

The Quantum Character of Physical Fields. Foundations of Field Theories

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Abstract: The existing field theories are based on the properties of closed exterior forms, which are invariant ones and correspond to conservation laws for physical fields. Hence, to understand the foundations of field theories and their unity, one has to know how such closed exterior forms are obtained. In the present paper it is shown that closed exterior forms corresponding to field theories are obtained from the equations modelling conservation (balance) laws for material media. It has been developed the evolutionary method that enables one to describe the process of obtaining closed exterior forms. The process of obtaining closed exterior forms discloses the mechanism of evolutionary processes in material media and shows that material media generate, discretely, the physical structures, from which the physical fields are formed. This justifies the quantum character of field theories. On the other hand, this process demonstrates the connection between field theories and the equations for material media and points to the fact that the foundations of field theories must be conditioned by the properties of material media. It is shown that the external and internal symmetries of field theories are conditioned by the degrees of freedom of material media. The classification parameter of physical fields and interactions, that is, the parameter of the unified field theory, is connected with the number of noncommutative balance conservation laws for material media.

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Introduction

Originally, beginning from the 17th century, the physics based on the differential equations, which describe physical processes. However, from the 20th century, the prob-

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lem of invariant (independent of the choice of the coordinate system) description of physical phenomena arose. As the result, the formalisms based on the tensor, group, variational methods, on the theories of symmetries, transformations and so on with the basic requirement of invariance were developed in physics.

This gave rise to building the field theories that enable one to describe physical fields and their interactions. Such theories were based on the postulates turned out practically not to be connected with the equations of mathematical physics, which describe physical processes. Just the absence of such a connection produced the emergence of the problems in field theories that are connected with investigation of general foundations of existing field theories, their unity, and constructing the general field theory.

In present paper it is shown that the connection of field theories with the equations describing physical processes in material media must lie at the basis of the general field theory.

The investigation of the foundations of field theories has been carried out using skew-symmetric differential forms. Skew-symmetric differential forms, which deal with differentials and differential expressions, can describe a conjugacy of various operators and objects. This is of principal importance for mathematical physics and field theories since the conjugated objects are invariants.

The properties of existing field theories are those of the closed exterior skew-symmetric differential forms [1,2], which are conjugated objects and correspond to conservation laws for physical fields. The properties of closed exterior forms explicitly or implicitly manifest themselves essentially in all formalisms of field theory, such as the Hamilton formalism, tensor approaches, group methods, quantum mechanics equations, the Yang-Mills theory and others. The gauge transformations (unitary, gradient and so on), the gauge symmetries and the identical relations of field theories are transformations, symmetries and relations of the theory of closed exterior forms.

Such connection between field theories and the theory of closed exterior forms enables one to understand the properties of field theories, which are common for all existing field theories.

However, this does not solve the basic problems of field theories. To understand the general foundations of field theories and their unity, one must know how the closed exterior forms connected with field theories are obtained.

It is known that the closed exterior forms are obtained from differential equations provided the requirements of integrability of these equations [1].

In the present paper it is shown, firstly, that the equations describing (balance) conservation laws for *material media* serve as the differential equations from which the closed exterior forms related to field theories and corresponding conservation laws for *physical fields* are obtained. And, secondly, it is developed the method, which is evolutionary one hence this method allows to find not only closed exterior forms, but also to describe the process of obtaining closed exterior forms.

The process of obtaining closed exterior forms, on one side, demonstrates the connection between field theories and the equations for material media, and, on other side,

discloses the mechanism of evolutionary processes in material media and shows that material media generate physical fields. This underlines the fact that the foundations of field theories, namely, the theories describing physical fields, must be conditioned by the properties of material media.

It has been possible to carry out the investigation of general foundations of field theories due to the skew-symmetric differential forms, which, unlike to the exterior forms, are defined on deforming manifolds and hence they possess the evolutionary properties. The mathematical apparatus of such forms includes some nontraditional elements such as nonidentical relations and degenerate transformation, and this enables one to describe the evolutionary processes, discrete transitions, quantum jumps, and generation of various structures.

In the first section the general properties of field theories are investigated with the help of closed exterior forms. In the next section the analysis of the equations of the balance conservation laws for material media, which describe the state of material system and the mechanism of generating physical structures forming physical fields. In the last section the general foundations of field theories obtained from the analysis of the equations for material media are discussed.

1. Connection Of Field Theories with the Theory of Closed Exterior Forms

Closed Exterior Forms and Conservation Laws

From the closure condition of the exterior form θ^p (p -form):

$$d\theta^p = 0 \quad (1.1)$$

one can see that the closed exterior form θ^p is a conserved quantity. This means that this can correspond to a conservation law, namely, to some conservative quantity. If the form is closed only on pseudostructure, i.e. this form is a closed inexact one, the closure conditions are written as

$$d_\pi \theta^p = 0 \quad (1.2)$$

$$d_\pi {}^* \theta^p = 0 \quad (1.3)$$

where ${}^* \theta^p$ is the dual form.

Condition (1.3), i.e. the closure condition for dual form, specifies the pseudostructure π . {Cohomology, sections of cotangent bundles, the eikonal surfaces, potential surfaces, pseudo-Riemannian and pseudo-Euclidean spaces, and others are examples of the pseudostructures and manifolds that are made up by pseudostructures.}

From conditions (1.2) and (1.3) one can see the following. The dual form (pseudostructure) and closed inexact form (conservative quantity) made up a conjugated conservative object that can also correspond to some conservation law. The conservation laws, to which physical fields are subject, are just such conservation laws.

The conservative object made up by the closed inexact exterior form and corresponding dual form is a differential-geometrical structure. (Such differential-geometrical structures are examples of G-structures.) The physical structures, which made up physical fields, and corresponding conservation laws are such differential-geometrical structures,

Properties of Closed Exterior Differential Forms

Invariance. Gauge transformations. It is known that the closed exact form is the differential of the form of lower degree:

$$\theta^p = d\theta^{p-1} \quad (1.4)$$

Closed inexact form is also a differential, and yet not total but interior on pseudostructure

$$\theta_\pi^p = d_\pi\theta^{p-1} \quad (1.5)$$

Since the closed form is a differential (a total one if the form is exact, or an interior one on the pseudostructure if the form is inexact), it is obvious that the closed form turns out to be invariant under all transformations that conserve the differential. The unitary transformations, the tangent, the canonical, the gradient transformations and so on are examples of such transformations. *These are gauge transformations of field theories.*

With the invariance of closed forms it is connected the covariance of relevant dual forms.

Conjugacy. Duality. Symmetries. The closure of exterior differential forms is the result of conjugating the elements of exterior or dual forms. The closure property of the exterior form means that any objects, namely, the elements of exterior form, the components of elements, the elements of the form differential, the exterior and dual forms, the forms of sequential degrees and others, turn out to be conjugated.

With the conjugacy it is connected the duality.

The example of a duality having physical sense: the closed exterior form is a conservative quantity corresponding to conservation law, and the closed form (as the differential) can correspond to a certain potential force.

The conjugacy is possible if there is one or another type of symmetry.

The gauge symmetries, which are interior symmetries of field theory and with which gauge transformations are connected, are symmetries of closed exterior differential forms. Symmetries of closed dual forms are exterior symmetries of the equations of field theory.

Identical relations of exterior forms. Since the conjugacy is a certain connection between two operators or mathematical objects, it is evident that, to express the conjugacy mathematically, it can be used relations. These are identical relations.

The identical relations express the fact that each closed exterior form is the differential of some exterior form (with the degree less by one). In general form such an identical relation can be written as

$$d_\pi\varphi = \theta_\pi^p \quad (1.6)$$

In this relation the form in the right-hand side has to be a *closed* one.

Identical relations of exterior differential forms are a mathematical expression of various types of conjugacy that leads to closed exterior forms.

Such relations like the Poincaré invariant, vector and tensor identical relations, the Cauchy-Riemann conditions, canonical relations, the integral relations by Stokes or Gauss-Ostrogradskii, the thermodynamic relations, the eikonal relations, and so on are examples of identical relations of closed exterior forms that have either the form of relation (1.6) or its differential or integral analogs.

The Analysis of the Properties of Field Theories Using Closed Exterior Forms

One can see that the properties of closed exterior differential forms correspond to the properties of field theories. Hence, the mathematical principles of the theory of closed exterior differential forms made up the basis of field theories that is common for all existing field theories. (It should be emphasized that the field theories are connected with the properties of *inexact* closed exterior forms.)

The connection between field theories and the theory of closed exterior forms is primarily explained by the fact that the closure conditions of exterior and dual forms correspond to conservation laws to which physical fields are subject. It is known that the conservation laws for physical fields are those that state an existence of conservative physical quantities or objects. The physical structures, which made up physical fields, and corresponding conservation laws are differential-geometrical structures formed by closed exterior forms and dual ones. [Below, using the evolutionary forms it will be shown that such physical structures arise in material media discretely.]

The properties of closed exterior and dual forms, namely, invariance, covariance, conjugacy, and duality, lie at the basis of the group, structural and other invariant methods of field theories.

The nondegenerate transformations of field theory are transformations of closed exterior forms. These are gauge transformations for spinor, scalar, vector, and tensor fields, which are transformations of closed (0-form), (1-form), (2-form) and (3-form) respectively.

The gauge, i.e. internal, symmetries of the field theory equations (corresponding to the gauge transformations) are those of closed exterior forms. The external symmetries of the equations of field theory are symmetries of closed dual forms.

The basis of field theory operators is connected with the nondegenerate transformations of exterior differential forms. If, in addition to the exterior differential, we introduce the following operators: (1) δ for transformations that convert the form of $(p+1)$ degree into the form of p degree, (2) δ' for cotangent transformations, (3) Δ for the $d\delta - \delta d$ transformation, (4) Δ' for the $d\delta' - \delta' d$ transformation, one can write down the operators in the field theory equations in terms of these operators that act on the exterior differential forms. The operator δ corresponds to Green's operator, δ' does to the canonical transformation operator, Δ does to the d'Alembert operator in 4-dimensional space, and

Δ' corresponds to the Laplace operator.

It can be shown that the equations of existing field theories are those obtained on the basis of the properties of the exterior form theory. The Hamilton formalism is based on the properties of closed exterior form of the first degree and corresponding dual form. The closed exterior differential form $ds = -Hdt + p_j dq_j$ (the Poincaré invariant) corresponds to the field equation related to the Hamilton system. The Schrödinger equation in quantum mechanics is an analog to field equation, where the conjugated coordinates are changed by operators. It is evident that the closed exterior form of zero degree (and dual form) correspond to quantum mechanics. Dirac's *bra*- and *ket*- vectors constitute a closed exterior form of zero degree [3]. The properties of closed exterior form of the second degree (and dual form) lie at the basis of the electromagnetic field equations. The Maxwell equations may be written as [4] $d\theta^2 = 0$, $d^*\theta^2 = 0$, where $\theta^2 = \frac{1}{2}F_{\mu\nu}dx^\mu dx^\nu$ (here $F_{\mu\nu}$ is the strength tensor). Closed exterior and dual forms of the third degree correspond to the gravitational field. (However, to the physical field of given type it can be assigned closed forms of less degree. In particular, to the Einstein equation [5] for gravitational field it is assigned the first degree closed form, although it was pointed out that the type of a field with the third degree closed form corresponds to the gravitational field.)

One can recognize that the gauge transformations as well as the symmetries and equations of field theories are connected with closed exterior forms of given degree. This enables one to introduce a classification of physical fields and interactions according to the degree of closed exterior form. (If denote the degree of corresponding closed exterior forms by k , the case $k = 0$ will correspond to strong interaction, $k = 1$ will correspond to weak interaction, $k = 2$ will correspond to electromagnetic interaction, and $k = 3$ will correspond to gravitational interaction.) This shows that there exists a commonness between field theories describing physical fields of different types. The degree of closed exterior forms is a parameter that integrates fields theories into unified field theory.

Thus one can see that existing invariant field theories are based on the properties of closed exterior forms. And such a connection also discloses the problems of existing invariant field theories.

There are no answer to the question of how closed inexact exterior forms, which correspond to physical structures and reflect the properties of conservation laws and on which properties field theories are based, are obtained.

Below we will show that the answer to these question may be obtained from the analysis of differential equations describing the conservation laws for material media. These are just the equations from which the closed exterior forms whose properties correspond to field theories are obtained.

The evolutionary method of investigating these equations applied in the present paper enables one to understand how physical fields are formed and what must lie at the basis of the general field theory. [The method that enables one to find the closed exterior forms (the invariants) had been proposed by Cartan [1]. The differential equations are imposed by the requirement of obeying the closure condition of exterior form made up by the derivatives of these equations (it is added the requirement that the external form differential vanishes), and next one finds the conditions

that these requirements are satisfied (the integrability conditions). This method enables one *to find* the closed exterior forms (the invariants) that can possess the equation under consideration. However, for the evolutionary equations of mathematical physics describing physical processes it is important not only *to find* closed forms, but it is also important to know how these forms *are obtained*, in other words, it is important to know how the closure conditions of exterior forms are realized evolutionary. For this a principally new *evolutionary* method is necessary.]

2. The Equations of Balance Conservation Laws for Material System: The Evolutionary Processes in Material Media. Origination of Physical Structures

The conservation laws for material media (material media will be considered as material systems) are the balance conservation laws for energy, linear momentum, angular momentum, and mass. They are described by differential equations [6-8]. [Material system is a variety of elements that have internal structure and interact to one another. Thermodynamic and gas dynamical systems, systems of charged particles, cosmic systems, systems of elementary particles and others are examples of material systems. Examples of elements that constitute the material system are electrons, protons, neutrons, atoms, fluid particles, cosmic objects and others. The conservation laws for material systems are balance ones. These are conservation laws that establish a balance between the variation of physical quantity of material system and the corresponding external action.]

Nonconjugacy of the Balance Conservation Law Equations: Noncommutativity of the Balance Conservation Laws

The conservation laws for material systems have a peculiarity, namely, they are non-commutative ones. To this it points out the analysis of the equations of the balance conservation laws. (Just the noncommutativity of the balance conservation laws is a moving force of the evolutionary processes in material media that lead to generation of physical fields.)

It turns out that, even without a knowledge of the concrete form of the equations for balance conservation laws, with the help of skew-symmetric differential forms one can see their specific features. To carry out such an investigation, in addition to exterior skew-symmetric forms the skew-symmetric differential forms, which possesses the evolutionary properties (and for this reason the author named those as "evolutionary" ones), will be used. These are skew-symmetric differential forms, which, unlike to exterior forms, are defined on deforming manifolds. Such skew-symmetric differential forms have a specific feature, namely, they cannot be closed. The differential of such form does not vanish. This differential includes the metric form differential of deforming manifold, which is obtained due to differentiating the basis and is nonzero. The evolutionary form commutator, in addition to the commutator made up by the derivatives of the coefficients of the form itself, includes (in contrast to the commutator of the exterior form) the metric form

commutator being nonzero.

(The role of evolutionary forms in mathematical physics and field theory is due to the fact that they, as well as exterior forms, correspond to conservation laws. However, these conservation laws are those not for physical fields but for material media.)

We will analyze the equations that describe the balance conservation laws for energy and linear momentum.

If firstly to write down these equations in the inertial reference system and next pass to the accompanying reference system (this system is connected with the manifold built by the trajectories of the material system elements), in the accompanying reference system the energy equation will be written in the form

$$\frac{\partial\psi}{\partial\xi^1} = A_1 \quad (2.1)$$

Here ψ is the functional specifying the state of material system (the action functional, entropy or wave function can be regarded as examples of such a functional), ξ^1 is the coordinate along the trajectory, A_1 is the quantity that depends on specific features of material system and on external (with respect to local domain made up by the element and its neighborhood) energy actions onto the system.

In a similar manner, in the accompanying reference system the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^\nu} = A_\nu, \quad \nu = 2, \dots \quad (2.2)$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of material system and on external force actions.

Eqs. (2.1) and (2.2) can be convoluted into the relation

$$d\psi = A_\mu d\xi^\mu, \quad (\mu = 1, \nu) \quad (2.3)$$

where $d\psi$ is the differential expression $d\psi = (\partial\psi/\partial\xi^\mu)d\xi^\mu$.

Relation (2.3) can be written as

$$d\psi = \omega \quad (2.4)$$

here $\omega = A_\mu d\xi^\mu$ is the skew-symmetric differential form of the first degree.

Since the balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

Relation (2.4) was obtained from the equation of the balance conservation laws for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be a form of the second degree. And in combination with the equation of the balance conservation law for mass this form will be a form of degree 3.

Thus, in general case the evolutionary relation can be written as

$$d\psi = \omega^p \quad (2.5)$$

where the form degree p takes the values $p = 0, 1, 2, 3$. The evolutionary relation for $p = 0$ is similar to that in the differential forms, and it was obtained from the interaction of time and energy of material system.

It could be noted that the degree p is connected with the number of interacting conservation laws that is equal to $(p + 1)$.

Relation obtained from the equation of the balance conservation laws has a specific feature, namely, this relation turns out to be nonidentical.

To justify this we shall analyze relation (2.4). This relation proves to be nonidentical since the left-hand side of the relation is a differential, which is a closed form, but the right-hand side of the relation involves the differential form ω , which is unclosed evolutionary form. The metric form commutator of the manifold, on which the form ω is defined, is nonzero since this manifold is an accompanying, deforming, manifold. The commutator made up by the derivatives of coefficients A_μ the form ω itself is also nonzero, since the coefficients A_μ are of different nature, that is, some coefficients have been obtained from the energy equation and depend on the energetic actions, whereas the others have been obtained from the equation for linear momentum and depend on the force actions.

In a similar manner one can prove the nonidentity of relation (2.5).

The nonidentity of evolutionary relation means that the balance conservation law equations are inconsistent (nonconjugated). This reflects the properties of the balance conservation laws that have a governing importance for the evolutionary processes in material media, namely, their *noncommutativity*. [The nonidentity of evolutionary relation points to the fact that *on the initial manifold* the equations of the balance conservation laws are nonintegrable ones: the derivatives of these equations do not make up the differential, that is, a closed form which can be directly integrated. This is explained by the fact that these equations, like any equations describing physical processes, include nonpotential terms.]

Physical Meaning of the Equations of Balance Conservation Laws: Description of the State of Material System

Nonequilibrium state of material system. The evolutionary relation obtained from the equations of balance conservation laws discloses a physical meaning of these equations – these equations describe the state of material system.

It is evident that if the balance conservation laws be commutative, the evolutionary relation would be identical and from that it would be possible to get the differential $d\psi$, this would indicate that the material system is in the equilibrium state.

However, as it has been shown, in real processes the balance conservation laws are noncommutative. The evolutionary relation is not identical and from this relation one cannot get the differential $d\psi$. This means that the system state is nonequilibrium. That

is, due to noncommutativity of the balance conservation laws the material system state turns out to be nonequilibrium. It is evident that the internal force producing such nonequilibrium state is described by the evolutionary form commutator. Everything that gives the contribution to the commutator of the form ω^p leads to emergence of internal force. (Internal force is a force that acts inside the local domain of material system, i.e. a domain made up by the element and its neighborhood.)

Nonidentical evolutionary relation also describes how the state of material system varies. This turns out to be possible due to the fact that the evolutionary nonidentical relation is a selfvarying one. This relation includes two objects one of which appears to be unmeasurable. The variation of any object of the relation in some process leads to variation of another object and, in turn, the variation of the latter leads to variation of the former. Since one of the objects is an unmeasurable quantity, the other cannot be compared with the first one, and hence, the process of mutual variation cannot stop. This process is governed by the evolutionary form commutator, that is, by interaction between the commutator made up by derivatives of the form itself and by metric form commutator of deforming manifold made up by the trajectories of material system. (This is an exchange between quantities of different nature, between physical quantities and space-time characteristics.)

[In essence, the evolutionary equation is a correlative relation. When changing the terms of this relation cannot become equal to one another, but in this case they correlate to one another. The terms of the evolutionary form commutator in nonidentical relation also correlate to one another.]

Selfvariation of nonidentical evolutionary relation points to the fact that the nonequilibrium state of material system turns out to be selfvarying. State of material system changes but holds nonequilibrium during this process.

Transition of material system from nonequilibrium state to the locally-equilibrium state. Origination of physical structure. The significance of the evolutionary relation selfvariation consists in the fact that in such a process it can be realized conditions under which the inexact, closed *on pseudostructure*, exterior form is obtained from the evolutionary form. This transition is possible only as the degenerate transformation, namely, a transformation that does not conserve the differential. The conditions of degenerate transformation are those that determine the direction on which interior (only along a given direction) differential of the evolutionary form vanishes. These are the conditions that defines the pseudostructure, i.e. the closure conditions of dual form, and leads to realization of the exterior form closed on pseudostructure.

As it has been already mentioned, the differential of the evolutionary form ω^p involved into nonidentical relation (2.5) is nonzero. That is,

$$d\omega^p \neq 0 \quad (2.6)$$

If the conditions of degenerate transformation are realized, it will take place the transition $d\omega^p \neq 0 \rightarrow$ (degenerate transformation) $\rightarrow d_\pi\omega^p = 0, d_\pi^*\omega^p = 0$

The relations obtained

$$d_\pi\omega^p = 0, d_\pi^*\omega^p = 0 \quad (2.7)$$

are the closure conditions for exterior inexact form and dual form. This means that it is realized the exterior form closed on pseudostructure.

In this case on the pseudostructure π evolutionary relation (2.5) converts into the relation

$$d_{\pi}\psi = \omega_{\pi}^p \quad (2.8)$$

which proves to be an identical relation. Since the form ω_{π}^p is a closed one, on the pseudostructure this form turns out to be a differential. There are differentials in the left-hand and right-hand sides of this relation. This means that the relation obtained is an identical one.

Here it should be emphasized that under degenerate transformation the evolutionary form remains to be unclosed and the evolutionary relation itself remains to be nonidentical one. (The evolutionary form differential vanishes only on pseudostructure: the differential, which equals zero, is an interior one, the total differential of the evolutionary form is nonzero.)

The transition from nonidentical relation (2.5) obtained from the balance conservation laws to identical relation (2.8) means the following. Firstly, the emergence of the closed (on pseudostructure) inexact exterior form (relation (2.7) and right-hand side of relation (2.8)) points to origination of physical structure. And, secondly, the existence of the state differential (left-hand side of relation (2.8)) points to the transition of material system from nonequilibrium state to the locally-equilibrium state. (*But in this case the total state of the material system turns out to be nonequilibrium.*)

Identical relation (2.8) points to the fact that the origination of physical structures is connected with the transition of material system to the locally-equilibrium state.

The origination of physical structures in material system manifests itself as an emergence of certain observable formations, which develop spontaneously. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, creating massless particles, and others. (One can see that the processes described also explain such phenomena as turbulence, radiation and others.)

Conditions of degenerate transformation: degrees of freedom of material system. The conditions of degenerate transformation that lead to emergence of closed inexact exterior form are connected with any symmetries. Since these conditions are closure conditions of dual (metric) form, they can be caused by symmetries of coefficients of the metric form commutator (for example, these can be symmetrical connectednesses).

Under describing material system the symmetries are conditioned by degrees of freedom of material system. The translational degrees of freedom, internal degrees of freedom of the system elements, and so on can be examples of such degrees of freedom.

The conditions of degenerate transformation (vanishing the dual form commutator) define the pseudostructure. These conditions specify the derivative of implicit function, which defines the direction of pseudostructure. The speeds of various waves are examples of such derivatives: the speed of light, the speed of sound and of electromagnetic waves, the speed of creating particles and so on. It can be shown that the equations for surfaces

of potential (of simple layer, double layer), integral surfaces, equations for one, two, . . . eikonals, of the characteristic and of the characteristic surfaces, the residue equations and so on serve as the equations for pseudostructures.

To the degenerate transformation it must correspond a vanishing of some functional expressions, such as Jacobians, determinants, the Poisson brackets, residues and others. Vanishing of these functional expressions is the closure condition for dual form.

And it should be emphasized once more that *the degenerate transformation is realized as a transition from the accompanying noninertial coordinate system to the locally inertial system*. The evolutionary form is defined in the noninertial frame of reference (deforming manifold). But the closed exterior form created is obtained with respect to the locally-inertial frame of reference (pseudostructure).

Characteristics and Classification of Physical Structures. Forming Pseudometric and Metric Spaces

Characteristics of physical structures. The physical structure is a differential-geometrical structure made up by the dual form and closed inexact form. This is a pseudostructure (dual form) with conservative quantity (closed inexact form). The conservative quantities describe certain charges.

Since the physical structures are generated by material media by means of the balance conservation laws, their characteristics are connected with the characteristics of material systems and with the characteristics of evolutionary forms obtained from the equations of balance conservation laws.

It was already mentioned that the pseudostructure is obtained from the condition of degenerate transformation, which is connected with the degrees of freedom of material system.

The total differential of evolutionary form, which holds to be nonzero, defines two another characteristics of physical structure.

The first term of the evolutionary form differential, more exactly, its commutator, determines the value of discrete change (the quantum), which the quantity conserved on the pseudostructure undergoes during transition from one pseudostructure to another. The second term of the evolutionary form commutator determines the bending of pseudostructure. The bending specifies the characteristics that fixes the character of the manifold deformation, which took place before physical structure emerged. (Spin is an example of such a characteristics).

The closed exterior forms obtained correspond to the state differential for material system. The differentials of entropy, action, potential and others are examples of such differentials.

As it was already mentioned, in material system the created physical structure is revealed as an observable formation. It is evident that the characteristics of the formation (intensity, vorticity, absolute and relative speeds of propagation of the formation), as well as those of created physical structure, are determined by the evolutionary form and its

commutator and by the material system characteristics.

Classification of physical structures. The connection of the physical structures with the skew-symmetric differential forms allows to introduce a classification of these structures in dependence on parameters that specify the skew-symmetric differential forms and enter into nonidentical and identical relation. To determine these parameters one has to consider the problem of integration of the nonidentical evolutionary relation.

Under degenerate transformation from the nonidentical evolutionary relation one obtains a relation being identical on pseudostructure. Since the right-hand side of such a relation can be expressed in terms of differential (as well as the left-hand side), one obtains a relation that can be integrated, and as a result he obtains a relation with the differential forms of less by one degree.

The relation obtained after integration proves to be nonidentical as well.

By sequential integrating the nonidentical relation of degree p (in the case of realization of corresponding degenerate transformations and forming the identical relation), one can get a closed (on the pseudostructure) exterior form of degree k , where k ranges from p to 0.

In this case one can see that after such integration the closed (on the pseudostructure) exterior forms, which depend on two parameters, are obtained. These parameters are the degree of evolutionary form p in the evolutionary relation and the degree of created closed forms k .

In addition to these parameters, another parameter appears, namely, the dimension of space.

What is implied by the concept “space”?

In the process of deriving the evolutionary relation two frames of reference were used and, correspondingly, two spatial objects. The first frame of reference is an inertial one, which is connected with the space where material system is situated and is not directly connected with material system. This is an inertial space, it is a metric space. (This space is also formed by the material systems.) The second frame of reference is a proper one, it is connected with the accompanying manifold, which is not a metric manifold.

While generating closed forms of sequential degrees $k = p, k = p - 1, \dots, k = 0$ the pseudostructures of dimensions $(n + 1 - k): 1, \dots, n + 1$ are obtained, where n is the dimension of inertial space. As a result of transition to the exact closed form of zero degree *the metric* structure of the dimension $n + 1$ is obtained.

The parameters of physical structures generated by the evolutionary relation depend on the degree of differential forms p and k and on the dimension of original inertial space n .

With introducing the classification by numbers p, k and n one can understand the internal connection between various physical fields. Since physical fields are the carriers of interactions, such classification enables one to see the connection between interactions.

Such a classification may be presented in the form of the table given below. This table corresponds to elementary particles.

[It should be emphasized the following. Here the concept of “interaction” is used in a twofold

meaning: an interaction of the balance conservation laws that relates to material systems, and the physical concept of “interaction” that relates to physical fields and reflects the interactions of physical structures, namely, it is connected with exact conservation laws].

TABLE

interaction	$k \setminus p, n$	0	1	2	3
					graviton
					↑
gravitation	3				electron
					proton
					neutron
					photon
				photon2	
				↑	
electro- magnetic	2			electron	photon3
				proton	
				neutrino	
			neutrino1		
			↑		
weak	1		electron	neutrino2	neutrino3
			quanta		
		quanta0	quanta1	quanta2	quanta3
strong	0	↑			
		quarks?			
particles					
material	exact	electron	proton	neutron	deuteron?
nucleons?	forms				
N		1	2	3	4
		time	time+	time+	time+
			1 coord.	2 coord.	3 coord.

In the Table the names of the particles created are given. Numbers placed near particle names correspond to the space dimension. Under the names of particles the sources of interactions are presented. In the next to the last row we present particles with mass (the elements of material system) formed by interactions (the exact forms of zero degree obtained by sequential integrating the evolutionary relations with the evolutionary forms of degree p corresponding to these particles). In the bottom row the dimension of the *metric* structure created is presented.

From the Table one can see the correspondence between the degree k of the closed forms realized and the type of interactions. Thus, $k = 0$ corresponds to strong interaction, $k = 1$ corresponds to weak interaction, $k = 2$ corresponds to electromagnetic interaction, and $k = 3$ corresponds to gravitational interaction.

The degree k of the closed forms realized and the number p connected with the number of interacting balance conservation laws determine the type of interactions and the type of particles created. The properties of particles are governed by the space dimension. The last property is connected with the fact that closed forms of equal degrees k , but obtained from the evolutionary relations acting in spaces of different dimensions n , are distinctive because they are defined on pseudostructures of different dimensions (the dimension of pseudostructure $(n + 1 - k)$ depends on the dimension of initial space n). For this reason the realized physical structures with closed forms of degrees k are distinctive in their properties.

The parameters p , k , n can range from 0 to 3. They determine some completed cycle. The cycle involves four levels, to each of which are assign their own values of p ($p = 0, 1, 2, 3$) and space dimension n .

In the Table one cycle of forming physical structures is presented. Each material system has his own completed cycle. This distinguishes one material system from another system. One completed cycle can serve as the beginning of another cycle (the structures formed in the preceding cycle serve as the sources of interactions for beginning a new cycle). This may mean that one material system (medium) proves to be imbedded into the other material system (medium). The sequential cycles reflect the properties of sequentially imbedded material systems. And yet a given level has specific properties that are inherent characteristics of the same level in another cycles. This can be seen, for example, from comparison of the cycle described and the cycle in which to the exact forms there correspond conductors, semiconductors, dielectrics, and neutral elements. The properties of elements of the third level, namely, of neutrons in one cycle and of dielectrics in another, are identical to the properties of so called "magnetic monopole" [9,10].

Forming pseudometric and metric spaces The mechanism of creating the pseudostructures lies at the basis of forming the pseudometric surfaces and their transition into metric spaces.

It was shown above that the evolutionary relation of degree p can generate (in the presence of degenerate transformations) closed forms of the degrees $p, p - 1, \dots, 0$. While generating closed forms of sequential degrees $k = p, k = p - 1, \dots, k = 0$ the pseudostructures of dimensions $(n + 1 - k)$: $1, \dots, n + 1$ are obtained. As a result of transition to the exact closed form of zero degree the metric structure of the dimension $n + 1$ is obtained. Under the effect of external actions (and in the presence of degrees of freedom) the material system can convert the initial inertial space of the dimension n into the space of the dimension $n + 1$.

Here the following should be pointed out. Physical structures are generated by local domains of material system. These are elementary physical structures. By combining

with one another they can form large-scale structures making up pseudomanifolds and physical fields.

Sections of the cotangent bundles (Yang-Mills fields), cohomologies by de Rham, singular cohomologies, pseudo-Riemannian and pseudo-Euclidean spaces, and others are examples of pseudostructures and spaces that are formed by pseudostructures. Euclidean and Riemannian spaces are examples of metric manifolds that are obtained when going to the exact forms.

What can be said about the pseudo-Riemannian manifold and Riemannian space?

The distinctive property of the Riemannian manifold is an availability of the curvature. This means that the metric form commutator of the third degree is nonzero. Hence, the commutator of the evolutionary form of third degree ($p = 3$), which involves into the proper metric form commutator, is not equal to zero. That is, the evolutionary form that enters into the evolutionary relation is unclosed, and the relation is nonidentical one.

When realizing pseudostructures of the dimensions 1, 2, 3, 4 and obtaining the closed inexact forms of the degrees $k = 3, k = 2, k = 1, k = 0$ the pseudo-Riemannian space is formed, and the transition to the exact form of zero degree corresponds to the transition to the Riemannian space.

It is well known that while obtaining the Einstein equations it was assumed that there are satisfied the conditions [4,8]: 1) the Bianchi identity is satisfied, 2) the coefficients of connectedness are symmetric, 3) the condition that the coefficients of connectedness are the Christoffel symbols, and 4) an existence of the transformation under which the coefficients of connectedness vanish. These conditions are the conditions of realization of degenerate transformations for nonidentical relations obtained from the evolutionary nonidentical relation with evolutionary form of the degree $p = 3$ and after going to the identical relations. In this case to the Einstein equation the identical relations with forms of the first degree are assigned.

From the description of evolutionary processes in material media one can see that physical fields are generated by material media. (And thus the causality of physical processes and phenomena is explained.)

Here it should be emphasized that the conservation laws for material media, i.e. the balance conservation laws for energy, linear momentum, angular momentum, and mass, which are noncommutative ones, play a controlling role in these processes. This is precisely the noncommutativity of the balance conservation laws produced by external actions onto material system, which is a moving force of evolutionary processes leading to origination of physical structures (to which exact conservation laws are assigned). [Non-commutativity of balance conservation laws for material media and their controlling role in evolutionary processes accompanied by emerging physical structures practically have not been taken into account in the explicit form anywhere. The mathematical apparatus of evolutionary differential forms enables one to take into account and to describe these points.]

3. On the Foundations of General Field Theory

The results of the analysis of the equations of conservation laws for material media shows the connection of physical fields with material media.

This points to the fact that the fields theories that describe physical fields must be connected with the equations that describe material systems.

Such a connection, which is common to all field theories, discloses the general foundations of field theories, their quantum character, justifies the unity of field theories and can serve as an approach to general field theory.

The theories of exterior and evolutionary skew-symmetric differential forms, which reflect the properties of conservation laws for physical fields and material media, allow to disclose and justify the general principles of field theories. In this case the properties of closed exterior forms demonstrate these principles, and the theory of evolutionary forms justifies this. Below we present certain of concepts that lie at the basis of field theories. (The results obtained with using the evolutionary forms are italicized).

1. Physical fields are formatted by physical structures that are described by closed inexact exterior and dual forms. *Physical structures are generated by material media. Characteristics of physical structures relate to the characteristics of material systems.*

2. *The conservation laws for physical fields, on which the field theories are based, are connected with the conservation laws for material media (with the balance conservation laws for energy, linear momentum, angular momentum, and mass and the analog of such law for the time).*

3. Internal and external symmetries of field theories are those of closed exterior and dual forms. *They are conditioned by the degrees of freedom of material media.*

4. *The origination of physical structures, from which physical fields are made up, proceeds **discretely** under realization of the degrees of freedom of material systems.* This explains the quantum character of field theories.

5. The gauge transformations of field theories are transformations of closed exterior forms. *They are connected with the degenerate transformations of the equations of conservation laws for material media.*

6. *The constants of field theory must be connected with the characteristics of material systems.*

7. The classification parameter of physical fields and interactions, that is, the parameter of the unified field theory, is the degree of closed exterior forms corresponding to conservation laws for physical fields. *This parameter is connected with the number of the equations of interacting noncommutative balance conservation laws. This connection justifies the parameter of the united field theory.*

The results obtained show that when building the general field theory it is necessary to take into account the connection of existing field theories (which are based on the conservation laws for physical fields) with the equations of noncommutative conservation laws for material media (the balance conservation laws for energy, linear momentum, angular momentum and mass and the analog of such laws for the time, which takes into

account the noncommutativity of the time and the energy of material system).

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