Lorentz Invariant Majorana Formulation of the Field Equations and Dirac-like Equation for the Free Photon

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Abstract: In this paper we present a new geometric formulation (Clifford algebra formalism) of the field equations, which is independent of the reference frame and of the chosen system of coordinates in it. This formulation deals with the complex 1-vector $\Psi = E - icB$ (i is the unit imaginary), which is four-dimensional (4D) geometric generalization of Majorana's complex 3D quantity $\mathbf{\Psi} = \mathbf{E} - ic\mathbf{B}$. When the sources are absent the field equations with the complex Ψ become Dirac-like relativistic wave equations for the free photon. In the frame of "fiducial" observers (the observers who measure fields are at rest) and in the standard basis the component form of the field equations with 4D Ψ reproduces the component form of Majorana-Maxwell equations with 3D field $\mathbf{\Psi}$. The important differences between the approach with the 4D Ψ and that one with the 3D $\mathbf{\Psi}$ are discussed.

1. Introduction

In Majorana formulation of electrodynamics the Maxwell equations are written in terms of complex combination of the three-dimensional (3D) vectors of the electric and the magnetic fields $\mathbf{E}$ and $\mathbf{B}$ respectively, $\mathbf{\Psi} = \mathbf{E} - ic\mathbf{B}$, see [1,2]. (The vectors in the 3D space will be designated in bold-face.) In terms of $\mathbf{\Psi}$ the Maxwell equations in vacuum can be cast in a Dirac-like form using the correspondence principle $W \to i\hbar\partial/\partial t$, $\mathbf{p} \to -i\hbar\nabla$. In that case $\mathbf{\Psi}^* \cdot \mathbf{\Psi} = \mathbf{E}^2 + c^2\mathbf{B}^2$ is proportional to the probability density function for a photon. An important advantage of Majorana formulation of electrodynamics is that it does not make use of the intermediate electromagnetic potentials but deals with

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observable quantities, the electric and the magnetic fields.

Covariant Majorana formulation is developed in [2]. There the covariant form of the complex field $\Psi_\mu = E_\mu - iB_\mu$ is introduced. The covariant Maxwell equation with Ψ_μ are written only for the free fields, i.e., when $j^\beta = 0$. It is worth noting that E_μ, B_μ and Ψ_μ are components that are determined in the specific system of coordinates, which we call Einstein's system of coordinates. In Einstein's system of coordinates the standard, i.e., Einstein's synchronization [3] of distant clocks and Cartesian space coordinates x^i are used in the chosen inertial frame. We also point out that in [2] E^μ and B^μ are treated as the "auxiliary fields," while the 3D vectors **E** and **B** are considered as the physical fields.

Further generalization of Majorana formulation is presented in [4]. There a geometric approach to special relativity is developed, which deals with tensors as 4D geometric quantities. We note that such geometric approach with tensors as geometric quantities is considered not only in [4] but in [5, 6] as well, while a similar treatment in which 4D geometric quantities are Clifford multivectors is presented in [7-10]. The approach to special relativity with 4D geometric quantities is called the invariant special relativity. In the the invariant special relativity one considers that the 4D geometric quantities are well-defined both theoretically and *experimentally* in the 4D spacetime, and not, as usual, the 3D quantities. All physical quantities are defined without reference frames, i.e., as absolute quantities (AQs) or, when some basis has been introduced, they are represented as 4D coordinate-based geometric quantities (CBGQs) comprising both components and a *basis*. It is shown in the mentioned references that such geometric approach is in a complete agreement with the principle of relativity and, what is the most important, with experiments, see [5] (tensor formalism) and [8-10] (geometric algebra formalism). In [4] Sec. 6.3 the invariant Majorana electromagnetic field Ψ^a is defined as $\Psi^a = E^a - icB^a$, where E^a, B^a and Ψ^a are the 4D AQs with definite physical meaning and not the "auxiliary fields". In the same section the field equation with Ψ^a is presented, which for $j^\beta = 0$ is reduced to the Dirac-like relativistic wave equation for the free photon.

In this paper we shall explore a similar Lorentz invariant Majorana formulation in which physical quantities will be represented by Clifford multivectors. To simplify the derivation of all important relations we shall employ recently developed axiomatic geometric formulation of electromagnetism [10] in which the primary quantity for the whole electromagnetism is the electromagnetic field F (bivector). New Lorentz invariant Majorana form of the field equations and Dirac-like equations for the free photon are reported. The similarities and the differences between our Lorentz invariant field equations with the 4D Ψ and Majorana-Maxwell equations with the 3D **Ψ** are discussed.

2. A brief summary of geometric algebra

In this paper the investigation with 4D geometric quantities will be done in the geometric algebra formalism, see, e.g., [11] and [12]. First we provide a brief summary of Clifford algebra with multivectors. Clifford vectors are written in lower case (a) and general multivectors (Clifford aggregate) in upper case (A). The space of multivectors is graded and multivectors containing elements of a single grade, r, are termed homogeneous and

often written A_r. The geometric (Clifford) product is written by simply juxtaposing multivectors AB. A basic operation on multivectors is the degree projection $\langle A \rangle_r$ which selects from the multivector A its $r-$ vector part (0 = scalar, 1 = vector, 2 = bivector, ...). We write the scalar (grade-0) part simply as $\langle A \rangle$. The geometric product of a grade-r multivector A_r with a grade-s multivector B_s decomposes into $A_r B_s = \langle AB \rangle_{r+s} + \langle AB \rangle_{r+s-2} ... + \langle AB \rangle_{|r-s|}$. The inner and outer (or exterior) products are the lowest-grade and the highest-grade terms respectively of the above series $A_r \cdot B_s \equiv \langle AB \rangle_{|r-s|}$, and $A_r \wedge B_s \equiv \langle AB \rangle_{r+s}$. For vectors a and b we have $ab = a{\cdot}b + a{\wedge}b$, where $a{\cdot}b \equiv (1/2)(ab+ba)$, and $a{\wedge}b \equiv (1/2)(ab-ba)$. In the case considered in this paper Clifford algebra is developed over the field of the complex numbers. Complex reversion is the operation which takes the complex conjugate of the scalar (complex) coefficient of each of the 16 elements in the algebra and reverses the order of multiplication of vectors in each multivector, $\overline{AB} = \overline{B}\,\overline{A}$, where, e.g., the complex reversion of A is denoted by an overbar $\overline{A}$.

Any multivector A is a geometric 4D quantity defined without reference frame, i.e., an AQ. When some basis has been introduced A can be written as a CBGQ comprising both components and a basis. Usually [11, 12] one introduces the standard basis. The generators of the spacetime algebra are taken to be four basis vectors $\{\gamma_\mu\}, \mu = 0, ...3$ (the standard basis) satisfying $\gamma_\mu \cdot \gamma_\nu = \eta_{\mu\nu} = diag(+---)$. This basis is a right-handed orthonormal frame of vectors in the Minkowski spacetime M^4 with γ_0 in the forward light cone. The γ_k ($k = 1, 2, 3$) are spacelike vectors. The basis vectors γ_μ generate by multiplication a complete basis for the spacetime algebra: $1, \gamma_\mu, \gamma_\mu \wedge \gamma_\nu, \gamma_\mu \gamma_5, \gamma_5$ (16 independent elements). γ_5 is the pseudoscalar for the frame $\{\gamma_\mu\}$.

Observe that the standard basis corresponds, in fact, to Einstein's system of coordinates. However different systems of coordinates are allowed in an inertial frame and they are all equivalent in the description of physical phenomena. For example, in [4], two very different, but physically completely equivalent, systems of coordinates, Einstein's system of coordinates and the system of coordinates with a nonstandard synchronization, the everyday (radio) ("r") synchronization, are exposed and exploited throughout the paper. In order to treat different systems of coordinates on an equal footing we have developed such form of the Lorentz transformations (LT) which is independent of the chosen system of coordinates, including different synchronizations, [4] (tensor formalism) and [7] (Clifford algebra formalism). Furthermore in [4] we have presented the transformation matrix that connects Einstein's system of coordinates with another system of coordinates in the same reference frame. For the sake of brevity and of clearness of the whole exposition, we shall write CBGQs only in the standard basis $\{\gamma_\mu\}$, but remembering that the approach with 4D geometric quantities holds for any choice of basis. All equations written with 4D AQs and 4D CBGQs will be manifestly Lorentz invariant equations.

As mentioned above a Clifford multivector A, an AQ, can be written as a CBGQ, thus with components and a basis. Any CBGQ is an invariant quantity under the passive Lorentz transformations; both the components and the basis vectors are transformed but the whole 4D geometric quantity remains unchanged, e.g., the position 1-vector x can be decomposed in the S and S' (relatively moving) frames and in the standard

basis $\{\gamma_\mu\}$ and some non-standard basis $\{r_\mu\}$ as $x = x^\mu \gamma_\mu = x'^\mu \gamma'_\mu = ... = x'^\mu_r r'_\mu$. The primed quantities are the Lorentz transforms of the unprimed ones. The invariance of some 4D CBGQ under the passive Lorentz transformations reflects the fact that such mathematical, invariant, geometric 4D quantity represents the same physical quantity for relatively moving observers.

3. The relations that connect F with E, B and with Ψ, $\overline{\Psi}$

In contrast to the usual Clifford algebra approaches [11, 12], and all other previous approaches, we have shown in [10] that the bivector field F, and not the 3D vectors **E** and **B** or the electromagnetic potentials, can be considered as the primary physical quantity for the whole electromagnetism. From the known F one can find different 4D quantities that represent the 4D electric and magnetic fields; they are considered in [8] and [9]. One of these representations, which is examined in [7-9], uses the decomposition of F into 1-vectors E and B

$$F = (1/c)E \wedge v + (B \wedge v)I,$$
$$E = (1/c)F \cdot v, \quad B = (1/c^2)(F \wedge v)I; \; E \cdot v = B \cdot v = 0, \tag{1}$$

where I is the unit pseudoscalar. (I is defined algebraically without introducing any reference frame, as in [13] Sec. 1.2.) The velocity v can be interpreted as the velocity (1-vector) of a family of observers who measures E and B fields. That velocity v and all other quantities entering into (1) are defined without reference frames, i.e., they are AQs.

It is proved in [8, 9] (Clifford algebra formalism) and [6] (tensor formalism) that the observers in relative motion see the same field, e.g., the E field in the S frame is the same as in the relatively moving S' frame; $E^\mu \gamma_\mu = E'^\mu \gamma'_\mu$, where all primed quantities are the Lorentz transforms of the unprimed ones. The LT transform the components E^μ from the S frame again to the components E'^μ from the S' frame, in the same way as for any other 1-vector. For example, the transformations for the components of the 1-vector E are

$$E'^0 = \gamma(E^0 - \beta E^1), E'^1 = \gamma(E^1 - \beta E^0), E'^2 = E^2, E'^3 = E^3, \tag{2}$$

and the same for the transformations of the components of the 1-vector B. Thus the Lorentz transformed E'^μ are not expressed by the mixture of components E^μ and B^μ of the electric and magnetic fields respectively from the S frame. This is in sharp contrast to all previous formulations of electromagnetism, starting with Einstein's work [3], in which the components E'_i of the 3D **E**′ are expressed by the mixture of components of E_i and B_i from the S frame. For example, the transformations for the components of the 3D **E** are

$$E'_x = E_x, \; E'_y = \gamma(E_y - \beta c B_z), \; E'_z = \gamma(E_z + \beta c B_y), \tag{3}$$

and similarly for the components of the 3D **B**, see, e.g., [14] Sec. 11.10. In all textbooks and papers treating relativistic electrodynamics these usual transformations of the components of the 3D **E** (3) and **B** (e.g., [14] Sec. 11.10) are considered to be the LT, but

the fundamental results obtained in [6] and [8, 9] exactly show that they drastically differ from the LT of the 4D quantities that represent the electric and the magnetic fields.

Next we introduce the complex fields, the 4D AQ Ψ and its complex reversion $\overline{\Psi}$. They are defined in terms of 1-vectors of the electric and magnetic fields E and B as

$$
\Psi = E - icB, \quad \overline{\Psi} = E + icB,
$$
$$
E = (1/2)(\Psi + \overline{\Psi}), \quad B = (i/2c)(\Psi - \overline{\Psi}); \; v \cdot \Psi = v \cdot \overline{\Psi} = 0. \tag{4}
$$

The complex Ψ and $\overline{\Psi}$ are homogeneous, grade-1, multivectors. The meanings of v and I are the same as in (1).

Using (1) we find that the F formulation and the complex Ψ formulation are connected by the relations

$$
F = (1/2c)\{(\Psi + \overline{\Psi}) \wedge v + i[(\Psi - \overline{\Psi}) \wedge v]I\}.
$$
$$
\Psi = (1/c)F \cdot v + (i/c)I(F \wedge v). \tag{5}
$$

We note that one can construct the formulation of electrodynamics with the complex 1-vectors Ψ and $\overline{\Psi}$ as 4D AQs, i.e., Lorentz invariant Majorana formulation of electrodynamics using the relations (5) and the work [10]. Such formulation is perfectly suited for the transition to the quantum electrodynamics.

4. Lorentz invariant Majorana form of the field equation and Dirac-like equation for the free photon

As already mentioned we shall use the F formulation [10] to find the field equation for Ψ. In the F formulation [10] the electromagnetic field is represented by a bivector-valued function $F = F(x)$ on the spacetime. The source of the field is the electromagnetic current j which is a 1-vector field and the gradient operator ∂ is also 1-vector. A single field equation for F was first given by M. Riesz [15] as

$$
\partial F = j/\varepsilon_0 c, \quad \partial \cdot F + \partial \wedge F = j/\varepsilon_0 c. \tag{6}
$$

The trivector part is identically zero in the absence of magnetic charge.

Using (5) and (6) we write the field equation in terms of the complex 1-vector Ψ as

$$
\partial \cdot (\Psi \wedge v) + i \left[\partial \wedge (\Psi \wedge v) \right] I = j/\varepsilon_0. \tag{7}
$$

This form of the field equation (in which $\overline{\Psi}$ does not appear) is achieved separating vector and trivector parts and then combining them to eliminate $\overline{\Psi}$. The equation (7) is the most general basic equation for the Lorentz invariant Majorana formulation of electrodynamics.

From this field equation with AQs one can get more familiar field equation with CBGQs that are written in the standard basis $\{\gamma_\mu\}$. Thus instead of (7) we have

$$
\partial_\alpha[(\delta^{\alpha\beta}{}_{\mu\nu} - i\varepsilon^{\alpha\beta}{}_{\mu\nu})\Psi^\mu v^\nu]\gamma_\beta = (j^\beta/\varepsilon_0)\gamma_\beta, \tag{8}
$$

where $\delta^{\alpha\beta}{}_{\mu\nu} = \delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} - \delta^{\alpha}_{\nu}\delta^{\beta}_{\mu}$. The equation (8) can be also written as

$$\partial_{\alpha}[(\Gamma^{\alpha})^{\beta}{}_{\mu}\Psi^{\mu}]\gamma_{\beta} = (j^{\beta}/\varepsilon_0)\gamma_{\beta},$$
$$(\Gamma^{\alpha})^{\beta}{}_{\mu} = \delta^{\alpha\beta}{}_{\nu\rho}v^{\rho}g^{\nu}{}_{\mu} + i\varepsilon^{\alpha\beta}{}_{\nu\mu}v^{\nu}. \tag{9}$$

We note that the same equation as (9) is obtained in the tensor formalism in [4]. Observe that our $(\Gamma^{\alpha})^{\beta}{}_{\mu}$ differ from the expression for the corresponding quantity $(\gamma^{\alpha})^{\beta}{}_{\mu}$, Eq. (30) in [2].

In the case when the sources are absent, $j = 0$, and when it is assumed that the velocity 1-vector v is independent of x, then the field equation with the 4D Ψ as AQ, (7), becomes

$$v(\partial \cdot \Psi) - (v \cdot \partial)\Psi + i\left[v \wedge (\partial \wedge \Psi)\right]I = 0. \tag{10}$$

Then we introduce a generalization of the correspondence principle that deals with 4D AQs

$$i\hbar\partial \rightarrow p. \tag{11}$$

Inserting (11) into (10) we reveal Dirac-like relativistic wave equation for the free photon, which is written with AQs

$$v(p \cdot \Psi) - (v \cdot p)\Psi + i\left[v \wedge (p \wedge \Psi)\right]I = 0. \tag{12}$$

If we write Eq. (10) with CBGQs in the standard basis $\{\gamma_{\mu}\}$ then we get an equation that is very similar to (9)

$$[(\Gamma^{\alpha})^{\beta}{}_{\mu}(\partial_{\alpha}\Psi^{\mu})]\gamma_{\beta} = 0. \tag{13}$$

Remember that v in (10) and (13) is independent of x, whereas $(\Gamma^{\alpha})^{\beta}{}_{\mu}$ is the same as in (9). When the generalized correspondence principle (11) is written with CBGQs in the $\{\gamma_{\mu}\}$ basis it takes the form

$$\gamma^{\alpha}i\hbar\partial_{\alpha} \rightarrow \gamma^{\alpha}p_{\alpha}. \tag{14}$$

Inserting (14) into Eq. (13) we find the following equation

$$[(\Gamma^{\alpha})^{\beta}{}_{\mu}(p_{\alpha}\Psi^{\mu})]\gamma_{\beta} = 0. \tag{15}$$

The equation (15) is Dirac-like relativistic wave equation for the free photon, but now written with CBGQs in the $\{\gamma_{\mu}\}$ basis.

It is clear from the form of the equations (8), (9) (with some general v^{μ}) and (13), (15) (with v^{μ} independent of x) that they are invariant under the passive LT, since every 4D CBGQ is invariant under the passive LT. The field equations with primed quantities, thus in a relatively moving S' frame, are exactly equal to the corresponding equations in S, which are given by the above mentioned relations. Thus these equations are not only covariant but also the Lorentz invariant field equations. The principle of relativity is automatically included in such formulation.

In addition let us briefly examine how one can get the field equations in the formulation with 1-vectors of the electric field E and the magnetic field B from the corresponding

Eq (7), when AQs are used, and (8) or (9), when CBGQs in the $\{\gamma_\mu\}$ basis are used. These equations are already obtained and discussed in detail in [9] and previously in [7]. Substituting the decomposition of Ψ into E and B from (4) into (7) one gets two equations with real multivectors

$$
\begin{aligned}
\partial \cdot (E \wedge v) + [\partial \wedge (cB \wedge v)]\, I &= j/\varepsilon_0. \\
\partial \cdot (cB \wedge v) - [\partial \wedge (E \wedge v)]\, I &= 0.
\end{aligned}
\tag{16}
$$

The equations (16) are the same as Eq. (39) in [9]. Similarly, starting with (8) we find

$$
\begin{aligned}
\partial_\alpha(\delta^{\alpha\beta}{}_{\mu\nu}E^\mu v^\nu + \varepsilon^{\alpha\beta\mu\nu}v_\mu cB_\nu)\gamma_\beta &= (j^\beta/\varepsilon_0)\gamma_\beta, \\
\partial_\alpha(\delta^{\alpha\beta}{}_{\mu\nu}v^\mu cB^\nu + \varepsilon^{\alpha\beta\mu\nu}v_\mu E_\nu)\gamma_\beta &= 0,
\end{aligned}
\tag{17}
$$

which is the same as Eq. (40) in [9]. Of course, these equations, (16) and (17), are also Lorentz invariant field equations but with 1-vectors E and B.

5. Comparison with Majorana-Maxwell equations with the 3D Ψ

Let us now see how our results can be reduced to Majorana-Maxwell equations with the 3D $\boldsymbol{\Psi}$. In the presence of sources these equations are

$$
div\boldsymbol{\Psi} = \rho/\varepsilon_0, \quad irot\boldsymbol{\Psi} = \mathbf{j}/\varepsilon_0 c + (1/c)\partial\boldsymbol{\Psi}/\partial t,
\tag{18}
$$

see, e.g., Eqs. (2) in [1]. When the sources are absent, $\rho = 0$, $\mathbf{j} = \mathbf{0}$, and when the correspondence principle $W \to i\hbar\partial/\partial t$, $\mathbf{p} \to -i\hbar\nabla$ is used in (18), then Eq. (18) with the 3D $\boldsymbol{\Psi}$ leads to the transversality condition and to Majorana-Maxwell equation in a Dirac-like form

$$
\mathbf{p} \cdot \boldsymbol{\Psi} = 0, \quad W\boldsymbol{\Psi} + i\mathbf{p} \times \boldsymbol{\Psi} = \mathbf{0},
\tag{19}
$$

see Eqs. (43) and (44) in [2].

As seen from (5) (or (4) and (1)) the complex 1-vectors Ψ and $\overline{\Psi}$ (or 1-vectors E and B) are not uniquely determined by F, but their explicit values depend also on v. Let us choose the frame in which the observers who measure Ψ and $\overline{\Psi}$, i.e., E and B, are at rest. For them $v = c\gamma_0$. This frame will be called the frame of "fiducial" observers (for that name see [16]), or the γ_0 - frame. All quantities in that frame will be denoted by the subscript f, e.g., Ψ_f, E_f and B_f. Furthermore, the standard basis $\{\gamma_\mu\}$ will be chosen in the γ_0 - frame. Then in that frame the velocity $v = c\gamma_0$ has the components $v^\alpha = (c, 0, 0, 0)$ and Ψ and $\overline{\Psi}$ (E and B) become Ψ_f and $\overline{\Psi}_f$ (E_f and B_f) and they do not have temporal components, $\Psi_f^0 = \overline{\Psi}_f^0 = 0$, $E_f^0 = B_f^0 = 0$. In the γ_0 - frame Eq. (8) becomes

$$
(\partial_i\Psi_f^i - j^0/\varepsilon_0 c)\gamma_0 + (i\varepsilon^{ki}{}_{0j}\partial_k\Psi_f^j - \partial_0\Psi_f^i - j^i/\varepsilon_0 c)\gamma_i = 0.
\tag{20}
$$

All terms in (20) are CBGQs that are written in the $\{\gamma_\mu\}$ basis. The equation (20) cannot be further simplified as a geometric equation. However if one compares the components

from Eq. (20) and the components from Majorana-Maxwell equations (18) then it is seen that they are the same. Hence it is the component form of Eq. (20) (the "fiducial" frame and the standard basis $\{\gamma_\mu\}$) which agrees with the component form of Majorana-Maxwell equations with the 3D $\boldsymbol{\Psi}$ (18).

In the case when $j^\mu = 0$, and with the replacement (14), Eq. (20) can be written as

$$p_i \Psi^i_f \gamma_0 + (-p_0 \Psi^i_f + i\varepsilon^{ki}{}_{0j} p_k \Psi^j_f)\gamma_i = 0. \tag{21}$$

The same result (21) follows from Eq. (15) when it is considered in the γ_0 - frame in which the $\{\gamma_\mu\}$ basis is chosen. The whole equation (21) is written with CBGQs in the standard basis $\{\gamma_\mu\}$ and cannot be further simplified as a geometric equation. Comparing that equation with Majorana-Maxwell equations (19) we again see that only component forms of both equations can be compared. From the first term (with γ_0) in Eq. (21) we find the component form of the transversality condition written with 4D p and Ψ_f, $p_i \Psi^i_f = 0$ (remember that in the γ_0 - frame $\Psi^0_f = 0$), which agrees with the component form of the transversality condition with the 3D $\mathbf{p}$ and $\boldsymbol{\Psi}$ from Eq. (19), $p_i \Psi^i = 0$. The second term (with γ_i) in Eq. (21) yields the component form of Dirac-like equation for the free photon that is written with 4D p and Ψ_f. It agrees with the component form of the corresponding equation (19) with the 3D $\mathbf{p}$ and $\boldsymbol{\Psi}$.

Similarly, in the frame of "fiducial" observers and in the $\{\gamma_\mu\}$ basis, we can derive the component form of the usual Maxwell equations with the 3D $\mathbf{E}$ and $\mathbf{B}$ from Eq. (17). This is discussed in detail in [9].

However, it is worth noting that there are very important differences between our Eqs. (20) and (21), or, better to say, our Eqs. (13) and (15), and Majorana-Maxwell equations (18) and (19). Our equations (20), (21), (13) and (15) are written with 4D CBGQs and the components are multiplied by the unit 1-vectors γ_μ, whereas Majorana-Maxwell equations (18) and (19) are written with 3D vectors and the components are multiplied by the unit 3D vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Only in the frame of "fiducial" observers and in the $\{\gamma_\mu\}$ basis the temporal component of the complex 1-vector Ψ is zero, but in all other relatively moving inertial frames this component is different from zero. Furthermore in any frame other than the γ_0 - frame the "fiducial" observers are moving and the velocity v has the spatial components as well.

The complex 1-vector Ψ transforms under the LT as every 1-vector transforms, e.g., the components transform as in Eq. (2), whereas the unit 1-vectors γ_μ transform by the inverse LT. This gives that the whole Ψ is unchanged, i.e., it holds that $\Psi^\mu \gamma_\mu = \Psi'^\mu \gamma'_\mu$ as for any other 4D CBGQ. On the other hand there is no transformation which transforms the unit 3D vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ into the unit 3D vectors $\mathbf{i}'$, $\mathbf{j}'$, $\mathbf{k}'$. Hence it is not true that, e.g., the 3D vector $\mathbf{E}' = E'_1 \mathbf{i}' + E'_2 \mathbf{j}' + E'_3 \mathbf{k}'$ is obtained by the LT from the 3D vector $\mathbf{E} = E_1 \mathbf{i} + E_2 \mathbf{j} + E_3 \mathbf{k}$. Namely the components E_i of the 3D $\mathbf{E}$ are transformed by the usual transformations (3), which differ from the LT (2), and, as said above, there is no transformation for the unit 3D vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. The same holds for the transformations of the 3D $\mathbf{B}$ and consequently for the transformations of the 3D $\boldsymbol{\Psi}$. This means that the correspondence of the 4D picture with complex 1-vector Ψ and the 3D picture with

Majorana 3D complex vector $\mathbf{\Psi}$ exists only in the frame of "fiducial" observers and in the $\{\gamma_\mu\}$ basis and not in any other relatively moving inertial frame, or in some nonstandard basis. Moreover, that correspondence in the γ_0 - frame and in the $\{\gamma_\mu\}$ basis refers only to the component forms of the corresponding equations. Our equations with 4D geometric quantities are the same in all relatively moving inertial frames, i.e., they are Lorentz invariant equations, whereas it is not true for Majorana-Maxwell equations with the 3D $\mathbf{\Psi}$.

Similarly it is proved in [9] that, contrary to the generally accepted opinion, Maxwell equations with the 3D $\mathbf{E}$ and $\mathbf{B}$ are not covariant under the LT. The field equations for the electric and magnetic fields that are Lorentz invariant are, e.g., the equations with 1-vectors E and B, Eqs. (17).

The situation with the physical importance of the 4D fields Ψ and $\overline{\Psi}$ and the corresponding 3D fields $\mathbf{\Psi}$ and $\mathbf{\Psi}^*$ is the same as it is the situation with the physical importance of the 4D fields E and B and the corresponding 3D fields $\mathbf{E}$ and $\mathbf{B}$. The comparison with experiments, the motional electromotive force in [8], the Faraday disk in [9] and the Trouton-Noble experiment in [10], strongly support our conclusions that the 4D fields E and B are not the "auxiliary fields," as explicitly considered in [2] and tacitly assumed in all previous works, but that an independent physical reality must be attributed to such 4D fields E and B (or even better to the electromagnetic field F, [10]) and not to the corresponding 3D fields $\mathbf{E}$ and $\mathbf{B}$. More generally, it is shown in [5] that there is a true agreement, which is independent of the chosen reference frame and the coordinate system in it, between the theory that deals with 4D geometric quantities and the well-known experiments which test special relativity, the "muon" experiment, the Michelson-Morley type experiments, the Kennedy-Thorndike type experiments and the Ives-Stilwell type experiments. It is also discovered in [5] that, contrary to the common opinion, there is no such agreement between Einstein's formulation of special relativity and the mentioned experiments.

6. Conclusions

The consideration presented in this paper reveals that in the 4D spacetime the complex fields, the 4D Ψ and its complex reversion $\overline{\Psi}$, are physically important and well-defined quantities that correctly transform under the LT, whereas it is not the case with the 3D complex field $\mathbf{\Psi}$ and its complex conjugate field $\mathbf{\Psi}^*$. In the 4D spacetime Majorana-Maxwell equations with the 3D $\mathbf{\Psi}$, (18) and (19), have to be replaced with our Lorentz invariant field equations with the 4D Ψ.

For $j \neq 0$ we have presented new Lorentz invariant field equation (7) in which only the 4D AQs are used. Eqs. (8) and (9) are the corresponding field equations with 4D CBGQs written in the standard basis $\{\gamma_\mu\}$. For $j = 0$ we have field equations for the 4D Ψ, (10) and (12) with 4D AQs, and (13) and (15) with 4D CBGQs.

A new generalization of the correspondence principle is introduced by Eq. (11), where the AQs are used, or by Eq. (14) with CBGQs.

The equations (12) (with AQs) and (15) (with CBGQs) are new forms for Dirac-like relativistic wave equations for the free photon, which are not yet reported in the literature, as I am aware. They will be the starting point for the construction of the observer independent stress-energy vector $T(n)$ (1-vector) and all other quantities that are derived from $T(n)$, as are the energy density U (scalar, i.e., grade-0 multivector), the Poynting vector S (1-vector), etc. All these quantities will be expressed by means of the 4D Ψ and $\overline{\Psi}$ in a complete analogy with the construction of these quantities in the axiomatic F formulation [10].

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