Matrix Theory and the Modified Space-Time Uncertainty

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Abstract: We consider the modified space-time uncertainty in the matrix theory point of view. First, we find a suitable theorem for the modified space-time uncertainty. Furthermore, this theorem is proved in the matrix theory compactifications

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There has been a great deal of interest in the idea that the matrix theory may be an effective tool in physical theories. This tool plays an important role in the high-energy physics and quantum field theory. In this paper, we consider the modified space-time uncertainty in the matrix theory compactifications. In the usual space-time uncertainty, a small distance should detect by going to high and higher momenta according to $\Delta x \geq \frac{\hbar}{\Delta p}$. In string theory regime, Witten [1] has shown that for detection of small distances by going to very high momenta, the behavior of "Heisenberg microscope" changes to,

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar}, \quad (1)$$

Relation (1) is a standard result of string theory and scale inversion symmetry,

$$\frac{\Delta p \sqrt{\alpha'}}{\hbar} \leftrightarrow \frac{\hbar}{\Delta p \sqrt{\alpha'}} \quad (2)$$

which relates large and small distances,

$$\frac{\alpha'}{R} \leftrightarrow R \quad (3)$$

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Relation (3) named T-duality. This symmetry relates large and small distance. Numerous calculation of the space-time uncertainty have been done from different point of view and all hint at the modified space-time uncertainty as the relation (1) \[1,3,4,5,6\]. An alternative solution of the modified space-time uncertainty is in the matrix theory.

Let begin with Schwarz inequality, for any \(|\delta|\), and \(|\gamma|\) we can write,

\[
|\gamma| = (\hat{A} - \langle A \rangle_{\alpha})|\alpha| \\
|\delta| = (\hat{B} - \langle B \rangle_{\alpha})|\alpha|
\]

In this framework the Schwarz inequality written as,

\[
<\gamma|\gamma >= <\delta|\delta >= |<\gamma|\delta|^2
\]

In accordance with relation (4), (5) and (6) we have,

\[
<\gamma|\gamma >= |<\alpha| (\hat{A} - \langle A \rangle_{\alpha}) (\hat{A} - \langle A \rangle_{\alpha})|\alpha| = |<\alpha| (\hat{A} - \langle A \rangle_{\alpha})|\alpha| = (\Delta A_{\alpha})^2
\]

and,

\[
<\delta|\delta >= |<\alpha| (\hat{B} - \langle B \rangle_{\alpha}) (\hat{B} - \langle B \rangle_{\alpha})|\alpha| = |<\alpha| (\hat{B} - \langle B \rangle_{\alpha})|\alpha| = (\Delta B_{\alpha})^2
\]

From Schwarz inequality and relation (7), (8) and (6) we conclude

\[
<\gamma|\gamma><\delta|\delta >= (\Delta A_{\alpha})^2(\Delta B_{\alpha})^2 \geq |<\alpha|T|\alpha| |^2 + |<\alpha|T'||\alpha| |^2
\]

Where \(T, T'\) are non-Hermitian operators \[1\] and,

\[
\hat{T} = (\hat{A} - A_{\alpha})(\hat{B} - B_{\alpha}) \\
\hat{T}' = (\hat{B} - B_{\alpha})(\hat{A} - A_{\alpha})
\]

Here \(\hat{T}, \hat{T}'\) must be the Hermitian operators. Hence, we can analyze eqs (10) as,

\[
\hat{T} = \hat{D} + \frac{1}{2}i\hat{C}, \hat{T}' = \hat{F} + \frac{1}{2}i\hat{K}
\]

Where \(\hat{C}, \hat{D}, \hat{F}, \hat{K}\) are the Hermitian operators.

**Theorem:** let \(A\) be coordinate and \(B\), be the canonical momenta. The modified uncertainty relation in matrix theory defined by,

\[
(\Delta A_{\alpha})(\Delta B_{\alpha}) \geq \frac{1}{2}(<C_{\alpha}^2 + <K_{\alpha}^2>)^{1/2}
\]

There is a non-commutativity relation between the coordinate and momenta, hence we can write,

\[
(\hat{A} - A_{\alpha})(\hat{B} - B_{\alpha}) \neq (\hat{B} - B_{\alpha})(\hat{A} - A_{\alpha})
\]
From eqs (13) we have,

\[ \hat{C} = \frac{(\hat{A} - \langle A \rangle)(\hat{B} - \langle B \rangle)}{i} - \frac{(\hat{B} - \langle B \rangle)(\hat{A} - \langle A \rangle)}{i} \]  
(14)

\[ \hat{D} = \frac{(\hat{A} - \langle A \rangle)(\hat{B} - \langle B \rangle)}{2} + \frac{(\hat{B} - \langle B \rangle)(\hat{A} - \langle A \rangle)}{2} \]  
(15)

\[ \hat{K} = \frac{(\hat{B} - \langle B \rangle)(\hat{A} - \langle A \rangle)}{i} - \frac{(\hat{A} - \langle A \rangle)(\hat{B} - \langle B \rangle)}{i} \]  
(16)

\[ \hat{F} = \frac{(\hat{B} - \langle B \rangle)(\hat{A} - \langle A \rangle)}{2} + \frac{(\hat{A} - \langle A \rangle)(\hat{B} - \langle B \rangle)}{2} \]  
(17)

we can write,

\[ (\Delta A_\alpha)^2(\Delta B_\alpha)^2 \geq |\langle T \rangle_\alpha|^2 + |\langle T' \rangle_\alpha|^2 = \frac{1}{4} \langle C \rangle^2_\alpha + \frac{1}{4} \langle K \rangle^2_\alpha \]  
(18)

Consequently, from eqs. (18) we conclude that,

\[ (\Delta A_\alpha)(\Delta B_\alpha) \geq \frac{1}{2} (\langle C \rangle^2_\alpha + \langle K \rangle^2_\alpha)^{1/2} \]  
(19)

Eqs (19) is the genuine modified space-time uncertainty principle in the matrix theory compactifications.

**Conclusion**

We incorporated the modified space-time uncertainly in the matrix theory by a theorem, and furthermore, provident of this theorem considered.
References