

# Conditions for the Generation of Causal Paradoxes from Superluminal Signals

Giuseppe Russo \*

Department of Physics and Astronomy, University of *Catania*  
and National Institute of Nuclear Physics, Section of Catania  
*Viale A. Doria 6, I-95123 Catania, Italy*

Received 17 October 2005, Published 20 December 2005

---

**Abstract:** We introduce a simple method to illustrate how the Lorentz transformation lead to causal loop paradoxes when they are applied to superluminal velocities.

© Electronic Journal of Theoretical Physics. All rights reserved.

*Keywords:* Special relativity - Superluminal signals - Causal paradoxes

*PACS (2003):* 03.30

---

## 1. Introduction

In the framework of the special theory of relativity (STR), recent theoretical [1] and experimental [2], [3],[4],[5], evidencies of superluminal<sup>2</sup> motions necessarily lead to unacceptable causal loop paradoxes. The problem arises because while all observers agree about the time ordering of events linked by a subluminal signal, for a superluminal signal different observers disagree on whether the signal was received after or before it was emitted. In other words, viewed in a certain class of inertial frames, a superluminal signal travels backward in time. Thus, the Tolman's paradox [6], namely the communication with the past, in principle, would become possible. Although the causal paradoxes of the STR are reported in the main introductory texts on relativity only a very limited space is dedicated to them. Because of the intriguing connection between causality violation and time machines construction known from the science fiction, such an argument still continues to captivate physicist' interest.

The aim of this paper is to show the connection between the superluminal signal existence and the causality violation by means of a simple mathematical derivation including

---

\* Giuseppe.Russo@ct.infn.it

<sup>2</sup> Hereafter, by "superluminal" we always mean "with a speed larger than the speed  $c$  of light in vacuum".

also an appropriate graphic representation.

## 2. Causal loop paradoxes

Before discussing causal paradoxes, we shall remind the reader of the Lorentz transformation relating space and time coordinates in different frames of reference. Let us consider two arbitrary inertial frames which will denote with  $\Sigma$  and  $\Sigma'$  and having a common origin at  $t=t'=0$ , but with the origin of  $\Sigma'$  moving along the x-axis of  $\Sigma$  at a relative speed  $v$ . Then, from the two Einstein's postulates of STR, the transformation equations between the two sets of space-time coordinates are

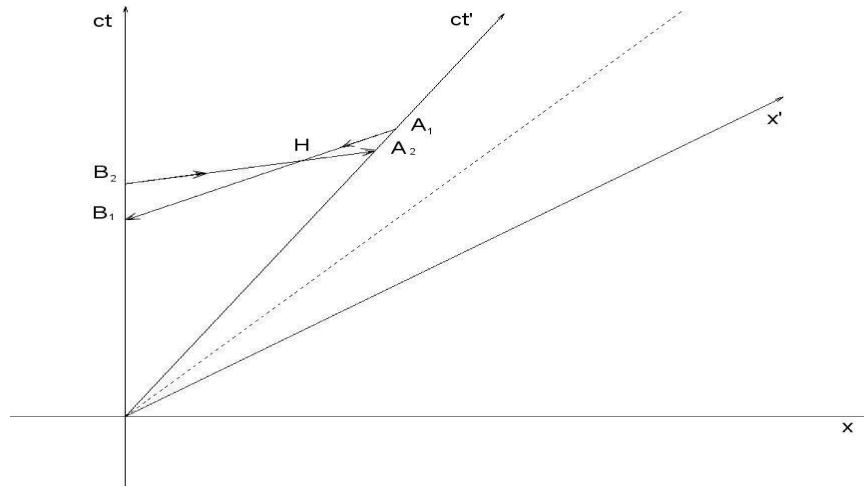
$$\begin{aligned}x' &= \gamma(x - \beta ct) \\y' &= y \\z' &= z \\ct' &= \gamma(ct - \beta x)\end{aligned}\tag{1}$$

with the usual meaning for  $\beta = v/c$  and  $\gamma = (1 - \beta^2)^{-1/2}$ . It is important to stress that the equations (1) are necessary consequences of the relativity postulates. In other words, any shift, however small, of the spatial and/or time coefficient away from its relativistic value necessarily implies the existence of a "preferred" frame. Within the STR one can show that, for the speed of particles and of all physical "signals",  $c$  should be an upper limit if we insist on the invariance of causality. This, in a certain sense, represents a further Einstein's postulate to always preserve the causal connections. Let us suppose now that exists any process in which an event A causes an event B at a "superlight" speed  $u > c$  relative to some frame  $\Sigma$ . Choose coordinates in  $\Sigma$  so that these events both occur on the x-axis and let their spatial and time separations be  $\Delta x > 0$  and  $\Delta t > 0$ . Then, in another inertial frame we have from the fourth of equations (1),

$$\Delta t' = \gamma\left(\Delta t - \frac{\beta}{c}\Delta x\right) = \gamma\Delta t\left(1 - \frac{\beta}{c}u\right)\tag{2}$$

For  $c/u < \beta$  we would have  $\Delta t' < 0$ . This means that would exist a class of inertial frames  $\Sigma'$  in which B precedes A, i.e. in which cause and effect are reversed or in which information flow from receiver to transmitter. Thus, we could have foreknowledge of future events and if we decide deliberately foil them, by manipulating the past, we would incur grave contradictions. In a bidimensional Minkowski diagram having space in abscissae and time in ordinates, the line of arguments as the use of superluminal signal velocities leads to a violation of the Einstein causality is illustrated in Figure 1. There are two arbitrary inertial frames of reference displayed.

At the time  $t'(A_1)$ , a superluminal signal relatively to  $\Sigma'$  (the question), is emitted from its origin toward the origin of  $\Sigma$ .  $B_1$  is the event associated to its arrival in  $\Sigma$ .



**Fig. 1** Spatial and time coordinates of two arbitrary inertial observers moving with a relative velocity  $v$ . The lines indicated by arrows represent two signals which are declared superluminal and propagate in the future with respect to the reference frame in which each signal is emitted.

The observer in  $\Sigma$ , after waiting for a time  $\Delta t > 0$ , decide to send, toward the origin of  $\Sigma'$ , a superluminal signal, relatively to  $\Sigma$  (event labelled with  $B_2$  in figure 1).  $A_2$  is the event associated with the arrival of the signal (the answer) to the origin of  $\Sigma'$ . The only constraint is that each signal should travel in the future with respect to the observer which has emitted it. Well, now we shall demonstrate that under both the conditions:

$$\tilde{\beta} > \frac{1}{\beta} \quad \text{and} \quad \beta^* > \frac{1}{\beta} \quad (3)$$

( $\tilde{\beta}$  and  $\beta^*$  being the velocities, in  $c$  units, of the signals in  $\Sigma'$  and  $\Sigma$  respectively), certainly exists a non-negative solution for the waiting time  $\Delta t$  for which the event labelled with  $A_2$  occurs before the  $A_1$  one with respect both the  $\Sigma$  and  $\Sigma'$  reference frames. Indeed, we have for the signal starting from  $A_1$

$$x'(B_1) = -\tilde{\beta}[ct'(B_1) - ct'(A_1)] \quad (4)$$

and also

$$x'(B_1) = -\beta ct'(B_1) \quad (5)$$

being  $B_1$  an event which happens on the  $x=0$  axis. Thus, being  $\tilde{\beta} > \beta$ , we get

$$ct'(B_1) = \frac{\tilde{\beta}}{\tilde{\beta} - \beta} ct'(A_1) \quad (6)$$

and using the fourth equation of the Lorentz transformation (1), we find the result:

$$ct(B_1) = \frac{\tilde{\beta}}{\gamma(\tilde{\beta} - \beta)} ct'(A_1) \quad (7)$$

After a time interval  $\Delta t \geq 0$ , the observer in the  $\Sigma$  frame decides to send his "answer". Let  $B_2$  be the event associated with the departure, from the origin of  $\Sigma$  of the superluminal

signal having a velocity which is  $\beta^*$  (in  $c$  units) with respect to the observer at rest in  $\Sigma$ . Thus, if  $A_2$  denote the event associated with the arrival of such a signal to the origin of  $\Sigma'$ , we have

$$x(A_2) = \beta^*[ct(A_2) - ct(B_2)] \quad (8)$$

$$x(A_2) = \beta ct(A_2) \quad (9)$$

Then, being also  $\beta^* > \beta$ , we get

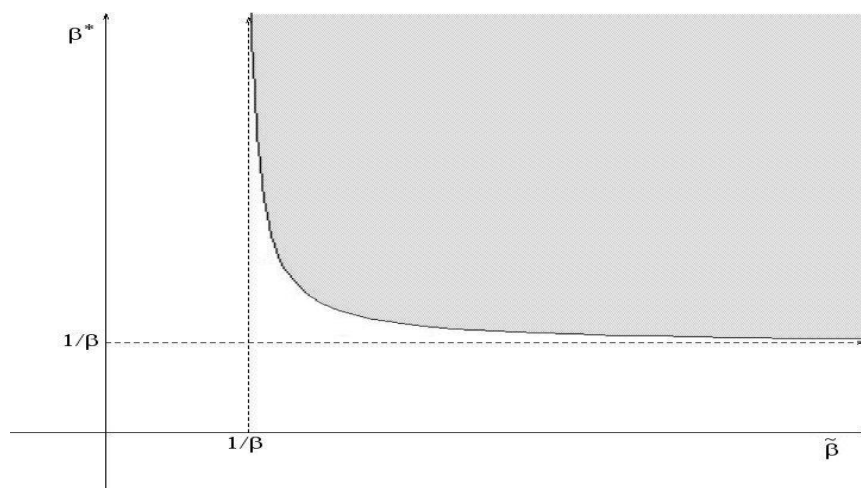
$$ct(A_2) = \frac{\beta^*}{\beta^* - \beta} ct(B_2) \quad (10)$$

and using the fourth equation of the inverse Lorentz transformation we find the result:

$$ct'(A_2) = \frac{\beta^*}{\gamma(\beta^* - \beta)} ct(B_2) \quad (11)$$

By using the (7), the equation (11) provides:

$$ct'(A_2) = \frac{\beta^*}{\gamma(\beta^* - \beta)} \left[ c\Delta t + \frac{\tilde{\beta}}{\gamma(\tilde{\beta} - \beta)} ct'(A_1) \right] \quad (12)$$



**Fig. 2** Graphic representation of the inequality (17) in the  $\tilde{\beta} - \beta^*$  plane. The shadowed indicates the region of values for  $\tilde{\beta}$  and  $\beta^*$  in which the causal paradoxes became possible.

Now, for non-negative values of  $\Delta t$ , the  $ct'(A_2) < ct'(A_1)$  inequality necessarily requires that the important condition

$$\frac{1}{\gamma^2} \frac{\tilde{\beta}\beta^*}{(\tilde{\beta} - \beta)(\beta^* - \beta)} < 1 \quad (13)$$

should be, at least, satisfied. This condition (13) has an interesting geometrical representation in the  $\tilde{\beta} - \beta^*$  plane. In fact, the inequality (13) also writes,

$$\tilde{\beta} + \beta^* - \beta\tilde{\beta}\beta^* - \beta < 0 \quad (14)$$

representing a region delimited by an hyperbola which, by means the of translation:

$$\tilde{\beta} = \tilde{\Lambda} + \frac{1}{\beta} \quad (15)$$

$$\beta^* = \Lambda^* + \frac{1}{\beta} \quad (16)$$

becomes:

$$\tilde{\Lambda}\Lambda^* > \frac{1}{(\gamma\beta)^2} \quad (17)$$

In figure 2 we displayed the region characterized from both  $\tilde{\Lambda} > 0$  and  $\Lambda^* > 0$  inequalities in which the (17) is satisfied, namely it is shown the region (shadowed) where the causality can be violated. It is worth noting that in such a region the superluminals signal have slopes which are strictly less than  $\beta$ . Finally, under the condition (17), from equation (12) we find also an interval of non-negative values for  $\Delta t$  which is limited by a maximum value given by:

$$\begin{aligned} \Delta t^{max} &= \frac{\gamma\beta^2}{\beta^*(\tilde{\beta} - \beta)} t'(A_1) [(\tilde{\beta} - \frac{1}{\beta})(\beta^* - \frac{1}{\beta}) - \frac{1}{(\gamma\beta)^2}] = \\ &= \frac{\gamma^2\beta^2}{\beta^*\tilde{\beta}} t(B_1) [\tilde{\Lambda}\Lambda^* - \frac{1}{(\gamma\beta)^2}] \end{aligned} \quad (18)$$

As outlined at the beginning of this section (see equation (2)), the signals  $A_1$ - $B_1$  and  $B_2$ - $A_2$  travel backwards in time for the  $\Sigma$  and  $\Sigma'$  frames respectively. Thus, we have a double meaning for the world lines of figure 1. In fact, under the conditions which assure the existence of the common H event, the  $H$ - $B_1$  and  $H$ - $A_2$  world lines represent two time-reversed return paths leading to the  $B_1$  and  $A_2$  events which lie in the past light-cones of the  $B_2$  and  $A_1$  respectively.

### 3. Conclusion and outlook

In conclusion we have shown, by means of elementary mathematical tools and an appropriate graphic representation, how the use of signals declared "superluminal" with respect to each inertial observer can lead to a causal loop paradox when the velocity of any signal is greater than  $c^2/v$ .

In order to avoid the causal paradoxes, different solutions are suggested in the literature. The first is based on the so-called Extended STR proposed by Recami and Mignani [7]. Within such an approach the causal paradoxes are avoided through "the switching procedure" also known as the "reinterpretation principle" [8]: a negative-energy superluminal particle moving backwards in time is actually a positive-energy superluminal antiparticle propagating forwards in time and viceversa. According to the "switching rule" the causality violation is avoided, assuming of course that negative-energy objects travelling forward in time do not exist.

Concerning the compatibility between superluminal velocities and STR, another solution was discussed by Nimtz and Haibel [9]. They have shown that, experimentally, the principle of causality could not be violated in consequence of the narrow frequency band width of all signals.

Finally, in the context of a more general theory, the causal paradoxes seem to be typical of a set of transformations: those so-called equivalent (ET) to the STR [10], [11] depending on the synchronization parameter, the coefficient of  $x$  in the transformation of time, which is in large part conventional and often indicated as  $e_1$ . One can also show [12] that for the subset of transformations characterized by  $e_1 \geq 0$ , the causal paradoxes are not permitted. Within such a class that corresponding to a particular  $e_1=0$  value, provides the so-called "inertial transformation" which transform time independently of space coordinates [11], [13]. Although these agree with the microscopic and macroscopic experimental evidences for time dilation, they are based on the absolute synchronization concept.

## References

- [1] K.Scharnhorst, "On propagation of light in the vacuum between plates", Phys. Lett. B **236**, 354-359 (1990). G. Barton, "Faster-than-c light between parallel mirrors. The Scharnhorst effect rederived", Phys. Lett. B **237**, 559-562 (1990). K. Scharnhorst and G. Barton, "QED between parallel mirrors: light signals faster than c, or amplified by the vacuum", J. Phys. A **26**, 2037-2046 (1993). K. Scharnhorst, "The velocities of light in modified QED vacua", Ann. Phys. **7** 700-709 (1998).
- [2] A. Enders and G. Nimtz, "On superluminal barrier traversal", J. Phys. **12**, 1693-1698 (1992). G. Nimtz, "Evanescent modes are not necessarily Einstein causal" Eur. Phys. J. B **7**, 523-525 (1999).
- [3] L.J. Wang, A. Kuzmich and A. Dogarlu, "Gain-assisted superluminal light propagation", Nature **406**, 277-279 (2000).
- [4] J. J. Carey, J. Zawadzka, D.A. Jaroszynski and K. Wynne, "Noncausal time response in frustated total internal reflection?", Phys. Rev. Lett. **84**, 1431-1434 (2000).
- [5] D.Mugnai, A. Ranfagni and R. Ruggeri, "Observation of superluminal behaviors in wave propagation", Phys. Rev. Lett. **84**, 4830-4833 (2000).
- [6] R.C. Tolman, "The theory of the Relativity of Motion" (Berkeley: University of California Press, 1917).
- [7] E. Recami and R. Mignani, "Classical theory of tachyons (Special relativity extended to superluminal frames and objects)", Rivista Nuovo Cim. **4**, 209-290 (1974); E398 and references therein.
- [8] E. Recami, "Classical tachyons and possible applications", Rivista Nuovo Cim. **9**, issue no.6 1-178 (1986) and references therein. E. Recami, "Tachyon mechanics and causality; A systematic thorough analysis of the tachyon causal paradoxes", Found. Phys. **17**, 239-296 (1987).
- [9] G. Nimtz and A. Haibel, "Basics of superluminal signals", Ann. Phys. (Leipzig) **11**, 163-171 (2002).
- [10] R. Mansouri and R. U. Sexl, "A test theory of special relativity: I. Simultaneity and clock synchronization Gen. Rel. Grav. **8**, 497-513 (1977).
- [11] F. Selleri, "Space, time and their transformations", Chin. J. Eng. Elect. **6**, 25-44 (1995). F. Selleri, "Noninvariant one-way velocity of light", Found. Phys. **26**, 641-664 (1996).
- [12] G. Russo, "On the equivalent theories to the special relativity", submitted for publication on Foundations of Physics (2005)
- [13] F. Selleri, "Space and Time should be preferred to spacetime 1 and 2" in Int. Work, Phys. for the 21st century, 5-9 june 2000, Natural Philosophy Association (Boston) and University of Connecticut.