU} = 2i\mathbf{W}; \quad \mathbf{UV} + \mathbf{VU} = 0 \quad (19)$$

and therefore

$$\langle \mathbf{U} \rangle = \langle \mathbf{U}^2 \rangle = 1; \quad \langle \mathbf{V} \rangle = 0; \quad \langle \mathbf{W} \rangle = 0 \quad (20)$$

as it should be.

It is however important to stress that in this approach one never needs to use the explicit form of the matrices, or to make reference to their matrix elements and eigenvectors. *Only expectation values enter in this formulation of the theory.* The classical relations (11) and (12) are now replaced by the quantum ones

$$p_{ab} = \left(\frac{1}{4}\right)[1 + (a\alpha + b\beta + ab\gamma)] \quad (21)$$

and

$$P_1 = p_{++} + p_{--} = \left(\frac{1}{2}\right)(1 + \gamma); \quad P_2 = p_{+-} + p_{-+} = \left(\frac{1}{2}\right)(1 - \gamma) \quad (22)$$

5. Conclusions

Our quantization procedure is therefore accomplished with the result that the mathematical nature of the quantum random variables (Hermitian matrices) turns out to be, rather than an arbitrary *a priori* assumption, a *consequence* of the physical postulate that *randomness is an essential irreducible feature of quantum events* (uncertainty principle). It is not only a matter of taste. In the conventional approach, in fact, one always risks to look invain for nonexisting physical aspects of purely mathematical concepts.

We have seen that a direct consequence of the quantum postulate is that the condition (14) implies that the quantum statistical probability distributions p_{ab} defined in eq. (11)

are no longer positive definite because x, y, z are always chosen outside the octahedron (3). This is often considered to be an obstacle to any direct probabilistic formulation of QM.

However, once that, following Feynman's suggestion, the strong prejudice against treating the "pseudoprobabilities" as legitimate expressions of the statistical properties of the quantum ensemble is overcome, it is easy to show that they lead to the experimentally well known "interference" patterns of a beam of quantum particles going through a Mach Zender interferometer. In fact the essential result of this reformulation of the phenomenon is that the quantum counting rates (22) of the counters D_1 and D_2 differ from the classical ones (12) for the absence of the factor u which, as repeatedly underlined, is always smaller than one.

The effects of the variation of the physical parameters of the setup (represented in fig.1 by the "shifter" S) shows up in fact both in the classical and in the quantum model through the functional dependence $\gamma = \cos\theta$ on the polar angle θ of the unit vector representing the ensemble on the sphere (14) *this means therefore that in quantum ensembles the counting rate of each counter always varies between 0 and 1 depending on the value of γ while in the corresponding classical ensemble it varies only in the limited range between $(1/2)(1 - u)$ and $(1/2)(1 + u)$.*

We expect therefore that a beam made of identical atoms with different values of internal degrees of freedom (excited states) would show the reduced interference pattern of the classical model.

Acknowledgment

I am grateful to my friends Marzio Cassandro and Gianni Jona Lasinio for their illuminating observations and suggestions which contributed substantially to the final version of this paper.

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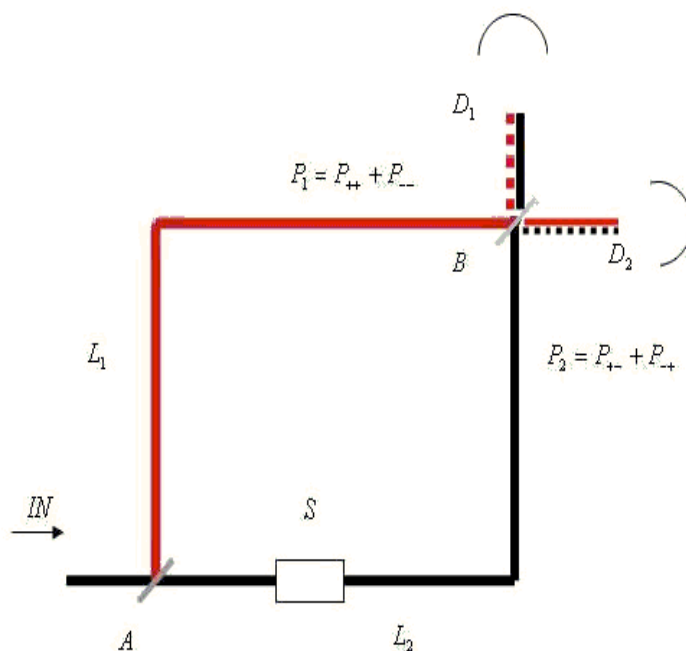


Fig. 1