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Matrix Theory and the Modified Space-Time Uncertainty

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Abstract: We consider the modified space-time uncertainty in the matrix theory point of view. First, we find a suitable theorem for the modified space-time uncertainty. Furthermore, this theorem is proved in the matrix theory compactifications

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There has been a great deal of interest in the idea that the matrix theory may be an effective tool in physical theories. This tool plays an important role in the high-energy physics and quantum field theory. In this paper, we consider the modified space-time uncertainty in the matrix theory compactifications. In the usual space-time uncertainty, a small distance should detect by going to high and higher momenta according to $\Delta x \geq \hbar/\Delta p$. In string theory regime, Witten [1] has shown that for detection of small distances by going to very high momenta, the behavior of "*Heisenberg microscope*" changes to,

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar}, \tag{1}$$

Relation (1) is a standard result of string theory and scale inversion symmetry,

$$\frac{\Delta p\sqrt{\alpha'}}{\hbar} \leftrightarrow \frac{\hbar}{\Delta p\sqrt{\alpha'}} \tag{2}$$

which relates large and small distances,

$$\frac{\alpha'}{R} \leftrightarrow R \tag{3}$$

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Relation (3) named T-duality. This symmetry relates large and small distance. Numerous calculation of the space-time uncertainty have been done from different point of view and all hint at the modified space-time uncertainty as the relation (1) [1,3,4,5,6]. An alternative solution of the modified space-time uncertainty is in the matrix theory.

Let begin with Schwarz inequality, for any $|\delta>$, and $|\gamma>$ we can write,

$$|\gamma>=(\hat{A}-<A_\alpha>)|\alpha> \tag{4}$$

$$|\delta>=(\hat{B}-<B_\alpha>)|\alpha> \tag{5}$$

In this framework the Schwarz inequality written as,

$$<\gamma|\gamma><\delta|\delta>\geq|<\gamma|\delta>|^2 \tag{6}$$

In accordance with relation (4), (5) and (6) we have,

$$<\gamma|\gamma>=|<\alpha|(\hat{A}-<A_\alpha>)(\hat{A}-<A_\alpha>)|\alpha>=|<\alpha|(\hat{A}-<A_\alpha>)^2|\alpha>|=(\Delta A_\alpha)^2 \tag{7}$$

and,

$$<\delta|\delta>=|<\alpha|(\hat{B}-<B_\alpha>)(\hat{B}-<B_\alpha>)|\alpha>=|<\alpha|(\hat{B}-<B_\alpha>)^2|\alpha>|=(\Delta B_\alpha)^2 \tag{8}$$

From Schwarz inequality and relation (7), (8) and (6) we conclude

$$<\gamma|\gamma><\delta|\delta>=(\Delta A_\alpha)^2(\Delta B_\alpha)^2\geq|<\alpha|\hat{T}|\alpha>|^2+|<\alpha|\hat{T}'|\alpha>|^2 \tag{9}$$

Where $\hat{T},\hat{T}'$ are non-Hermitian operators [1] and,

$$\begin{aligned}\hat{T}&=(\hat{A}-<A>_\alpha)(\hat{B}-<B>_\alpha),\\ \hat{T}'&=(\hat{B}-<B>_\alpha)(\hat{A}-<A>_\alpha)\end{aligned} \tag{10}$$

Here $\hat{T},\hat{T}'$ must be the Hermitian operators. Hence, we can analyze eqs (10) as,

$$\hat{T}=\hat{D}+\frac{1}{2}i\hat{C},\hat{T}'=\hat{F}+\frac{1}{2}i\hat{K}. \tag{11}$$

Where $\hat{C},\hat{D},\hat{F},\hat{K}$ are the Hermitian operators.

Theorem: let A be coordinate and B, be the canonical momenta. The modified uncertainty relation in matrix theory defined by,

$$(\Delta A_\alpha)(\Delta B_\alpha)\geq\frac{1}{2}(<C>_\alpha^2+<K>_\alpha^2)^{1/2}. \tag{12}$$

There is a non-commutativity relation between the coordinate and momenta, hence we can write,

$$(\hat{A}-<A>_\alpha)(\hat{B}-<B>_\alpha)\neq(\hat{B}-<B>_\alpha)(\hat{A}-<A>_\alpha) \tag{13}$$

From eqs (13) we have,

$$\hat{C} = \frac{(\hat{A}- < A >_\alpha)(\hat{B}- < B >_\alpha)}{i} - \frac{(\hat{B}- < B >_\alpha)(\hat{A}- < A >_\alpha)}{i} \tag{14}$$

$$\hat{D} = \frac{(\hat{A}- < A >_\alpha)(\hat{B}- < B >_\alpha)}{2} + \frac{(\hat{B}- < B >_\alpha)(\hat{A}- < A >_\alpha)}{2} \tag{15}$$

$$\hat{K} = \frac{(\hat{B}- < B >_\alpha)(\hat{A}- < A >_\alpha)}{i} - \frac{(\hat{A}- < A >_\alpha)(\hat{B}- < B >_\alpha)}{i} \tag{16}$$

$$\hat{F} = \frac{(\hat{B}- < B >_\alpha)(\hat{A}- < A >_\alpha)}{2} + \frac{(\hat{A}- < A >_\alpha)(\hat{B}- < B >_\alpha)}{2} \tag{17}$$

we can write,

$$(\Delta A_\alpha)^2(\Delta B_\alpha)^2 \geq | < T >_\alpha |^2 + | < T' >_\alpha |^2 = \frac{1}{4} < C >_\alpha^2 + \frac{1}{4} < K >_\alpha^2 \tag{18}$$

Consequently, from eqs. (18) we conclude that,

$$(\Delta A_\alpha)(\Delta B_\alpha) \geq \frac{1}{2}(< C >_\alpha^2 + < K >_\alpha^2)^{1/2} \quad . \tag{19}$$

Eqs (19) is the genuine modified space-time uncertainty principle in the matrix theory compactifications.

Conclusion

We incorporated the modified space-time uncertainly in the matrix theory by a theorem, and furthermore, provident of this theorem considered.

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