EJTP **10** (2006) 1–10 *Electronic Journal of Theoretical Physics*

Non-Minimal Coupling Effects of the Ultra-Light Particles on Photons Velocities in the Radiation Dominated Era of the Universe

El-Nabulsi Ahmad Rami [*]

Plasma Application Laboratory, Department of Nuclear and
Energy Engineering and Faculty of Mechanical, Energy and Production Engineering,
Cheju National University, Ara-dong 1, Jeju 690-756,
Korea

Received 6 September 2005 , Accepted 27 November 2005, Published 25 May 2006

Abstract: The effect of the ultra-light masses of the order of the Hubble constant, implemented in Einstein's field equations *from non-minimal coupling and supergravities arguments*, on photons velocities in the radiation dominated epoch of the Universe within the framework of non-minimal interaction of electromagnetic fields with gravity is developed and discussed in details.

Keywords: Non-minimal coupling, ultra-light masses, effective cosmological constant, photons velocity.
PACS (2006): 04.40 Nr, 98.80 -k, 95.30 SI

1. Introduction

A standard result of Einstein's gravity is that massless particles, in particular photons, move at light celerity 'c'. A question worth examining is whether the velocity of massless photons is shifted when these later propagate in exotic background filled of ultra-light particles (**ULP**) of tiny masses in the order of the Hubble constant ($m \approx H$). This could have important cosmological and astrophysical implications. In fact, the possibility of shifting photon propagation (**SPP**) in gravitational fields (*or non-trivial topologies*) is an interesting prediction of quantum field theory in curved space-time. It appears that photon propagation may depend on their direction and polarisation, travel with speeds exceeding the normal speed of light 'c' [**1**]. It is a quantum effect induced by vacuum

---

[*] atomicnuclearengineering@yahoo.com

polarisation (*allowing the photons to exist as a virtual* e^+e^-*pair so that at the quantum level it is characterized by the Compton wavelength of the electron*) and implies that the Principle of Equivalence does not hold for interacting quantum field theories such as **QED**. The propagation of photon in Schwarzschild, Robertson-Walker, gravitational wave, de Sitter backgrounds, charged black hole were done and remarkable results were discovered [**2**]. In each case (*except the totally isotropic de Sitter space-time*) it was possible to find directions and polarisations for which the photon velocity exceeds 'c'. Generalization to neutrino propagation in a Robertson-Walker metric using the Weinberg-Salam model was done in [**3**]. In a gravitational field, the photon propagation is sensitive to an anisotropic space-time curvature and may depend on this later [**4,5**]. Recently, a series of papers has appeared in which the light velocity varies in the early Universe and this solves the horizon, monopole and the flatness problems in standard cosmology [**6,7,8,9**]. In this work, we will investigate further the consequences of non-minimal coupling on light velocity in the presence of ultra-light particles.

2. Non-minimal Coupling, Supergravities Arguments and Einstein Fields Equations

We start with the non-minimal interaction of electromagnetic fields with gravity in the following form $\tilde{L} = \sqrt{g}\xi R_{\mu\nu}F^{\mu}_{\alpha}F^{\nu\mu}$, ξ being the coupling constant, $R_{\mu\nu}$ the Riemann tensor, g the metric scalar and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic strength (A_μ *is the vector potential*). Terms like RF^2 are neglected here. In this way, the field equations read [**10,11,12**]:

$$D_\nu \left[F^{\mu\nu} - 2\xi \left(R^{\mu}_{\alpha}F^{\alpha\nu} - R^{\nu}_{\alpha}F^{\alpha\mu}\right)\right] = 0$$

where D_ν is the covariant derivative. Let us now restrict our attention to the form of the Riemann tensor we will choose for this work. In a recent paper [**13**], we introduced, for some scalar field , a non-minimal coupling between the scalar curvature and the density of the scalar field in the following form $L = -\xi\sqrt{g}R\phi^*\phi, \xi = 1/6$. R is the scalar curvature and is the complex conjugate of . From a view point of quantum field theory in curved space-time, it is natural to consider such a non-minimal coupling. In fact, the conformal case results in an extension of the property of conformal invariance for massless fields, which is attractive from physical point of view. This parameter describes the strength of the coupling between the curvature of spacetime and the inflation. Minimal coupling corresponds to $\xi = 0$. It was shown that in this case and for a particular scalar negative complex potential field $V(\phi\phi^*) = 3/4m^2(\omega\phi^2\phi^{*2} - 1)$,$\omega$ being a tiny parameter inspired from supergravity inflation theories, ultra-light masses 'm' are implemented naturally in Einstein field equations (**EFE**), leading to a cosmological constant 'Λ' in accord with observations[2]. In matter-free background, the scalar curvature was found to be $=4\bar{\Lambda}$

---

[2] *It has been argued that a non-minimal coupling term-generated by quantum corrections-is to be expected whenever the space-time curvature is large; in most theories that describe inflationary scenarios, it turns out that a value of* ξ *different from zero is unavoidable. As a matter of fact, it seems sensible to consider an explicit non-minimal coupling in the supergravities inflationary paradigm.*

where $\bar{\Lambda} = \Lambda - 3/4m^2$ is the *effective cosmological constant*[3] (*in natural units,* $m \approx \hbar H/c^2$ *where '*$\hbar$*' is the Planck constant and '*c*' being the celerity of light*). As a result, one candidate Einstein field equations is($\hbar = c = 1$):

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\bar{\Lambda}g_{\mu\nu} \tag{1}$$

$\bar{\Lambda} = \Lambda - 3m^2/4 \equiv \Lambda_1 + \Lambda_2$ is the effective cosmological constant ($\Lambda_2 = -3\lambda\!\!\!^{-}{}_C^{-2}$ *where* $\lambda_C = \hbar/mc$ *is the Compton wavelength in natural units*). Remark that for $\Lambda_1 = 0$, the scalar curvature is negative and the space-time is not Minkowskian. Other field equations exist also but correspond only for $p = \rho/3$ (*radiation era*):

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi G\left[\left(p + \rho + \frac{3m^2}{8\pi G}\right)u_\mu u_\nu + pg_{\mu\nu}\right] \tag{2a}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G\left[\left(p + \rho + \frac{3m^2}{8\pi G}\right)u_\mu u_\nu + \left(p + \frac{\Lambda}{8\pi G}\right)g_{\mu\nu}\right] \tag{2b}$$

$$= -8\pi G\left[\left(p + \bar{\rho}\right)u_\mu u_\nu + \left(p + \frac{\Lambda}{8\pi G}\right)g_{\mu\nu}\right] \tag{2c}$$

$$\equiv -8\pi G\left[T_{\mu\nu}\left(p,\rho\right) + \frac{\Lambda}{8\pi G}g_{\mu\nu} + t_{\mu\nu}\left(m\right)\right] \tag{2d}$$

$$\equiv -8\pi G\left[T_{\mu\nu}\left(p,\rho\right) + T_{\mu\nu}\left(\Lambda,m\right)\right] \tag{2e}$$

$$\equiv -8\pi G\sum T_{\mu\nu} \tag{2f}$$

p and ρ are the pressure and density of matter,$\bar{\rho} = \rho + 3m^2/8\pi G$, $t_{\mu\nu} = 3m^2/8\pi G u_\mu u_\nu$ and:

$$T_{\mu\nu}\left(p,\rho\right) = \left(p + \rho\right)u_\mu u_\nu + pg_{\mu\nu} \tag{3}$$

$$T_{\mu\nu}\left(\Lambda,m\right) = t_{\mu\nu} + \frac{\Lambda}{8\pi G}g_{\mu\nu} = \frac{1}{8\pi G}\left[3m^2u_\mu u_\nu + \Lambda g_{\mu\nu}\right] \tag{4}$$

Contracting equations (2) with $g^{\mu\nu}$ using $g^{\mu\nu}u_\mu u_\nu = -1$ yields of course. In this way, the ultra-light masses and the cosmological constant are parts of the matter contents of the Universe rather than geometrical entities. The radiative field equations (**RFE**) (2-a,b,c,d,e,f) are identical to that of Einstein standard ones but with an additional energy density $\rho_m = 3m^2/8\pi G(m \leq H)$. One can also refer to equation (4) as the stress-energy tensor of vacuum and light particles, which is a "*microscopic stress-energy tensor*". In fact, the conservation law holds and we have:

$$\nabla^\nu\sum T_{\mu\nu} \equiv \nabla^\nu T_{\mu\nu}\left(p,\rho\right) + \nabla^\nu T_{\mu\nu}\left(\Lambda,m\right) = 0 \tag{5}$$

When $T_{\mu\nu}\left(p,\rho\right) = 0$, the microscopic stress-energy tensor will behave as the macroscopic one if we assume that:

$$P\left(\Lambda,m\right) \equiv P_\Lambda = \Lambda/8\pi G \tag{6}$$

---

[3] *In* [**13**], $8\pi G \equiv \kappa$ *was set equal to unity.*

$$\rho\left(\Lambda,m\right)=\frac{3m^2-\Lambda}{8\pi G} \tag{7}$$

That is, in the microscopic version, if $\Lambda=0$, $P=0$ but the density is positive. While for $\Lambda>3m^2$, the pressure is positive and the density is negative. If $0<\Lambda<3m^2$, then both the pressure and the density are positive.

Before treating the non-minimal coupling scenario, we will discuss briefly the implications of equations (2) in standard cosmology. For this, we consider a homogenous and isotropic Universe in the radiation dominated epoch described by Friedman-Robertson-Walker line element with scale factor $a\left(t\right)$[**14**]. The radiative field equations read:

$$\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}=\frac{8\pi G\rho}{3}+\frac{\Lambda}{3}+m^2 \tag{8}$$

$$\frac{\ddot{a}}{a}=-\frac{8\pi G\rho}{3}+\frac{\Lambda}{3}-\frac{m^2}{2} \tag{9}$$

$k=-1,0,+1$ is the curvature constant for open, flat or closed space-time and $\dim\left(\Lambda\right)=\dim\left(m^2\right)=length^{-2}$. If the cosmological constant and the ultra-light masses are assumed to be constant with time, then from the energy conservation law: $\rho\propto a^{-4}$. For zero density, $2\Lambda>\left(<\right)3m^2$ and the acceleration of the Universe accelerate (*decelerate*) with time($\ddot{a}>\left(<\right)0$). Combining equation (8) and (9), we can eliminate the density factor and obtains:

$$\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}=\frac{2\Lambda}{3}+\frac{m^2}{2}=\frac{2}{3}\Lambda\left(1+\frac{3m^2}{4\Lambda}\right)\equiv\frac{2}{3}\hat{\Lambda} \tag{10}$$

where $\hat{\Lambda}=\Lambda\left(1+3m^2/4\Lambda\right)$.

If we restrict ourselves to spatially flat universes ($k=0$) and we use the definition $H\equiv\dot{a}/a=H_0\left(\frac{da}{adT}\right)$ where time is assumed to be measured in units of Hubble times $T\equiv H_0t$, then:

$$\frac{dH}{dT}+2\frac{H^2}{H_0}=\frac{2\hat{\Lambda}}{3H_0} \tag{11}$$

Assuming that $\hat{\Lambda}\propto T^{-\alpha}$, that is the cosmological constant and the ultra-light masses decrease with time, this model was found to be singular but can significantly be older than models with constant Λ and m^2 [**15,16,17,18,19**]. For $m^2<<\Lambda$, $\hat{\Lambda}\to\Lambda$, while for $m^2>>\Lambda$, $\hat{\Lambda}\to3/4m^2$. If for instance, we assume that $m^2=\beta/a^2$and $\Lambda=\delta\dot{a}^2/a^2+\eta\ddot{a}/a,\beta,\delta,\eta$ are constants [**20,21**], than from equation (10)[4]:

$$\frac{\ddot{a}}{a}\left(1-\frac{2\eta}{3}\right)+\frac{\dot{a}^2}{a^2}\left(1-\frac{2\delta}{3}\right)+\left(k-\frac{\beta}{2}\right)\frac{1}{a^2}=0 \tag{12}$$

which gives:

$$\dot{a}^2=\frac{3\left(\beta-2k\right)}{2\left(3-2\delta\right)}+Da^{-\frac{2(3-2\delta)}{3-2\eta}},D=const. \tag{13}$$

---

[4] *The fact that the two terms Λ and m^2 play the role of two cosmological constant in the theory, we have the freedom to choose $\Lambda=\delta_1\dot{a}^2/a^2+\eta_1\ddot{a}/a+\beta_1/a^2$ and $m^2=\delta_2\dot{a}^2/a^2+\eta_2\ddot{a}/a+\beta_2/a^2$where $\delta_{1,2},\eta_{1,2},\beta_{1,2}$ are constants. In this work, we simplified our assumptions just to have at the beginning a simple idea about the effects of the ultra-light masses in the theory.*

For $D = 0$ which corresponds to singular solutions, one finds for flat space-time($k = 0$):

$$a = \sqrt{\frac{3\beta}{2\left(3 - 2\delta\right)}t} \tag{14}$$

where $\delta < 3/2, \beta > 0$. In this way :

$$m^2 = \frac{2\left(3 - 2\delta\right)}{3t^2} \tag{15}$$

$$\Lambda = \frac{\delta}{t^2} \tag{16}$$

From equation (8), we find:

$$\rho = \frac{3\left(\delta - 1\right)}{8\pi G t^2} \tag{17}$$

with $1 < \delta < 3/2$. In this way, we don't have an inflationary phase and no horizon problem appears. From the above equations, we see that the ultra-light masses, the cosmological constant and the density are independent of the value of β and η. The Hubble parameter is $H = \dot{a}/a = 1/t$ and the density matter of the Universe is given by $\Omega^r = \rho/\rho_c = \delta - 1 < 1/2$ where $\rho_c = 3H^2/8\pi G$ is the critical density. The deceleration parameter is $q \equiv -\ddot{a}a/\dot{a}^2 = 0$. The density parameter due to vacuum contribution is $\Omega^\Lambda = \Lambda/3H^2 = \delta/3$ and that due to ultra-light particles contributions is $\Omega^m = m^2/H^2 = 2\left(3 - 2\delta\right)/3$. In this way $\Omega^{Total} = \Omega^r + \Omega^m + \Omega^\Lambda = 1$ as required by inflation [**22**]. The ultra-light particles than contribute to the total energy density and their masses decrease as inverse to time. Note from equations (15) and (16) that $\Lambda = 3\delta m^2/2\left(3 - 2\delta\right) < 9m^2/4$. Finally, note that when the 'Λ' and 'm^2' terms dominate the dynamics of equation (8) with the assumption that the Universe undergoes a long period of evolution during which the celerity of light changes as $c = c_0 a^n$, c_0, n =constants [**8**]:

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2\left(t\right)}{3} + m^2 c^2\left(t\right) \rightarrow \left(\frac{\Lambda}{3} + m^2\right) c_0^{2n} a^{2n} \tag{18}$$

So at large times, we have $a \propto t^{-1/n}$ and it was found in [**8**] that for negative "n", there is a solution to the quasi-lambda problem.

In order to have a very simple idea about the role of the ultra-light masses in the theory, we suppose that the space-time is flat, that is $k = 0$ with the following behavior of the ultra-light masses $m^2 = \beta/a^2$and the cosmological constant $\Lambda = \delta/a^2, \beta, \eta$=constants (*see footnote 4*) [**23,24,25**]. In this case, when 'Λ' and 'm^2' terms dominate at large times the dynamics of equation (8):

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2\left(t\right)}{3} + m^2 c^2\left(t\right) = \left(\beta + \frac{\delta}{3}\right) c_0^2 a^{2n-2} \tag{19}$$

That is $a \propto t^{-1/n-1}$ and from [**8,26**], it is required that $n < 0$ and $c = c_0 t^{-n/n-1}$. In summary, $m^2 \propto t^{2/n-1}$ and as a result $mc \propto 1/t$. Another way to study shifting and time-varying photons velocities is by using the non-minimal coupling of electromagnetic fields and gravity.

3. Varying Photons Velocities From Non-Minimal Coupling

Following [**10,11**], we admit the existence of a surface S represented by $\phi\left(x\right)=0$. The wavenumber of the photon trajectories is given by the gradient of its phase $k_\lambda=\nabla_\lambda\phi$ where the Faraday tensor vanishes at its hypersurface, that is $(F_{\mu\nu})_S=0$. Its derivative defines a function $\phi_{\mu\nu}$ such that:

$$\left(\partial_\lambda F_{\mu\nu}\right)_S=\left(D_\lambda F_{\mu\nu}\right)_S=k_\lambda\phi_{\mu\nu} \tag{20}$$

As a consequence, equation (1) takes the form:

$$\left[\phi^{\mu\nu}-2\xi\left(R^\mu_\alpha\phi^{\alpha\nu}-R^\nu_\alpha\phi^{\alpha\mu}\right)\right]k_\nu=0 \tag{21}$$

In the radiation dominated era, it follows that:

$$\begin{aligned}\phi^{\mu\nu}k_\nu-2\xi\Bigg\{&\left[\phi^{\alpha\nu}.\left(-\chi\right)\left\{\left\{\frac{4\rho}{3}+\rho_m\right\}u^\mu u_\nu+\left\{\frac{\rho}{3}+\frac{\rho_m}{2}-\frac{\Lambda}{\chi}\right\}\right\}\delta^\mu_\nu\right]\\&+\left[-\phi^{\alpha\mu}.\left(-\chi\right)\left\{\left\{\frac{4\rho}{3}+\rho_m\right\}u^\nu u_\alpha+\left\{\frac{\rho}{3}+\frac{\rho_m}{2}-\frac{\Lambda}{\chi}\right\}\right\}\delta^\nu_\alpha\right]\Bigg\}k_\nu=0\end{aligned} \tag{22}$$

where $\chi=8\pi G\quad(\hbar=c=1)$. For simplicity, we let $k_0=k_\mu u^\mu$ and we use the antisymmetric fact of $(\phi^{\mu\nu}-\phi^{\nu\mu}=2\phi^{\mu\nu})$ as well as Maxwell equations:

$$\phi_{\mu\nu}k_\lambda+\varphi_{\nu\lambda}k_\mu+\phi_{\lambda\mu}k_\nu=0 \tag{23}$$

By contracting by k^λ the last equation, equation (23) reduces to:

$$\phi^{\mu\nu}k_\nu=\frac{-2\xi\chi\left(\frac{4\rho}{3}+\rho_m\right)}{1+4\chi\left(\frac{\rho}{3}+\frac{\rho_m}{2}-\frac{\Lambda}{\chi}\right)\xi}k_0\phi^{\mu\nu}u_\nu\equiv\left(N\right)k_0\phi^{\mu\nu}u_\nu \tag{24}$$

Replacing (24) into (23), then:

$$\phi_{\mu\nu}k^2+N\left(-\phi_{\mu\nu}k_\lambda-\varphi_{\lambda\mu}k_\nu-k_\nu\phi_{\mu\lambda}\right)k_0u^\lambda=0 \tag{25}$$

The antisymmetric of $\phi_{\mu\nu}$eliminates all the terms in the parentheses of equation (25) and we are left with:

$$\phi_{\mu\nu}\left(k^2-Nk_0^2\right)=0\Rightarrow k^2-Nk_0^2=0,\forall\phi_{\mu\nu} \tag{26}$$

The effective photons velocity, in case all$\rho,\rho_m,\Lambda\neq0,\forall\xi$ is then given by [**6**]:

$$v^2=\frac{\left|k_ik^i\right|}{k_0^2}=\left|1+N\right|=\left|\frac{1-4\xi\Lambda-4\xi\chi\frac{\rho}{3}}{1+4\xi\chi\left(\frac{\rho}{3}+\frac{\rho_m}{3}-\frac{\Lambda}{\chi}\right)}\right| \tag{27}$$

and the light velocities is not equal to 'c'. Adopting equations (15), (16) and (17), equation (27) takes the form in normal units:

$$v^2=\left|\frac{1-4\frac{\xi}{t^2}\left(2\delta-1\right)}{1+4\frac{\xi}{t^2}\left(5-4\delta\right)}\right| \tag{28}$$

with $1 < \delta < 3/2$. As a result, for $\xi > (<)\, 0$, $v_{photons} < (>)\, c$ (*light celerity*). Adopting the fact $c = c_0 a^n$ with $n < 0$, than the photons velocities decreases with time whatever is the sign of the coupling constant.

An interesting case is when the background is '*free from matter*' . From (27) we get:

$$v^2 = \left| \frac{1 - 4\xi\Lambda}{1 + 4\xi\left(m^2 - \Lambda\right)} \right| \tag{29}$$

If $m^2 = 0$, than $v^2 = 1$ which is light celerity in units ($\hbar = c = 1$). Assuming $m^2 = \beta/t^2$, $4\Lambda = 3m^2$ or $\bar{\Lambda} = 0$ and as a result $R = 0$. In this case, equation (28) gives:

$$v^2 = \left| \frac{1 - 3m^2\xi}{1 + m^2\xi} \right| = \left| \frac{1 - \xi\frac{3\beta}{t^2}}{1 + \xi\frac{\beta}{t^2}} \right| = \left| \frac{t^2 - 3\xi\beta}{t^2 + \xi\beta} \right| \tag{30}$$

Again, if $\xi > (<)\, 0$, $v_{photons} < (>)\, c$ (*light celerity*). As a result, the velocity of photons is affected and shifted by the presence of the ultra-light tiny masses and depends on the sign of the coupling constant. It doesn't correspond in fact to null geodesics as in the standard case. Positive coupling constant corresponds to friction and negative one corresponds to superluminal case [**27,28**]. If we adopt the fact $c = c_0 a^n$, then the photons velocities not only is shifted but also decrease with time if $n < 0$ and increase if $n > 0$.

The constancy of the speed of light is not preserved in this analysis. It depends on how is filled the background space and how is used a coupling constant different of zero that modifies presentation of the Einstein's Field Equations (*EFT*), with an additional term. It is important to notice that the environment where speed of light reaches its maximum value is the lightest one: the empty space, all because of the constancy of the speed of light law, which in time, originates the fourth time-coordinate. In our case, the red-shift coefficient 'z' *varies with time* according to $cz = Hr$ combined to equations (29) or (30) for a matter free background. 'r' is supposed to be the distance form the galaxy to the earth [**14**]. If the coupling constant is assume to be positive, one can than have a cosmological model based on interpretation of the red shift by decrease of the light speed with time everywhere in the universe beginning with a certain moment of time in the past. Of course, the agreement with the fundamental physics laws will be completed by introducing in a future work the evolution of other fundamental constants synchronously with the variation of the light speed [**29**].

Finally, we note that recently, growing amount of astrophysical data show important evidence for statistical and apparent physical association between low-redshift galaxies and high-redshift quasi-stellar objects suggesting noncosmological origin of their redshift and as a result failure of classical quasar explanation [**30**]. The author found analytical solution of Einstein equations describing bubbles made from axions with periodic interaction potential considered as one of the leading dark matter candidate. Remember that in our model [**13**], the ultra-light masses implemented in Einstein field equations enabled us to solve the '*missing mass problem*' and as result considered as dark matter candidate. In *Minkowski space*, objects at constant proper distance with respect to an observer have zero redshift. However, in an expanding universe special relativistic concepts do not generally apply. In fact, a galaxy with zero total velocity does not have zero redshift even in the empty universe case. This demonstrates that cosmological redshifts are not special relativity Doppler shifts [**31,32**]. It was also proved that Minkowski coordinate and the Robertson-Walker coordinates (*FRW universe*) are interchangeable descriptions for an empty universe. However, velocities in the Minkowski universe are not equivalent to velocities in the *FRW* universe because of the different definitions of time and distance in these two models. A coordinate transform relates velocities in the Minkowski universe to velocities in the *FRW* universe. Superluminal recession velocities in the *FRW* universe do not violate special relativity because they are not in the observer's inertial frame.

4. Conclusions:

In this work, we used the Einstein's field equations with effective cosmological constant inspired from non-minimal coupling and supergravities arguments to study the consequences of non-minimal coupling between electromagnetic fields and gravity on light velocity in the presence of ultra-light particles at the radiation dominated epoch of the Universe. We showed that if the cosmological constant and the ultra-light square masses varies as $\Lambda = \delta\dot{a}^2/a^2 + \eta\ddot{a}/a$ and $m^2 = \beta/a^2$, then at singular solutions and for a flat spacetime, Λ, m^2 and ρ decreases with time as $1/t^2$, the Hubble parameter vary as $H = 1/t$, the deceleration parameter is zero and $\Omega^{Total} = \Omega^r + \Omega^m + \Omega^\Lambda = 1$ as required by inflation. As a result, the ultra-light particles than contribute to the total energy density and their masses decrease as inverse to time. When the 'Λ' and 'm^2' terms dominate the dynamics of our field equations with the assumption that the Universe undergoes a long period of evolution during which the celerity of light changes as $c = c_0 a^n$, it was found that at large times $a \propto t^{-1/n}$ and that for negative 'n', there is a solution to the quasi-lambda problem. Finally, we studied varying light velocities from non-minimal coupling. We found that photons velocities depends on the coupling constant and only on $\delta \in (1, 3/2)$in a way that $\xi > (<)\, 0$, $v_{photons} < (>)\, c$ with $m^2 > 0$.

The model described in this paper could have important implications in various systems, in particular cosmological scenarios, black hole physics and quantum interactions [**33,34,35,36,37**]. It is important to discuss the impact of the assumptions established to reach the model and the fact that a non-Minkowskian space is necessary to obtain speeds greater than that of light, and that photon could be represented by a positron-electron pair for characterizing it with the electron's Compton wavelength.

To further investigate all these issues, further studies will be necessary and work is in progress.

Acknowledgments:

The author is grateful for the referees for their useful comments and suggestions.

References

[1] I.T. Drummond and S.J. Hathrell, *Phys. Rev.* **D** 22, 343 (1980).

[2] R.D. Daniels and G.M. Shore, *Nucl. Phys.* **B** 425,634 (1994).

[3] Y. Ohkuwa, *Prog. Theor. Phys.* 65, 1058 (1981).

[4] R. G. Cai, *Nucl. Phys.* **B** 524,639 (1998).

[5] G. M. Shore, *Nucl. Phys.* **B** 460, 379 (1996).

[6] J. D. Barrow, *astro-ph/9811022.*

[7] A. Albrecht and J. Maguiejo, *Phys. Rev D 59, 043516 (1999) 043516.*

[8] J. D. Barrow and J. Maguiejo, *Phys. Lett. B 443, 104-110 (1998).*

[9] P. Teyssandier, *Annales de la Fondation Louis de Broglie*, Volume 29 no 1-2, (2004) (*gr-qc/0303081*).

[10] M. Novello, L. A. R Olivera and J. M. Salim J.M., *Class. Quant. Grav.* 7, 51 (1990).

[11] M. Novello and S. D. Jorda, *Mod. Phys. Lett.* **A**, vol 4, n19, 1809, (1989).

[12] 1-I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, *Effective Action in Quantum Gravity*, IOP-Publishing Ltd, (1982).

[13] R. A. El-Nabulsi, *Phys. Letts.* **B**, Vol. 619, 26 (2005).

[14] S. Weinberg, *Gravitation and Cosmology*, NY, Wiley (1972).

[15] J. M. Overduin and F. I. Cooperstock, *astro-ph/9805260.*

[16] J. W. Moffat, *Int. J. Mod. Phys.* **D**2, 351 (1993).

[17] J. W. Moffat, *Found. Of Phys.*, 23, 411 (1993).

[18] M. A. Clayton and J. W. Moffat, *Phys. Lett.***B** 460, 263 (1999).

[19] M. A. Clayton and J. W. Moffat, *Phys. Lett.***B** 477, 269 (2000).

[20] A. S. Al-Rawaf, *Mod. Phys. Lett.* **A** 13, 429 (1998).

[21] A. I. Arbab, *Spacetime and Substance 1(6), 39 (2001).*

[22] A. H. Guth, *Phys. Rev.* **D** 23, 347 (1981).

[23] M. S. Berman, *Nuovo Cimento* 74**B**, 182 (1983).

[24] M. S. Berman, *Gen. Rel. Grav.* 23, 465 (1991).

[25] M. S. Berman and F. M. Gomide, Gen. Rel. *Grav.* 20, 191 (1988).

[26] M. S. Berman and L. A. Trevisan, *gr-qc/0112011.*

[27] D. Syer, *astro-ph/9404063.*

[28] T. Chiba and K. Kohri, *Prog. Theor. Phys.* 107, 631 (2002).

[29] V. S. Troitskii, *Astrophysics and Space Science* 139, 389 (1987).

[30] A. A. Svidzinsky, *astro-ph/0409064.*

[31] Davis T. M., Lineweaver C. H., 2001, in Durrer R., Garcia-Bellido J., Shaposhnikov M., eds, *Cosmology and Particle Physics* 2000, American Institute of Physics conference proceedings, Volume 555. New York, p. 348.

[32] Davis T. M., Lineweaver C. H., Webb J. K., Am. J. Phys., 71, 358 (2003)

[33] C. F. E. Holzhey and F. Wilzeck, *Nucl. Phys.* **B** 380, 447 (1992).

[34] R. A. El-Nabulsi, *Chinese Phys. Lett. 23* No **5**, 1124-1127 (2006).

[35] R. A. El-Nabulsi, *Elec. J. Theor. Phys.* **7**, 27-46 (2005).

[36] R. A. El-Nabulsi, *Elec. J. Theor. Phys.* **9**, 1-5 (2006).

[37] R. A. El-Nabulsi, Supergravity, Non-mimimal Coupling and Eternal Accelerated Expansion: Some important Cosmological Features of Ultra-Light Particles and Induced Cosmological Constant, Submitted to *Elec. J. Theor. Phys.*