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Analytical One-Photon Double Differential Spectrum From In-Flight Decay of Scalar Neutral Mesons

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Abstract: We introduce a direct simple method to evaluate the one-photon double differential spectrum from the decay of pseudo-scalar neutral mesons. The analytical distributions of the opening angle and of the ratio of energies of the two gammas are then straightforwardly deduced. The physical interest is also outlined.

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1. Introduction

The study of meson decay processes provides powerful tools in the understanding of various phenomena in low- and high-energy nuclear and particle physics [1, 2].

In heavy-ion reactions at intermediate energies, the subthreshold pion production may carry important information about the initial stage of the nucleus-nucleus collision, i.e. when the projectile and target surface just begin to overlap. Theoretically, the subthreshold meson production can be described as a classical radiative process having a collective character, *i.e.* in terms of cooperative effects involving the contribution from several nucleons of the projectile and target [3]. Alternatively, in the framework of transport theory, the meson production at energies as low as 25 $MeV/nucleon$ can be interpreted as due to nucleon-nucleon collisions whose time evolution is described by the Boltzmann-Nordheim-Vlasov equation, usually solved by means of test-particle techniques. The comparison with experimental data has indicated that, at intermediate energies, the subthreshold π^0-production seems to be originated from an incoherent superposition of individual nucleon-nucleon contributions,and therefore due to Fermi momentum coupling to the relative motion of the colliding nuclei [4].

In particle physics, the pion-nucleon scattering was the first (1970s) tool adopted for the study of meson-nucleon interaction at intermediate and high energies. The performed experiments gave an estimation for the scattering probability and, moreover, the evidence for the existence of instable intermediate states, the baryonic resonances [5]. These latter can be identified by a partial-wave expansion of the interacting meson's wave-function. The existence of baryonic resonances has been also confirmed in electromagnetic scattering on the nucleon, e.g. in the following scalar meson photoproduction reactions:

$$\gamma + N \longrightarrow N + \pi^0 \ , \tag{1}$$

$$\gamma + N \longrightarrow N + \eta \ , \tag{2}$$

where N is a nucleon (either n or p). The extraction of photo-excitation amplitudes beyond the first resonance region ($E_{tot}^{cm} \simeq 1.5\ GeV$) constitutes a powerful tool in the search for the so-called 'missing' resonances; these resonant baryonic states are predicted by QCD-inspired models, such as, for example, $SU(6) \otimes O(3)$-symmetric quark models, but not observed in conventional $\pi N \longrightarrow \pi N$ experiments. Moreover, photoproduction experiments can provide an effective criterium for the choice between the different theoretical models [6]. In the following we will focus our attention on the two-gamma decay of π^0 and η mesons, *i.e.* their dominant decay process, with branching ratios of 98% and 38% in the two cases respectively. Because of their very small mean free paths, the neutral mesons are detected indirectly, by reconstructing their kinematics from the decay products, usually via the invariant mass method. For this purpose, large efficiency detector systems with high granularity are currently operating in various laboratories [7, 8]. The differential spectra for the aforementioned processes are generally numerically deduced within Monte Carlo simulation approaches, which, as it's widely known, don't require any knowledge about the analytical expressions. Although this could be the only resource in cases of very complex calculations, we'll show here that their analytical expressions can be in general achieved by simple and straightforward mathematical considerations.

2. Double Differential Photon Spectrum

Let us consider a reaction producing a scalar neutral meson X (either π^0 or η), which in turns decays in two photons:

$$X \longrightarrow \gamma + \gamma$$

By choosing the polar axis along the direction of the X meson momentum in the laboratory-frame, the spin-0 nature of the X meson assures the process to be obviously isotropic and symmetrical in the X-center-of-mass frame, each produced photon carrying one-half of the rest energy of the X meson:

$$\theta_1' + \theta_2' = \pi \tag{3}$$

$$E_{\gamma i}' = \frac{m_X c^2}{2} \equiv E_0 \quad , \quad i = 1, 2 \ , \tag{4}$$

the primed quantities denoting the kinematical variables in the X-center-of-mass frame. The double-differential spectrum for the single decay photon is then trivially obtained as:

$$\frac{\partial^2 N_\gamma}{\partial E'_\gamma \partial \Omega'_\gamma} = \frac{1}{2\pi}\delta(E'_\gamma - E_0) \quad , \tag{5}$$

which is normalized to 2 in order to account for the fact that each decay process produces two photons.

By making use of (5), and of the usual Lorentz transformations for the four-momentum, the expression for the single-photon spectrum in the laboratory frame reads:

$$\frac{\partial^2 N_\gamma}{\partial E_\gamma \partial \Omega_\gamma} = \frac{E_\gamma}{E'_\gamma}\frac{\partial^2 N_\gamma}{\partial E'_\gamma \partial \Omega'_\gamma} = \frac{1}{2\pi}\frac{E_\gamma}{E'_\gamma}\delta(E'_\gamma - E_0) \tag{6}$$

Now, let us note that:

$$E'_\gamma = \gamma E_\gamma (1 - \beta\cos\theta) \ , \tag{7}$$

where γ and β have the usual meanings as in relativistic kinematics. This allows us to rewrite the (6) in terms of laboratory-frame kinematical variables only:

$$\frac{\partial^2 N_\gamma}{\partial E_\gamma \partial \Omega_\gamma} = \frac{1}{2\pi\gamma(1-\beta\cos\theta)}\delta[\gamma E_\gamma(1-\beta\cos\theta) - E_0] \tag{8}$$

2.1 One-photon energy spectrum

By observing that:

$$\delta[\gamma E_\gamma(1-\beta\cos\theta) - E_0] = (\gamma\beta E_\gamma)^{-1}\delta\left[\cos\theta - \frac{1}{\beta}\left(1 - \frac{E_0}{\gamma E_\gamma}\right)\right] \quad , \tag{9}$$

the integration of (8) over the whole solid angle is straightforwardly accomplished as:

$$\frac{dN_\gamma}{dE_\gamma} = (\gamma^2\beta E_\gamma)^{-1}\int_{-1}^{1}\frac{\delta\left[\cos\theta - \frac{1}{\beta}\left(1 - \frac{E_0}{\gamma E_\gamma}\right)\right]}{(1-\beta\cos\theta)}d\cos\theta = \frac{1}{\gamma\beta E_0}\Theta\left[1 - \frac{1}{\beta}\left|1 - \frac{E_0}{\gamma E_\gamma}\right|\right] \tag{10}$$

where Θ is the step-function (namely it is zero for negative argument and one otherwise). From this latter, and by making use of (4), we can infer the range of allowed values for the energy of the single decay photon to be:

$$E_- \leq E_\gamma \leq E_+ \ ,$$

where we defined:

$$E_\pm \equiv E_0\gamma(1\pm\beta) = \frac{E_X}{2}(1\pm\beta) \ ,$$

E_X being the relativistic total energy of the X meson.

The obtained flat energy spectrum (10) can easily be rewritten in a compact form as:

$$\frac{dN_\gamma}{dE_\gamma} = \frac{2}{E_X\beta}\Theta[(E_+ - E_\gamma)(E_\gamma - E_-)] \tag{11}$$

2.2 One-photon angular distribution

Let us note that:

$$\delta[\gamma E_\gamma(1-\beta\cos\theta)-E_0] = \frac{1}{\gamma(1-\beta\cos\theta)}\delta\left[E_\gamma - \frac{E_0}{\gamma(1-\beta\cos\theta)}\right] \tag{12}$$

By inserting this latter into (8), and integrating over the allowed interval of energy, we obtain:

$$\frac{dN_\gamma}{d\Omega_\gamma} = \int_{E_-}^{E_+} \frac{\partial^2 N_\gamma}{\partial E_\gamma \partial\Omega_\gamma} dE_\gamma = \frac{1}{2\pi}\frac{1-\beta^2}{(1-\beta\cos\theta)^2} \quad , \tag{13}$$

the following relation holding:

$$E_- \le \frac{E_0}{\gamma(1-\beta\cos\theta)} \le E_+$$

In *fig.*1 the dependence of (13) upon θ and β is shown.

3. Distribution of angular separation and of disparity of the photon pair

3.1 Angular separation distribution

Let us call θ the opening angle, in the laboratory frame, between the directions of the emitted photons, θ_1 and θ_2 being their deflection polar angles respectively. As the azimuthal angle is not affected by relativistic aberration, and the obvious relation $\phi_2 - \phi_1 = \pi$ holding, one easily infers:

$$\cos\theta = \frac{2\beta^2 - 1 - \beta^2\cos^2\theta'}{1-\beta^2\cos^2\theta'} \quad , \tag{14}$$

where we made use of the usual Lorentz transformation between θ_i' and θ_i and of (3); moreover, we briefly defined $\theta' \equiv \theta_i$ for a chosen value of i . From (14) we get:

$$\cos^2\theta' = \frac{1}{\beta^2}\left(1 - \frac{1-\beta^2}{\sin^2\frac{\theta}{2}}\right) \quad , \tag{15}$$

for any value of θ such that:

$$0 \le \cos\frac{\theta}{2} \le \beta$$

From this latter (in which we took into account that $0 \le \theta \le \pi$), the minimum value of the opening angle is easily deduced:

$$\theta_{min} = 2\arccos\beta \tag{16}$$

and, from the (15):

$$d\cos\theta' = \frac{1}{4\beta\gamma^2\sin^3\frac{\theta}{2}\sqrt{\beta^2-\cos^2\frac{\theta}{2}}} d\cos\theta \tag{17}$$

It is perhaps not superflous, now, to observe, from (15), that each value of $\cos\theta$ corresponds to two (opposite) values of $\cos\theta'$, and so the expression for the angular distribution of photon pairs in the laboratory frame is retrieved as:

$$\frac{dN_{\gamma\gamma}}{d\Omega} = 2\frac{dN_{\gamma\gamma}}{d\Omega'}\frac{d\cos\theta'}{d\cos\theta} = \frac{1-\beta^2}{8\pi\beta\sin^3\frac{\theta}{2}\sqrt{\beta^2-\cos^2\frac{\theta}{2}}} \ , \qquad (18)$$

where $[\theta_{min}, \pi]$ is the allowed range of values for θ.

In *fig.* 2 we plot the (18) for two different values of β. We can remark a huge rise and compression of the distribution when θ approaches θ_{min} as $\beta \to 0$; this behaviour is shown to be:

$$\lim_{\beta\to 0}\frac{dN_{\gamma\gamma}}{d\Omega} = \frac{1}{2\pi\sin\theta}\delta(\theta-\pi)$$

3.2 Disparity distribution

In order to obtain the distribution of disparity (i.e. the ratio of energies) of the two photons from the X-meson decay, let us define $u \equiv E_{\gamma\,1}/E_{\gamma\,2}$.

Then, the number of photon pairs characterized by an energy ratio not higher than ε is given [9] by:

$$\begin{aligned} N_{\gamma\gamma}(\varepsilon) &= \frac{1}{2}\int\int_{u\le\varepsilon}\frac{\partial^2 N_\gamma}{\partial E_{\gamma\,1}\partial E_{\gamma\,2}}dE_{\gamma\,1}dE_{\gamma\,2} = \\ &= \frac{1}{2}\int_{E_-}^{E_+}dE_{\gamma\,2}\int_{E_-}^{\varepsilon E_{\gamma\,2}}\frac{\partial^2 N_\gamma}{\partial E_{\gamma\,1}\partial E_{\gamma\,2}}dE_{\gamma\,1} = \\ &= \frac{1}{2}\int_{E_-/E_{\gamma\,2}}^{\varepsilon}du\int_{E_-}^{E_+}E_{\gamma\,2}\left(\frac{\partial^2 N_\gamma}{\partial E_{\gamma\,1}\partial E_{\gamma\,2}}\right)_{E_{\gamma\,1}=uE_{\gamma\,2}}dE_{\gamma\,2} \end{aligned} \qquad (19)$$

Therefore,

$$\frac{dN_{\gamma\gamma}}{d\varepsilon} = \frac{1}{2}\int_{E_-}^{E_+}E_{\gamma\,2}\left(\frac{\partial^2 N_\gamma}{\partial E_{\gamma\,1}\partial E_{\gamma\,2}}\right)_{E_{\gamma\,1}=\varepsilon E_{\gamma\,2}}dE_{\gamma\,2} \qquad (20)$$

In our case, the relation $E_{\gamma\,1}+E_{\gamma\,2}=E_X$ holding, one has:

$$\frac{\partial^2 N_\gamma}{\partial E_{\gamma\,1}\partial E_{\gamma\,2}} = \frac{dN_\gamma}{dE_{\gamma\,1}}\delta[E_X - E_{\gamma\,1} - E_{\gamma\,2}] \qquad (21)$$

By inserting this latter in (20), and by observing that:

$$\begin{aligned} \delta(E_X - E_{\gamma\,1} - E_{\gamma\,2}) &= \delta[E_X - (1+\varepsilon)E_{\gamma\,2}] = \\ &= (1+\varepsilon)^{-1}\delta\left(E_{\gamma\,2} - \frac{E_X}{1+\varepsilon}\right) \ , \end{aligned}$$

we deduce:

$$\frac{dN_{\gamma\gamma}}{d\varepsilon} = \frac{1}{2}\frac{E_X}{(1+\varepsilon)^2}\left(\frac{dN_\gamma}{dE_{\gamma\,1}}\right)_{E_{\gamma\,1}=E_X\varepsilon/(1+\varepsilon)} \ , \qquad (22)$$

from which, by using (11), we finally get:

$$\frac{dN_{\gamma\gamma}}{d\varepsilon} = \frac{1}{\beta(1+\varepsilon)^2}\Theta[(\varepsilon_+ - \varepsilon)(\varepsilon - \varepsilon_-)] \ , \tag{23}$$

where $\varepsilon_\pm \equiv (1 \pm \beta)/(1 \mp \beta)$.

In *fig.* 3 the (23) is displayed for $\beta = 0.5$.

4. Conclusions

In this paper we have introduced a direct way to retrieve analitical expressions for the energy spectrum and the angular distribution of photons from the in-flight decay of neutral scalar mesons. These have been used in order to get the angular separation and energy ratio distributions of the two decay photons. The importance of the meson production mechanisms in heavy-ion collisions at intermediate energies and in particle physics has been also outlined. In this latter context, our presented analytical expressions can be fruitfully used both in the simulation and experimental data analysis for neutral-meson experiments relying upon large solid angle acceptance and high granularity detectors.

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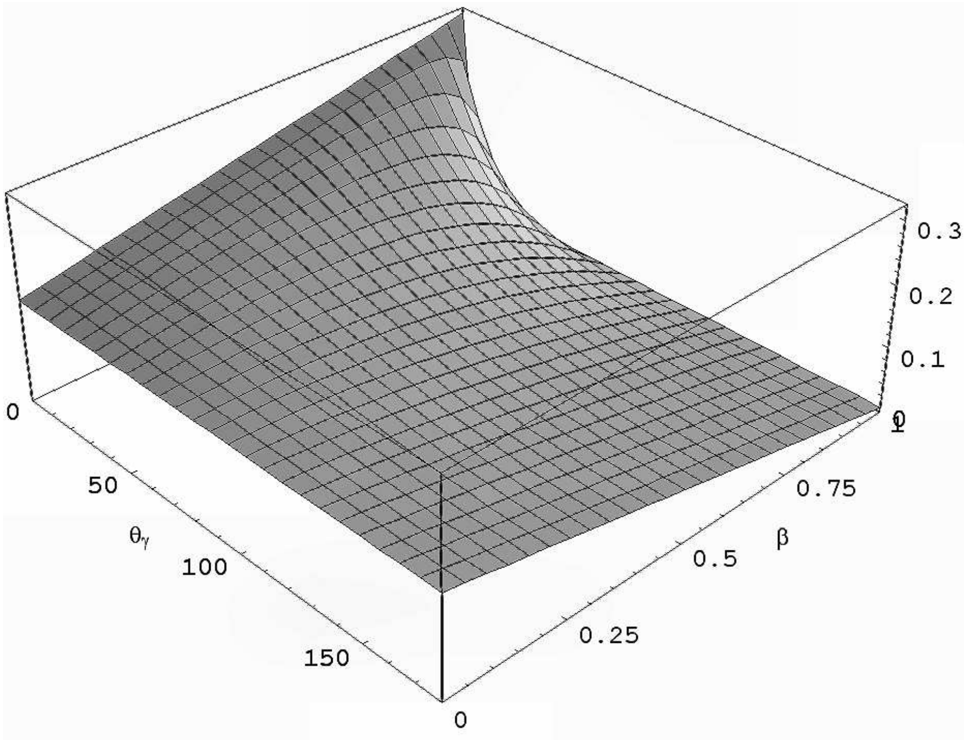

Fig. 1 One-photon angular distribution from the scalar meson decay as a function of β .

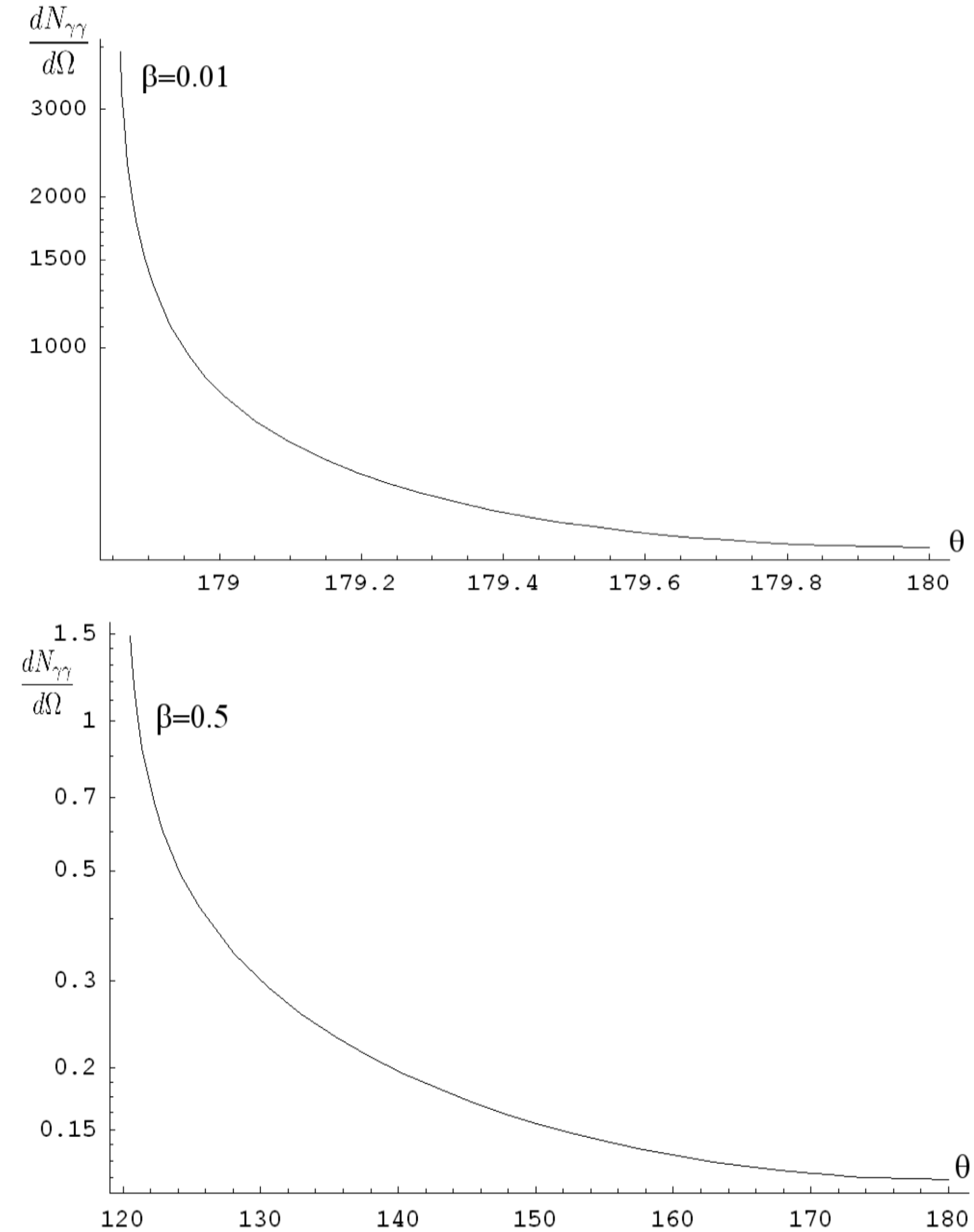

Fig. 2 Angular separation distribution of photon pairs in the laboratory frame for two different values of β.

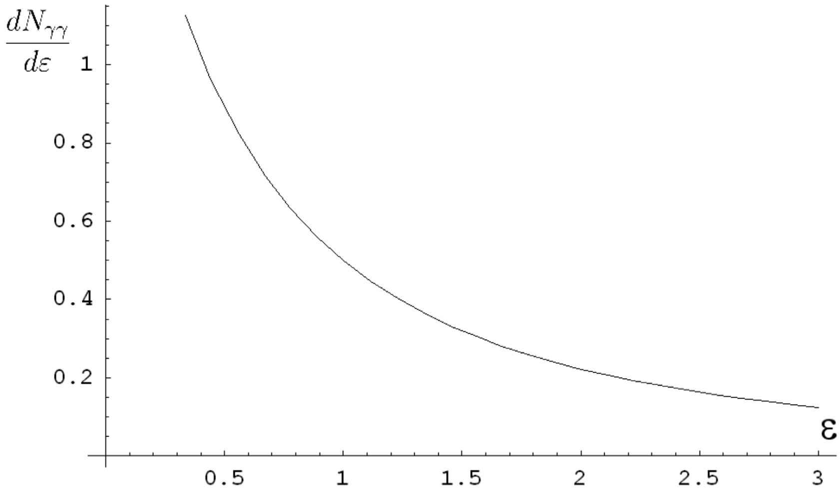

Fig. 3 Distribution of disparity for the photon pair, with $\beta = 0.5$.