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Physics of Currents and Potentials
IV. Dirac Space and Dirac Vectors in the Quantum Relativistic Theory

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Abstract: There has been presented an attempt to transfer the fundamental ideas of physics of continual currents and potentials, described in the previous articles of this series [1], [2], [3], from the classical theory to the quantum relativistic theory.
The concept of multidimensional Dirac space, which should contain wave equations of the relativistic quantum theory, has been introduced. Dirac space dimension d is determined by Yang-Mills multiplicity of the sector of physics: d = 8 for the singlet (quantum electrodynamic states); d = 20 for the two-sector singlet-triplet states; d = 52 for the three-sector singlet - triplet-octuplet states. It has been shown that the quantum relativistic state can not be described by the unique wave function (four-component Dirac vector). Singlet states are described by a pair of Dirac vectors, two-sector singlet-triplet states are described by four Dirac vectors, eight Dirac vectors are necessary for description of the three-sector singlet-triplet-octuplet states.
It has been shown that the necessity to consider the Riemann curvature of space causes additional difficulties in the process of construction of the quantum relativistic theory.

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1 Introduction. From Classical Field Theory to Quantum Relativistic Theory. Dirac Space

The classical field theory with continual currents, presented in the preceding articles of this series [1], [2], [3], did not require in its formulation any radical deviation from the

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fundamental ideas formed in theoretical physics of the XXth century. The main broadening of the well-established ideas in these articles consisted of rejection of the mechanical interpretation of 4-currents appearing in the theory: the space-like 4-current is not generated by the motion of the material ("ponderable", as Albert Einstein would have put it) charge carrier; the space-like current is primary and can not be reduced to any other, more simple entity.

It took the physicists of the XIXth century several decades to abandon the attempts to provide mechanical interpretation of the electromagnetic field. J. Maxwell was persistently searching for these interpretations; the traces of this search can be found in H. Hertz's works, but the works of G.A. Lorenz are already free from the attempts of mechanical interpretation of field: field is primary and can not be reduced to anything simpler.

Future historians of science may find it difficult to understand why the rejection of mechanical interpretation of the second half of the electromagnetic dyad, 4-current, was a full century late. This broadening of Lorentz approach to electromagnetic theory seems quite natural and expected. Undoubtedly, it becomes natural and even trivial only with the simultaneous recognition of dyadic nature of a field: "a current and a potential" as the two inseparable and equal halves of the continual field dyad, the yin and yang of electrodynamics.

The field thought of the most outstanding representative of the "field ideology" of the XXth century physics, Albert Einstein, was distinctively ***monadic***. Einstein perceived the field as a monad described by a single physical-geometric object, and he hoped to see the particles as bunches of field energy; perhaps – as the field singularities, characteristics and motion of which are completely determined by the field and can not be set arbitrarily.

For thirty-six years of thinking and working on this series of articles, I have often asked myself a question: "Would Albert Einstein have accepted the embodiment of Maxwell field program, which is presented in the first article of this series [1]?" Now I believe, that after the period of "monadic resistance" and stern grumbling, Einstein would have supported such version of electrodynamics. This is really a completely field and totally classical theory, and, besides, it incorporates the Einstein gravitation theory.

In a sense, the theory presented in articles [1], [2], [3], is the embodiment of Einstein's ideal of the classical field theory achieved at the cost of the rejection of monadic description of fields and the rejection of visual and naive mechanical interpretation of currents. Even if the field pattern of physical reality, presented in articles [1], [2], [3], is not the final version of Einstein's ideal, it, anyway, indicates a decisive step towards this ideal. And, after all, does not the idea of the primary nature of space-like 4-current open not less exciting intellectual perspective for theoretical physics than it once opened – ungrounded and arising from nowhere – Louis de Broglies idea of matter waves; the idea, which was decisively and immediately supported by Einstein.

Albert Einstein was convinced that the nonlinearity of field equations was necessary for existence of particles as some bunches of field energy. However, this is true only for the monadic theory.

Dyadic field electrodynamics, presented in article [1], is non-linear (even if we omit the requirement of the Riemann nature of geometry) despite the linearity of field equations: the boundary conditions for currents at the boundaries of current zones are non-linear, and the mere presence of previously unknown boundaries of current zones in the theory makes the problem nonlinear (even if the boundary conditions were linear).
In the quantum version of physics of currents and potentials, we have to turn decisively off the beaten "field" track of physics of the XXth century. In the classical version of the theory, presented in the preceding articles of this series, we were just slightly complementing this beaten track dating back to Faraday, Maxwell and Lorentz. We tried to use the "classical" asphalt to connect this track of electrodynamics with the "branch" of Yang-Mills theory which for decades had been positioning itself as purely quantum pathway that had emerged from quantum wasteland, without any classical basis.
Consistent, shrunk into itself, free from any self-contradictions and divergences, the quantum formulation of the field theory requires a rather radical break from the existing approach of modern theoretical physics - such a radical break that perhaps it could not have been approved by any of the genius founding fathers of quantum mechanics and quantum field theory. The essence of this break is to change the interpretation of the concept of physical reality itself.
For classical relativistic physics, field dyads in the three sectors of physics are the physical reality which is immersed in the four-dimensional space-time continuum, the Riemann geometry of which obeys the Einstein equations. In quantum relativistic physics, we apparently have to assume that semi-components of the field dyads (currents or potentials), along with the space-time coordinates, are not unknown physical quantities, but just the arguments of the Dirac wave functions. In other words, during the transition from classical to quantum version of the theory, the fields are no longer an element of physical reality and they turn into the ***capacitance for reality*** – the reality of an ensemble of field realizations; the capacitance supplementing the space-time or momentum four-dimensional continuum with its dimensions.
Mathematically, the ensemble is described by some set of wave functions - the Dirac spinors. These spinors in quantum electrodynamics depend on eight arguments: four space-time coordinates (***or*** four coordinates in the momentum space) plus four current components (***or*** four potential components). In the complete three-sector problem of physics ("the problem of the Standard Model"), each spinor has 52 arguments: 4 space-time coordinates (***or*** 4 coordinates in the momentum space) plus four components of each of the twelve currents of the Standard Model (***or*** each of the twelve potentials, ***or*** arbitrary combination of currents and potentials, constructed so that spinor arguments contain only one semi-component – current ***or*** potential – of each of the field sector dyad of the three sectors of physics). Wave equations for the Dirac spinors should be formulated as nonlinear equations in partial derivatives in this fifty-two-dimensional space. On the account of the missing of any meaningful term that would adequately describe this 52-dimensional continuum, we shall call it ***the Dirac space***. In singlet (electrodynamic) states, dimension d of the Dirac space is equal to eight; in pure triplet Yang-Mills states

$d = 16$, in mixed singlet-triplet (electroweak) states $d = 20$, in pure octuplet (chromodynamical) Yang-Mills states $d = 36$; in total three-sector states $d = 52$.
For quantum version of the theory, in contrast to classical version, it is impossible to introduce the concept of field tensor – the operation of potential differentiation by coordinates does not have any physical sense: coordinates and potentials are equal arguments of the Dirac spinors in the Dirac space. Accordingly, the quantum versions of the Maxwell and Yang-Mills equations can only be some operator relations. Basic wave equations for the Dirac spinors must be constructed as some radical generalizations of the Dirac equations. The Dirac equation itself, losing the status of the exact equation of physics, in the framework of this theory can be some approximate relation which appears after some procedure of an approximate integration of the wave equations by the field arguments, like in the classical electrodynamics of continual currents described in article [1], the Lorenz equation (equation of motion of a point charge carrier) appears as an ***approximate*** equation after the ***approximate*** integration of the equations of continual theory by the volume occupied by currents, in the approximation of a "weak external field" (see [1]).
We did not manage to construct the basic wave equations of the theory in this work. Moreover, according to the "rule of Tridentine prudence" [3], we would prefer to avoid premature discussion of the difficult questions of geometry: what space should the equations of gravitation, controlling the metric, be entered in – in the usual four-dimensional continuum , or in 52-dimensional Dirac continuum? How should the energy-momentum tensor be constructed in the theory which does not have a field tensor?, etc.
Within the framework of this theory, we have an intention to describe the method, by means of which it is possible to construct the Dirac spinors on the grounds of observational data[1] and to describe the connection between spinors.
We shall preface this description with presentation of non-relativistic quantum theory in such formulation which allows a natural transfer to the relativistic realm. Discussion of the foundations of quantum mechanics, learnt already at the University freshman class, can make a skilled reader bored and irritated. By putting up with this, the author hopes that this form of presentation of a well-known non-relativistic scheme will make it easier for the reader to further perceive the relativistic relations in the Dirac multi-dimensional space.

2 Non-relativistic Scheme (Born-Heisenberg-Schrodinger Program)

2.1 Born Density. De Broglie and Born Postulates

Non-relativistic quantum mechanics is based on the Newtonian concept of physical system as an object consisting of point particles n, coupled by forces of a long-range interaction, but, in contrast to Newtonian mechanics, in quantum mechanics, the n – particle system

[1] Here it would be appropriate to remind of the uncompromising Einsteinian maxim: "Only the theory determines what can be observable".

itself is not an element of reality allowing a mathematical description. The **ensemble** of identical to each other n-particle systems is the element of reality, i.e. an infinite number of the systems, from which it is possible to extract individual copies of such systems for making instantaneous coordinate measurements of all the particles of the system. Coordinate measurement in the frames of non-relativistic conception of reality can be, in principle, made with unlimitedly high precision. The process of this measurement destroys the system completely, and it is impossible to make further measurements with ***the same*** copy of the system.

In contrast to Newtonian mechanics, where ***only*** point ***coordinates*** are observable quantities, quantum mechanics is based on the postulate of the existence of one more observable quantity for each particle – momentum vector $\mathbf{k}$, which has the dimension $[\text{length}]^{-1}$. Within the framework of the non-relativistic concept, particle momentums can be measured, in principle, with unlimited precision. The process of this measurement destroys the system completely, and it is impossible to make further measurements with ***the same copy*** of the system.

The fundamental postulate of measurability of particle momentum and existence of an infinite three-dimensional momentum space with Euclidean metric, apparently, should be formulated explicitly and should be called de Broglie postulate. This postulate transforms geometric spatial monad of Newtonian mechanics into space-momentum dyad of quantum mechanics $\{\mathbf{x}|\mathbf{k}\}$. This is an important step on the way of relativistic transformation of space-time four-dimensional Minkowski monad x_ν into the dyad consisting of 4-coordinates x_ν and 4-momentums k_ν: $\{x_\nu\, k_\nu\}$. The essential feature of relativistic quantum theory is that we can not interpret semi-components of this dyad – x_ν and k_ν – as coordinates and momentums of the point object: point objects do not exist in relativistic physics. In quantum relativistic physics we should just talk about coordinate space x_ν and momentum space k_ν and assert that with corresponding measurements, we have an opportunity to discover ***something*** in the neighborhood of point x_ν (or point k_ν) on a small but finite element of the oriented three-dimensional hyper-surface σ in four-dimensional coordinate space (or momentum space).

But let us revert to non-relativistic physics.

For our purposes it is sufficient to concentrate on the accurate formulation of mathematical apparatus which is needed to describe the non-relativistic ensemble of one-particle systems (n=1).

Let us suppose that at some instant of time t, we made instantaneous coordinate measurement $\underset{\mathbf{x}}{N_t}$ of copies of the system, and let us assume that $\Delta\underset{\mathbf{x}}{N_t}$ of these measurements have registered the presence of the particle in some small but finite volume $\Delta\underset{\mathbf{x}}{\mathrm{V}}$ in the neighborhood of the point with radius-vector $\mathbf{x}$ in a pre-selected inertial frame of reference. We postulate that with the unlimited growth of a number of measurements of $\underset{\mathbf{x}}{N_t}$ there is limit $\underset{\mathbf{x}}{\mathrm{R}}{}_t$ of the relation $\Delta\underset{\mathbf{x}}{N_t}/\underset{\mathbf{x}}{N_t}$. Due to arbitrariness of the shape and volume of $\Delta\underset{\mathbf{x}}{\mathrm{V}}$ this limit can be expressed as $\Delta\underset{\mathbf{x}}{\mathrm{V}}$ volume integral of some function $\underset{\mathbf{x}}{\rho}(\mathbf{x},\, t)$ that depends on coordinates $\mathbf{x}$ at the instant of time t:

$$\underset{\mathbf{x}}{\mathrm{R}}{}_t = \lim_{\underset{\mathbf{x}}{N}_t\to\infty}\left(\frac{\Delta \underset{\mathbf{x}}{N}_t}{\underset{\mathbf{x}}{N}_t}\right) = \int\limits_{\Delta \underset{\mathbf{x}}{\mathrm{V}}} \underset{\mathbf{x}}{\rho}(\mathbf{x},\, t)\mathrm{d}\underset{\mathbf{x}}{\mathrm{V}}. \tag{1}$$

It is obvious that function $\underset{\mathbf{x}}{\rho}(\mathbf{x},\, t)$, determined by relation (1), is nonnegative, integrable with any small volume $\Delta\underset{\mathbf{x}}{\mathrm{V}}$ into the neighborhood of any point $\mathbf{x}$ and normalized per unit under integration into (1) over the entire infinite three-dimensional coordinate space. The dimension of $\underset{\mathbf{x}}{\rho}(\mathbf{x},\, t)$ is cm^{-3}.

In an absolutely similar way, making measurements of momentum for $\underset{\mathbf{k}}{N}_t$ copies of a one-particle system ***at the same instant of time*** t [2], we can discover that $\Delta\underset{\mathbf{k}}{N}_t$ of these measurements register the presence of a particle in a small but final volume $\Delta\underset{\mathbf{k}}{\mathrm{V}}$ of momentum space in the neighborhood of some momentum $\mathbf{k}$. The equipment for measuring momentum $\mathbf{k}$ is meant to be fixed relative to the same inertial system in which coordinates $\mathbf{x}$ are measured and to agree in the orientation of coordinate axes with the equipment that measures coordinates. We postulate that with the unlimited growth of a number of measurements of $\underset{\mathbf{k}}{N}_t$ there is a limit $\underset{\mathbf{k}}{\mathrm{R}}{}_t$ for relation $\Delta\underset{\mathbf{k}}{N}_t/\underset{\mathbf{k}}{N}_t$. Due to arbitrariness of the shape and size of volume $\Delta\underset{\mathbf{k}}{\mathrm{V}}$, this limit can be expressed as a volume $\Delta\underset{\mathbf{k}}{\mathrm{V}}$ integral of some function $\underset{\mathbf{k}}{\rho}(\mathbf{k},\, t)$ that depends on momentums $\mathbf{k}$ at the instant of time t:

$$\underset{\mathbf{k}}{\mathrm{R}}{}_t = \lim_{\underset{\mathbf{k}}{N}_t\to\infty}\left(\frac{\Delta \underset{\mathbf{k}}{N}_t}{\underset{\mathbf{k}}{N}_t}\right) = \int\limits_{\Delta \underset{\mathbf{k}}{\mathrm{V}}} \underset{\mathbf{k}}{\rho}(\mathbf{k},\, t)\mathrm{d}\underset{\mathbf{k}}{\mathrm{V}}. \tag{2}$$

Function $\underset{\mathbf{k}}{\rho}(\mathbf{k},\, t)$, determined by relation (2), is nonnegative, integrable with any small volume $\Delta\underset{\mathbf{k}}{\mathrm{V}}$ into the neighborhood of any momentum $\mathbf{k}$ and normalized per unit under integration into (2) over the entire infinite three-dimensional momentum space. The dimension of $\underset{\mathbf{k}}{\rho}(\mathbf{k},\, t)$ is cm^3.

We shall call functions $\underset{\mathbf{x}}{\rho}$ and $\underset{\mathbf{k}}{\rho}$[3], determined by relations (1) and (2), the Born densities: $\underset{\mathbf{x}}{\rho}$ is the Born density in coordinate space, $\underset{\mathbf{k}}{\rho}$ is the Born density in momentum space.

Let us call the assertion of the existence of limits (1) and (2) and, respectively, the existence of the Born densities, as the ***Born postulate***.

The use of terms, established in quantum mechanics, requires the obligatory use of the term "probability" in naming functions $\underset{\mathbf{x}}{\rho}$ and $\underset{\mathbf{k}}{\rho}$: $\underset{\mathbf{x}}{\rho}$ is the "probability density of finding the particle in the coordinate space", and $\underset{\mathbf{k}}{\rho}$ is the "probability density of finding a particle in the momentum space".

[2] It is obvious that this expression is principally non-relativistic; it can not be given Lorentz-invariant sense. However, it does not cause any difficulties, since the choice of instant of time for measurements in momentum space does not matter for the ensemble of isolated systems.

[3] The indications on the arguments of these functions $(\mathbf{x},\, t)$ or $(\mathbf{k},\, t)$ will further be omitted.

We suppose that the term "probability" in definition of the Born density is redundant. This term refers to the artificial, archaic and unlawful (or subconscious) preservation of Newton's idea of a separate copy of the system as an element of physical reality in quantum mechanics. In quantum mechanics, infinite ***ensemble*** of systems is physical reality: the measurements are made on some individual copies of the system, but mathematical description is only possible for ensemble. In fact, individual system does not have "being-in-time" which is comprehensible for a macro-observer, but ensemble has such being.
All physical information on the ensemble is enclosed in a pair of the Born densities $\left\{\underset{\mathbf{x}}{\rho}, \underset{\mathbf{k}}{\rho}\right\}$. We shall call this pair "the Born pair".

2.2 Heisenberg Postulate

Awareness of the Born pair allows to calculate different characteristics of the ensemble, such as the ensemble mean value of i-component of radius vector $\langle x_i\rangle$:

$$\langle x_i\rangle = \int \underset{\mathbf{x}}{\rho}\, x_i \mathrm{d}\underset{\mathbf{x}}{\mathrm{V}}, \tag{3}$$

or the ensemble mean value of i-component of momentum $\langle k_i\rangle$:

$$\langle k_i\rangle = \int \underset{\mathbf{k}}{\rho}\, k_i \mathrm{d}\underset{\mathbf{k}}{\mathrm{V}}. \tag{4}$$

Integration by the entire three-dimensional coordinate space (3) or by the entire three-dimensional momentum space (4) is implied in these relations.
In the same way we can calculate the mean ensemble value of the square of i-Cartesian component of radius-vector $\langle x_i^2\rangle$, or the mean ensemble value of the square of i-Cartesian momentum component $\langle k_i^2\rangle$:

$$\langle x_i^2\rangle = \int \underset{\mathbf{x}}{\rho}\, x_i^2 \mathrm{d}\underset{\mathbf{x}}{\mathrm{V}},$$
$$\langle k_i^2\rangle = \int \underset{\mathbf{k}}{\rho}\, k_i^2 \mathrm{d}\underset{\mathbf{k}}{\mathrm{V}}$$

as well as the dispersion value of each coordinate $\underset{\mathbf{x}}{\sigma_i}$ and each momentum component $\underset{\mathbf{k}}{\sigma_i}$:

$$\underset{\mathbf{x}}{\sigma_i} = \sqrt{\left\langle (x_i - \langle x_i\rangle)^2\right\rangle} = \sqrt{\langle x_i^2\rangle - \langle x_i\rangle^2},$$
$$\underset{\mathbf{k}}{\sigma_i} = \sqrt{\left\langle (k_i - \langle k_i\rangle)^2\right\rangle} = \sqrt{\langle k_i^2\rangle - \langle k_i\rangle^2}.$$

Let us form 3×3-Heisenberg matrix H_{ij} of quantities of dispersions:

$$\mathrm{H}_{ij} = \underset{\mathbf{x}}{\sigma_i}\, \underset{\mathbf{k}}{\sigma_j}. \tag{5}$$

It is obvious that the elements of matrix (5) are nonnegative. However, for diagonal elements, a stronger Heisenberg inequality is also valid:

$$\mathrm{H}_{ij} \geq \frac{1}{2}\, \delta_{ij}. \tag{6}$$

This inequality expresses a fundamental postulate of non-relativistic quantum physics of ensembles:
There are only such Born pairs $\left\{\underset{\mathbf{x}}{\rho},\ \underset{\mathbf{k}}{\rho}\right\}$ ***that satisfy Heisenberg inequality*** (6).
("The Heisenberg postulate").
We shall call the Born pair which satisfies inequality (6), the Born ensemble.

2.3 Schrödinger Theorem

The validity of the statement, which we will call ***Schrödinger theorem***, follows from the Heisenberg Postulate:
For each Born ensemble there is a pair of real functions $\underset{\mathbf{x}}{\mathrm{S}}(\mathbf{x}, t)$ and $\underset{\mathbf{k}}{\mathrm{S}}(\mathbf{k}, t)$, (the "phase pair") such one that the complex functions $\underset{\mathbf{x}}{\psi}$ and $\underset{\mathbf{k}}{\psi}$, determined by the relations

$$\underset{\mathbf{x}}{\psi} = \sqrt{\underset{\mathbf{x}}{\rho}}\, e^{i\underset{\mathbf{x}}{\mathrm{S}}}, \tag{7}$$

$$\underset{\mathbf{k}}{\psi} = \sqrt{\underset{\mathbf{k}}{\rho}}\, e^{i\underset{\mathbf{k}}{\mathrm{S}}}, \tag{8}$$

are Fourier images of each other .

$$\underset{\mathbf{x}}{\psi} \overset{\mathrm{FT}}{=} \underset{\mathbf{k}}{\psi}. \tag{9}$$

In relation (9) symbol "$\overset{\mathrm{FT}}{=}$" describes the procedure of non-relativistic three-dimensional Fourier transform, connecting the functions in the coordinate space with the functions in the momentum space:

$$\begin{aligned} \underset{\mathbf{x}}{\psi} &= \frac{1}{(2\pi)^{3/2}} \int \underset{\mathbf{k}}{\psi}\, e^{i\,\mathbf{k}\cdot\mathbf{x}}\, \mathrm{d}\underset{\mathbf{k}}{\mathrm{V}}, \\ \underset{\mathbf{k}}{\psi} &= \frac{1}{(2\pi)^{3/2}} \int \underset{\mathbf{x}}{\psi}\, e^{-i\,\mathbf{k}\cdot\mathbf{x}}\, \mathrm{d}\underset{\mathbf{x}}{\mathrm{V}}. \end{aligned} \tag{10}$$

Integrals in (10) are taken over the entire momentum space or, respectively, over the entire coordinate space. The two equations (10) are not independent: if the first one is satisfied, the second one is also satisfied.
From the determination of complex functions ψ (7) and (8) follows that:

$$\begin{aligned} \underset{\mathbf{x}}{\rho} &= \underset{\mathbf{x}}{\psi}\,\underset{\mathbf{x}}{\psi^*}, \\ \underset{\mathbf{k}}{\rho} &= \underset{\mathbf{k}}{\psi}\,\underset{\mathbf{k}}{\psi^*}, \end{aligned} \tag{11}$$

where asterisk $(^*)$ is a symbol of complex conjugation.
After substitution of expressions (7) and (8) into relations (10), and after separation of the real and imaginary parts, these relations form a system of two nonlinear real integral equations relative to a pair of unknown real phase functions $\underset{\mathbf{x}}{\mathrm{S}}$ and $\underset{\mathbf{k}}{\mathrm{S}}$.
Schrödinger theorem is the statement of the existence and uniqueness[4] of a solution to

[4] With the accuracy to within the arbitrary, time-dependent term.

this system of nonlinear integral equations.
Let us name complex functions $\underset{\mathbf{x}}{\psi}$ and $\underset{\mathbf{k}}{\psi}$, determined by relations (7) and (8), the wave functions of Schrödinger or, to be more precise, the ***Fourier-doublet of Schrödinger wave functions***.
Under conjugation (9), existing between the components of Fourier-doublet, only one of the two components of the doublet is enough to describe the Born ensemble: the second one can be calculated by formula (9).
The Schrödinger theorem proving is unknown to the author; let us leave the burden of its proving to mathematicians. Modern computational mathematics does not contain any clear and simple procedure for constructing a phase pair $\left\{\underset{\mathbf{x}}{\mathrm{S}}, \underset{\mathbf{k}}{\mathrm{S}}\right\}$ by the Born ensemble $\left\{\underset{\mathbf{x}}{\rho}, \underset{\mathbf{k}}{\rho}\right\}$.
The statement, converse to the Schrödinger theorem, (we shall name it Weyl theorem), is well known, and its proving can be found in any textbook on quantum mechanics: for any Fourier -doublet of Schrödinger wave functions, the Heisenberg inequalities (6) are satisfied[5].

2.4 Ensemble Mass. Planck Postulate

The Born densities are continuous and differentiable time functions which allows to study the dependence of any of the ensemble characteristics on time. Velocity of ensemble $\mathbf{v}$ is an important characteristic of the one-particle ensemble:

$$\mathbf{v} = \langle \mathbf{x} \rangle^{\bullet} = \int \mathbf{x}\, \partial_t \underset{\mathbf{x}}{\rho} \, \underset{\mathbf{x}}{\mathrm{dV}}, \tag{12}$$

where $\partial_t \underset{\mathbf{x}}{\rho}$ is the partial time derivative of the Born density $\underset{\mathbf{x}}{\rho}$.
Velocity $\mathbf{v}$ is the ensemble characteristic. It makes no sense to speak about "particle velocity" – this notion has no representation in the apparatus of quantum mechanics.
Relation (12) allows us to formulate the postulate of existence of the Born ensemble mass: For any of the Born ensemble there is a positive constant m, such one that

$$\mathbf{v} = \frac{1}{m} \langle \mathbf{k} \rangle. \tag{13}$$

The statement of quantity constancy is the hidden definition of the term "uniform time". The physical content of equality (13) is the following: if we choose the clock ("uniform time") so that quantity m is the constant for some single one-particle ensemble, for any

[5] Since student years, the author of this article was indignant by the very fact of Weyl ***derivability*** of fundamental and empirically irrefutable uncertainty relation (6) from arbitrarily constructed complex-valued apparatus of quantum mechanics. The scheme of construction of non-relativistic quantum mechanics, presented here, allows us to derive a ***mathematical fact*** of the existence of complex wave function from the Heisenberg ***physical postulate***: understanding of physics precedes the construction of mathematical apparatus. We tend to use the same approach in the relativistic area.

other one-particle ensemble its mass m, which is determined by (13), will be also constant. In fact, relation (13) contains three physical laws: the mass is scalar, the mass is positive, the mass is constant. As well as velocity, mass is the ensemble characteristic. It makes no sense to speak about the mass of a single particle: this concept does not have a representation in the apparatus of quantum mechanics[6].

In accordance with (13), the quantity, reciprocal to mass, has a dimension of diffusion coefficient:

$$\left[\frac{1}{m}\right] = \mathrm{cm}^2/\mathrm{sec}.$$

In Newtonian mechanics, as Ernst Mach already shown, mass is the characteristic of a system ***of two particles*** (we mean the mass ratio in the two-particle system, i.e., the dimensionless characteristic). To fix the numerical value of the mass of some particle in classical mechanics, it is necessary to choose an arbitrary standard of mass: mass in classical mechanics has independent dimension.

In quantum mechanics, mass characterizes ***a one-particle*** ensemble (there is no need to create an ensemble of two-particle systems) and it does not have independent dimension (there is no need for existence of mass standard).

Comparison of classical and quantum mass is possible in cases of making not very accurate and not destructive measurements – for example, in the cloud chamber – which allow to repeat the observations of the same copy of the system over time. During such measurements, the classical particle mass m_c can be determined with some limited accuracy. The connection of classical quantity m_c and ensemble characteristic m is fixed by the following postulate (the correspondence principle or Planck's postulate): there is a universal constant $\hbar$ (Planck's constant), such one that for any one-particle Born ensemble, the following relation is satisfied

$$m_c = m\hbar. \tag{14}$$

Planck's constant appears only in Planck's postulate[7], and in other related statements. It establishes the connection between not very accurate – in essence, not very accurate

[6] Undoubtedly, this statement would have caused the protest of many physicists, not just those persistent opponents of quantum mechanics, as Albert Einstein, Erwin Schrodinger and Louis de Broglie. Preparation of an ensemble of ***identical*** one-particle systems requires confidence in the fact that we do include the same particles into it – in particular, with the same masses. Description of actual procedures for preparation of the ensemble, as well as description of procedures for comparing actual observations that have a ***limited accuracy***, with predictions of quantum mechanics, which apparatus presupposes ***infinite accuracy*** of the measurement – it would be better to leave such description to the competent physicists-experimenters. This description requires the explicit formulation of specific hypotheses about the relationship of micro-and macro- world, which implies not only good knowledge of the capabilities of measuring equipment, but also a philosophical mind of the appropriate writer.

[7] Perhaps from the point of view of historical correctness, formulas (13) and (14) should be to associated with the name of Louis de Broglie. But we have already used the term "de Broglie postulate" before. However, in theoretical physics, any "I" is secondary; surnames can "stir up the contents with randomness obscuring the pure image of the truth", as the religious philosopher Pavel Florensky once said. Good works in any field, including theoretical physics, are similar to ancient Greek tragedies, "they are not written by whoever wishes to, or whenever he wishes to do it", as the same Florensky said.

– individual macro-measurements and ensemble characteristics which imply, in principle, unlimited precision of measurements and calculations. The use of Planck's constant in the apparatus of quantum mechanics is not necessary.
Formula (13) does not allow to determine the mass of a one-particle ensemble in the rest frame of the ensemble, in which $\langle \mathbf{k} \rangle = 0$ and $\mathbf{v} = 0$. In this system, the mass of the ensemble can be determined by the velocity of diffuse spreading of the ensemble in the coordinate space. Perhaps this is the basic physical meaning of the non-relativistic concept "one-particle ensemble mass": the characteristic of the ensemble spreading velocity.

2.5 One-particle Ensemble Dynamics. Wave Equation

The Born ensemble as a dynamic system must be described by a pair of equations that determine time derivatives of the Born densities by the Born densities known at this point of time:

$$\begin{aligned} \partial_t \underset{\mathbf{x}}{\rho} &= \underset{\mathbf{x}}{F}\left(\underset{\mathbf{x}}{\rho},\, \underset{\mathbf{k}}{\rho}\right), \\ \partial_t \underset{\mathbf{k}}{\rho} &= \underset{\mathbf{k}}{F}\left(\underset{\mathbf{x}}{\rho},\, \underset{\mathbf{k}}{\rho}\right). \end{aligned} \tag{15}$$

The right sides of equations (15) are non-linear in both of their functional arguments, and each of them is nonlocal in the conjugated argument: function $\underset{\mathbf{x}}{F}$ contains the integral operator over the momentum space, function $\underset{\mathbf{k}}{F}$ contains the integral operator over the coordinate space.
Let us name relation (15) the Born dynamic equation, and functions $\underset{\mathbf{x}}{F}$ and $\underset{\mathbf{k}}{F}$ – the Born generators. The overt form of the two Born generators is unknown for any of the ensembles[8].
For one-particle ensemble the following can be taken as a postulate:

$$\underset{\mathbf{k}}{F} = 0. \tag{16}$$

The Born density of one-particle ensemble in the momentum space, according to (15) and (16) is determined only by the initial conditions. Formula (16) contains both the momentum conservation law of the ensemble, and the energy conservation law of the ensemble.
Ignorance of the general forms of the Born generators $\underset{\mathbf{x}}{F}$ and $\underset{\mathbf{k}}{F}$ makes it impossible to give the explicit real formulation of the equations of quantum mechanics in form (15) – the mechanics, interpreted as "the theory of dynamic systems in the Born ensembles space".
However, God, while creating the quantum world, was so forgiving that we do not need to know the explicit form of the Born generators.

[8] It is easy to write down the expressions for Born generators if we allow the use of phase functions $\underset{\mathbf{x}}{\mathrm{S}}$ and $\underset{\mathbf{k}}{\mathrm{S}}$ in the notation, – but the explicit form of dependence of the phase functions on the Born densities is unknown.

We can formulate the following postulate of autonomy and linearity ("Schrödinger postulate"):
The equation, which each of Fourier-doublet components of Schrödinger wave functions obeys to ("Schrödinger equation" or "wave equation"), is autonomous relative to the second component of the doublet, linear and homogeneous.
This postulate, which should be called the postulate of the divine indulgence, has one convenient consequence: it is sufficient to know only one Schrödinger equation – for example, for the momentum component of Fourier-doublet; but, according to (9), we will obtain the equation for the coordinate component through the Fourier transform of the equation for momentum component.
The Schrödinger postulate has a purely mathematical formulation. It disguises its true physical sense and creates anxiety. God is such a perfect mathematician, that, possibly, He does not exploit math at all, and, anyway, He does not care about our human problems connected with the solution of nonlinear integral-differential dynamic Born equations (15). Why did God need such indulgence to our limited mathematical abilities?
It would be great if there was a formal proof of the fact that "Schrödinger postulate" is a mathematical consequence of the Heisenberg postulate – i.e. "Schrödinger Postulate" is actually not a postulate, but a theorem. In this case, God had just no choice: he could not create quantum ensembles with nonlinear Schrodinger equations. The author does not know such proof.
If "Schrödinger postulate" is independent of the Heisenberg postulate, God did have a choice, and the choice made by Him, should be formulated not in the language of a fictitious mathematical object – Fourier-doublet of complex Schrödinger wave functions - but in a more meaningful language , the language of Born generators $\underset{\mathbf{x}}{F}$ and $\underset{\mathbf{k}}{F}$, so that the statement of the linearity of Schrödinger equation would be a trivial consequence of this formulation.
Leaving the problem of linearity of Schrödinger equation unclear up to the end in a "deep non-relativistic rear", we face the question: should we expect the same divine indulgence in the relativistic theory? Should equation for the wave functions be also linear in relativistic physics?
For the one-particle Born ensemble considered here, Schrödinger equation in momentum space, as it is known, has the following form:

$$i\,\partial_t \underset{\mathbf{k}}{\psi} = \frac{\mathbf{k}^2}{2m}\,\underset{\mathbf{k}}{\psi}. \tag{17}$$

It is possible to create a certain illusion of the derivability of (17) based on non-consecutive semi-classical ideas, but it would be better to consider wave equation (17) the quantum postulate.
The Schrödinger equation in coordinate space arises from (17) by means of the Fourier transform:

$$i\,\partial_t \underset{\mathbf{x}}{\psi} = -\frac{1}{2m}\,\Delta\underset{\mathbf{x}}{\psi}, \tag{18}$$

where Δ is the Laplace operator.
Of course, we do not have a necessity to solve equation (18): it is enough to solve (17) and perform the Fourier transform for the obtained solution.
A definite and, in a sense, insuperable difficulty is connected with the need to satisfy the initial conditions for equations (17) and (18). Keeping to the framework of rational physical interpretation, we should specify the initial conditions in terms of the Born pair $\left\{\underset{\mathbf{x}}{\rho},\ \underset{\mathbf{k}}{\rho}\right\}$. Transferring these Born conditions to the initial conditions for Schrödinger wave functions, we should, at least at the initial moment of time, solve nonlinear integral equations (9) which, in accordance with formulas (7) and (8), determine the phase pair $\left\{\underset{\mathbf{x}}{\mathrm{S}},\ \underset{\mathbf{k}}{\mathrm{S}}\right\}$. Direct specifying of the initial conditions for Schrödinger's Fourier-doublet of the wave functions, apparently, should be considered impossible: we are preparing the ensemble as a certain Born pair, and not as Schrödinger's Fourier-doublet.
Therefore, the non-linear and nonlocal Born nature of quantum mechanics, is seen through Schrödinger's linearity at least as the problem of the initial conditions[9].

2.6 Born Pair and Schrodinger Fourier-doublet for the Ensemble of Two-particle Systems

The ensemble of two-particle systems is prepared from two ensembles of one-particle systems, which masses, m_1 and m_2, are known from the previous measurements made on these one-particle ensembles. A single measurement of coordinates $\mathbf{x}$ (or, respectively, of momentums $\mathbf{k}$) gives the group of two radius vectors $(\mathbf{x}_1,\ \mathbf{x}_2)$, or two momentums $(\mathbf{k}_1,\ \mathbf{k}_2)$. However, we can not associate a particular radius vector (or momentum) with a particular particle, having number 1 or 2, and, respectively, mass m_1 or m_2: point particles do not have any tags attached, and they are indistinguishable under individual measurements. Therefore, we have to turn each coordinate (or momentum) measurement into two possible descriptions of the measurement results, going over both of the possible options of assigning numbers to particles. A number of $\underset{\mathbf{x}}{N_t}$ coordinate measurements made over the ensemble at some instant of time t, turn into $2^{\underset{\mathbf{x}}{N_t}}$ sets of measurement descriptions. For each of these sets, we determine a number $\Delta \underset{\mathbf{x}}{N_t}$ – the number of measurements, at which the availability of particles 1 and 2 is registered in a small but finite volume $\Delta \underset{\mathbf{x}}{\mathrm{V}}$ of the six-dimensional configuration space in the vicinity of a point in this space, which is specified by an ordered pair of three-dimensional radius vectors $(\mathbf{x}_1,\ \mathbf{x}_2)$. Then we can calculate the ratio $\Delta \underset{\mathbf{x}}{N_t} / \underset{\mathbf{x}}{N_t}$ and study the behavior of this ratio under an unlimited increase in the number of measurements $\underset{\mathbf{x}}{N_t}$.
If the following two conditions are satisfied:
(1) for some sets of measurement descriptions, this ratio with the growth of $\underset{\mathbf{x}}{N_t}$ changes

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[9] It would be appropriate to ask physicist-experimenters the following question: is it possible to prepare the Born ensemble with a specified initial Schrödinger wave function – or, is it only the process of preparing the ensemble with two specified initial Born densities that is physically implemented?

chaotically: there is no limit for this ratio at $\underset{\mathbf{x}}{N_t} \to \infty$;

(2) for other sets of measurement descriptions, such limit $\underset{\mathbf{x}}{\mathrm{R}}{}_t$ exists, and it is the only one, – the two particles in two-particle ensemble are named statistically distinguishable[10], and limit $\underset{\mathbf{x}}{\mathrm{R}}{}_t$ determines the Born density $\underset{\mathbf{x}}{\rho}(\mathbf{x}_1, \mathbf{x}_2, t)$ in the six-dimensional configuration space:

$$\underset{\mathbf{x}}{\mathrm{R}}{}_t = \lim_{\underset{\mathbf{x}}{N_t} \to \infty} \left(\frac{\Delta \underset{\mathbf{x}}{N_t}}{\underset{\mathbf{x}}{N_t}} \right) = \iint\limits_{\Delta \underset{\mathbf{x}}{\mathrm{V}}} \underset{\mathbf{x}}{\rho}(\mathbf{x}_1, \mathbf{x}_2, t) \mathrm{d}\underset{\mathbf{x}}{\mathrm{V}_1}\, \mathrm{d}\underset{\mathbf{x}}{\mathrm{V}_2}. \tag{19}$$

In formula (19), the integrals are taken over all the coordinates of the first particle and all the coordinates of the second particle within the allocated six-dimensional volume element in the configuration space.

Similar relation, with replacement of coordinates $\mathbf{x}$ with momentums $\mathbf{k}$ under satisfying the conditions of statistical distinctiveness of the ensemble particles, determines the Born density $\underset{\mathbf{k}}{\rho}(\mathbf{k}_1, \mathbf{k}_2, t)$ in the six-dimensional momentum space of the two-particle ensemble:

$$\underset{\mathbf{k}}{\mathrm{R}}{}_t = \lim_{\underset{\mathbf{k}}{N_t} \to \infty} \left(\frac{\Delta \underset{\mathbf{k}}{N_t}}{\underset{\mathbf{k}}{N_t}} \right) = \iint\limits_{\Delta \underset{\mathbf{k}}{\mathrm{V}}} \underset{\mathbf{k}}{\rho}(\mathbf{k}_1, \mathbf{k}_2, t) \mathrm{d}\underset{\mathbf{k}}{\mathrm{V}_1}\, \mathrm{d}\underset{\mathbf{k}}{\mathrm{V}_2}. \tag{20}$$

Two real, positive, normalized per unit Born densities, $\underset{\mathbf{x}}{\rho}$ and $\underset{\mathbf{k}}{\rho}$, form the Born pair $\left\{ \underset{\mathbf{x}}{\rho}, \underset{\mathbf{k}}{\rho} \right\}$, for which we can calculate two 3×3-Heisenberg's matrices $\underset{1}{\mathrm{H}}{}_{ij}$, $\underset{2}{\mathrm{H}}{}_{ij}$ (5) for each particle of the two-particle ensemble.

Heisenberg inequality (6) is postulated for each particle of the two-particle ensemble. For the Born two-particle ensemble that satisfies the two inequalities (6), as well as for the one-particle situation, we can formulate the Schrödinger theorem – the statement that there is a pair of real phase functions $\underset{\mathbf{x}}{\mathrm{S}}(\mathbf{x}_1, \mathbf{x}_2, t)$ and $\underset{\mathbf{k}}{\mathrm{S}}(\mathbf{k}_1, \mathbf{k}_2, t)$, such one that the Fourier-doublet components of Schrödinger wave functions $\underset{\mathbf{x}}{\psi}(\mathbf{x}_1, \mathbf{x}_2, t)$ and $\underset{\mathbf{k}}{\psi}(\mathbf{k}_1, \mathbf{k}_2, t)$, determined by relations (7) and (8), are the Fourier images of each other:

$$\begin{aligned} \underset{\mathbf{x}}{\psi} &= \frac{1}{(2\pi)^3} \iint \underset{\mathbf{k}}{\psi}\, e^{i\,(\mathbf{k}_1 \cdot \mathbf{x}_1 + \mathbf{k}_2 \cdot \mathbf{x}_2)}\, \mathrm{d}\underset{\mathbf{k}}{\mathrm{V}_1}\, \mathrm{d}\underset{\mathbf{k}}{\mathrm{V}_2}, \\ \underset{\mathbf{k}}{\psi} &= \frac{1}{(2\pi)^3} \iint \underset{\mathbf{x}}{\psi}\, e^{-i\,(\mathbf{k}_1 \cdot \mathbf{x}_1 + \mathbf{k}_2 \cdot \mathbf{x}_2)}\, \mathrm{d}\underset{\mathbf{x}}{\mathrm{V}_1}\, \mathrm{d}\underset{\mathbf{x}}{\mathrm{V}_2}. \end{aligned} \tag{21}$$

Integrals in (21) are taken over the entire six-dimensional space.

Wave functions $\underset{\mathbf{x}}{\psi}$ of the two-particle ensemble, as it is known, satisfy the Schrödinger equation:

$$i\, \partial_t\, \underset{\mathbf{x}}{\psi} = -\frac{1}{2m_1} \underset{1}{\Delta} \underset{\mathbf{x}}{\psi} - \frac{1}{2m_2} \underset{2}{\Delta} \underset{\mathbf{x}}{\psi} + \frac{\alpha_{12}}{r_{12}} \underset{\mathbf{x}}{\psi}, \tag{22}$$

[10] These two conditions will knowingly be satisfied if, for example, a two-particle ensemble is made of one-particle ensembles of different masses.

where $\underset{1}{\Delta}$ is the Laplacian by components of vector $\mathbf{x}_1$; $\underset{2}{\Delta}$ is the Laplacian by components of vector $\mathbf{x}_2$; $r_{12} = |\mathbf{x}_2 - \mathbf{x}_1|$; α_{12} is the coefficient of the coulomb interaction in the two-particle system – a non-zero phenomenological constant characterizing the two-particle ensemble[11]. In a sense, (22) should be regarded as the definition of this constant.

Considering successively three two-particle systems (1,2), (2,3), (3,1), formed by three one-particle ensembles (1), (2), (3), any two of which are statistically distinguishable from each other (for example, proton, electron and muon), we can formally attribute electrical charges e_1, e_2, e_3 to each ensemble, defining these charges as solutions to the system of three algebraic equations:

$$\begin{aligned} e_1 \cdot e_2 &= \alpha_{12}, \\ e_2 \cdot e_3 &= \alpha_{23}, \\ e_3 \cdot e_1 &= \alpha_{31}. \end{aligned} \tag{23}$$

It is obvious that the solution to system (23) exists if the following condition is satisfied:

$$\alpha_{12} \cdot \alpha_{23} \cdot \alpha_{31} > 0. \tag{24}$$

Condition (24) should be taken as a postulate (”Coulomb postulate”). The physical sense of the Coulomb postulate is simple: we live in the world where electrical charges of the same sign repel each other, and charges with opposite sign attract each other. One can imagine the non-relativistic world with a different sign of inequality (24). (In such world, in algebraic equations (23), which determine electric charges, there would appear a minus sign: $e_1 \cdot e_2 = -\alpha_{12}$, etc.). If we assume, following Albert Einstein that the only interesting question of physics is: ”Did God have a choice?” the inequality (24), postulated and having no justification within the non-relativistic scheme, shows that God did have a choice. It is easy to see that the solution to system (23), under satisfying Coulomb inequality (24), exists and it is unique (accurate within the arbitrary choice of the charge sign for a single charged particle in the Universe).

But if the conditions of statistical distinguishability of two particles of the two-particle ensemble are not satisfied, both the method of constructing the Born densities (there is not one, but two different limits $\underset{\mathbf{x}}{\mathrm{R}}{}_t$ and two different limits $\underset{\mathbf{k}}{\mathrm{R}}{}_t$) and the method of connection of Fourier-doublet of Schrödinger wave functions with the Born densities, are complicated.

Description of the two-particle ensemble of two statistically indistinguishable particles requires the introduction of a spin as a phenomenological parameter which is explicitly included into the mathematical description of the Born ensemble.

Further descending to particulars of non-relativistic quantum system is irrelevant here.

[11] Of all kinds of fundamental interactions, non-relativistic quantum mechanics allows to describe only Coulomb electrostatic interaction as a non-relativistic rudiment of relativistic electromagnetic interaction. Any other interactions, traditionally considered in quantum mechanics, either have a conventional, model character (”quantum oscillator”) or are unlawful semi-relativistic approximations that destroy non-relativistic logical harmony and conceptual clarity of non-relativistic theory – but improve the agreement of predictions of the theory with high-precision experimental data of atomic and molecular spectroscopy.

Relativistic quantum theory, the discussion of which is our objective, as well as classical relativistic theory described in previous articles [1], [2], [3], does not contain particles at the "entry" to the theory. The particles should appear at the "exit" as an accurate description of some stationary states, or as an approximate description of some transitional non-stationary states. Spin can not remain phenomenological input parameter, but must also appear at the output of the theory.

3 Principles and Problems of Transition to Quantum Relativistic Theory

What can we decidedly borrow from non-relativistic quantum system to transfer to relativistic quantum theory?

- ***De Broglie Postulate***, which in the relativistic formulation states that four-dimensional momentum space $\{k_\nu\}$ exists along with four-dimensional coordinate space $\{x_\nu\}$. These two spaces form a dyad $\{x_\nu|k_\nu\}$. The semi-components of this dyad are complementary to each other by Niels Bohr: the description of the ensemble of fields is possible either in $\{x_\nu|$ or in $|k_\nu\}$.
- ***Born postulate of*** the existence of a relativistic analogue of the Born densities.
- Relativistic analogues of the ***Heisenberg postulate*** and the ***Schrödinger theorem*** of the existence of Fourier doublets of relativistic wave functions (Dirac spinors).
- Perhaps, some analogue of the ***linearity postulate*** of relativistic wave equations.

What can we transfer to the quantum relativistic theory from the classical three-sector field theory with continuum currents, which has been presented in the previous articles of this series?

- The idea of existence of the three independent sectors of physics – singlet, triplet and octuplet.
- The idea of dyadic nature of each sector in the form of a dyad current—potential. Dyad semi-components in the quantum version of the theory should be considered as complementary to each other by Niels Bohr: description of the ensemble of fields in each sector is possible either in terms of current or in terms of potential.
- The idea of the Riemann geometry of four-dimensional spatial continuum $\{x_\nu|$. Construction of the theory of particles, which have a rest mass, as intrinsic states of the theory, is apparently impossible without considering the Riemann curvature of space[12].

Transferring of the concept of Riemann geometry of coordinate 4-continuum $\{x_\nu|$ to quantum theory immediately gives rise to a number of painful questions, the correct

[12] "Does gravitational field play any role in the matter structure and should a continuum inside the atomic nucleus be considered perceptibly non-Euclidean?" – this is the rhetorical question which Albert Einstein asked himself and the readers of "Nature" during the epoch of the grandiose triumph of general relativity theory [4]. But he never turned to this subject any more. Neither Yukawa's meson, no isospin, no neutrino made Einstein recognizes the existence of fields other than gravitational or electromagnetic.

answers to which we just have to guess[13].

- What should be the geometry of the space of momentums $|k_\nu\}$ that are the second semi-component of spatial-momentum dyad $\{x_\nu|k_\nu\}$, be like?
- Should we make similar "non-Minkowski" demands to the geometry of multidimensional spaces of currents and potentials?
- Should we consider the metric tensor of the four-dimensional coordinate space $\{x_\nu|$ "attached"" to each unique, irreproducible field realization or is it appropriate to consider it a part of the description of the ensemble of field realizations?[14]
- If we accept, following Einstein, the minimalist" interpretation of physical geometry: "Ricci tensor depends linearly on the tensor of field energy-momentum", how should the right-hand sides of Einstein's equations be constructed? Should there be present a certain procedure of the field variables (currents or potentials) integration? How can a physical quantity, the integration of which by field variables generates the energy-momentum tensor, be constructed from Dirac spinors?
- How should Fourier-transform procedure, necessary for the introduction of Dirac spinors by formulas that have to be a relativistic analogue of formulas (9) and (10) of the non-relativistic quantum scheme, be formulated in the curved Riemannian space?

Leaving aside for some time these difficult questions, we can ask ourselves a more simple question – what can we transfer to the continuum relativistic quantum theory from the version of quantum field theory (but without its point fermions) which was developed in the twentieth century? Undoubtedly, there are two "transportable" ideas:

- The idea of the Dirac spinors as the principal instrument of the mathematical description of physical reality.
- The Dirac equation – as some prototype, as a heuristic base for the construction of equations of the new theory. For this theory, the existing Dirac equation should be an approximate consequence of the theory equations eventuating after some procedure of the approximate integration by field variables.

4 Dirac Space and Dirac Vectors in Quantum Electrodynamics

4.1 Born Fluxes

In quantum electrodynamics, we are dealing with two independent dyads: "space / momentum" – $\{x_\nu|k_\nu\}$, and "currents / potentials" – $\{\mathrm{J}_\nu|\mathrm{A}_\nu\}$. Combining semi-components of these dyads, we get 4 sets of measurement procedures: $\langle x|\mathrm{J}\rangle$, $\langle x|\mathrm{A}\rangle$, $\langle k|\mathrm{J}\rangle$, $\langle k|\mathrm{A}\rangle$. In this notation, the Lorentz indices are omitted, and the notation itself has the following

[13] Hardly can these answers allow a direct empirical test.

[14] From philosophical point of view, we have to put up with the multiplicity of field realizations, but the space (and momentum), four-dimensional capacitance for these realizations exists, of course, only in a single copy. In this sense, it is not very easy to understand what those physicists who speak of "quantization of gravitation" mean.

meaning:
$\langle x|\mathrm{J}\rangle$ – is the measurement procedure in the space-time continuum and in the space of currents;
$\langle x|\mathrm{A}\rangle$ – is the measurement procedure in the space-time continuum and in the space of potentials;
$\langle k|\mathrm{J}\rangle$ – is the measurement procedure in the momentum continuum and in the space of currents;
$\langle k|\mathrm{A}\rangle$ – is the measurement procedure in the momentum continuum and in the space of potentials.[15]
It is convenient to construct the following system of dyadic notation. We will call the space-momentum dyad $\{x|k\}$ the first dyad and in the bracketed notations of measurement procedures we will put the information on the element of the first dyad into the left side, before the dividing line. The current-potential dyad will be called the second dyad, and information on the element of the second dyad will be presented to the right side from the dividing line in the bracket symbol of the measuring procedure. For each dyad, instead of alphabetic characters it is convenient to use dyadic index which takes two possible values: $a = 1$ or $a = 2$.
Index 1 corresponds to the first semi-component of the dyad (coordinates x in dyad $\{x|k\}$, current J is in dyad $\{\mathrm{J}|\mathrm{A}\}$) , index 2 corresponds to the second semi-component of the dyad (momentum k in dyad $\{x|k\}$, potential A is in dyad $\{\mathrm{J}|\mathrm{A}\}$). Notation $\langle 1|2\rangle$ can be used with these dyadic indices to describe, for example, measuring procedure $\langle x|\mathrm{A}\rangle$. If some physical quantity q is determined by measurement procedure of type $\langle a|b\rangle$ (a, b $=$ 1 or 2), this information can be expressed by using the subscript dyadic indices: $\underset{ab}{\mathrm{q}}$.[16]
Let us assume that in space-time continuum $\{x|$ there is selected some infinite non-closed oriented three-dimensional hyper-surface σ with time-like unit normal vector n^μ at each point ($n^\mu n_\mu = +1$, $n^0 > 0$). This surface should contain space-like infinity[17].
Let us suppose that at sufficiently large number of points of this hyper-surface, there

[15] We can hardly present any explicit descriptions of these procedures, even in Einstein's genre of his favorite "mental experiments" – eventually, measurements ***inside the electron*** must be considered. In quantum version of physics of currents and potentials, something, apparently, has to be left unfinished, just like Leonardo left unfinished the head of Christ in his painting "The Last Supper" in Milan Dominican convent of Santa Maria delle Grazie, believing, according to Giorgio Vasari, that "he would not be able to express in it all the heavenly divinity, required by the image of Christ; but Leonardo gave splendor and simplicity to apostles' heads". A theoretical physicist, as well as an artist, must strive for "splendor and simplicity" in something that allows to describe itself, not daring to claim to describe the "heavenly divinity".
[16] The right-hand subscripts and superscripts are traditionally used as Lorentz indices; over-letter superscripts are used in [2] and [3] as Yang-Mills indices: for the dyadic indices there is a space left under the letter denoting a physical quantity.
[17] In non-relativistic situation, we would describe it in more simple terms, for example, – "let us assume that at some instant of time t measurements have been made over the entire infinite three-dimensional space".

have been made measurements in the current space. Let the total number of all measurements be equal to $\underset{11}{N}$. Let us suppose that $\Delta\underset{11}{N}$ is a number of such measurements that something is found in a small but finite 3-volume $\Delta\sigma$ of surface σ in the vicinity of some point x and meanwhile, in a small but finite 4-volume $\Delta\Omega$ of space of currents in the vicinity of some point of this space.

We state that there is a ratio limit $\Delta\underset{11}{N}/\underset{11}{N}$ at $\underset{11}{N} \to \infty$, and this limit can be expressed as an integral of a certain quantity $\underset{11}{\xi}$:

$$\lim_{\underset{11}{N}\to\infty}\left(\frac{\Delta\underset{11}{N}}{\underset{11}{N}}\right) = \int\limits_{\Delta\sigma}\int\limits_{\Delta\Omega}\underset{11}{\xi}\,\mathrm{d}\sigma\mathrm{d}\Omega. \tag{25}$$

In formula (25) $\mathrm{d}\sigma$ is a Lorentz-invariant scalar element of 3-volume of hyper surface σ:

$$\mathrm{d}\sigma^\nu = n^\nu\mathrm{d}\sigma;\ \mathrm{d}\sigma = \sqrt{\mathrm{d}\sigma^\nu\,\mathrm{d}\sigma_\nu},$$

and $\mathrm{d}\Omega$ is the element of 4-volume in the current space. The integral in (25) is sevenfold. Statement (25) is the relativistic analogue of the Born postulate (1). Quantity $\underset{11}{\xi}$ can be called $\langle 1|1\rangle$-Born density. This density is determined on an arbitrarily chosen hyper-surface σ. It should be assumed that the description of physical phenomena does not depend on the choice of σ, and locally does not depend on local normal vector n^ν, therefore, there is a 4-vector $\underset{11}{\rho}^\nu$, such one that

$$\underset{11}{\xi} = \underset{11}{\rho}^\nu n_\nu. \tag{26}$$

4-vector $\underset{11}{\rho}^\nu$ is ***a field*** in mathematical sense, i.e. depends on point x, but does not dependent on vector n^ν. But Born density $\underset{11}{\xi}$ itself is not a field: it depends not only on x, but also on n^ν. Let us name 4-vector $\underset{11}{\rho}^\nu$ "the Born flux" in representation $\langle 1|1\rangle$. This vector depends on two 4-vectors x^ν and J^ν, i.e. is the function of a point in the eight-dimensional measurement space (the Dirac space).

In accordance with the above, the four variants of measurement procedures $\langle a|b\rangle$ (a, b = 1 or 2) generate four Born fluxes $\underset{ab}{\rho}^\nu$, each of which is determined in its own version of the eight-dimensional Dirac space.

Consequently, ***in relativistic quantum electrodynamics, the Born postulate is a statement of the existence of the four Born fluxes*** $\underset{ab}{\rho}^\nu$.[18] Relativistic Born ensemble is the quadruplet of fluxes: sixteen functions liable to determination in the four inter-related realizations of the eight-dimensional Dirac space instead of two non-relativistic scalar Born densities in the two three-dimensional spaces – coordinate and momentum... The radical increase in a number of unknown functions and a number of their arguments...

[18] In some situations, this statement is too optimistic and requires clarification, for example, in the situation of collision of two electrons we can not distinguish in which of the two electrons the given point is located. In this case there is more than one limit (25). But, undoubtedly, it is reasonable to avoid discussing such clarifications at such an early phase of the theory construction.

4.2 Dirac Vectors

To move forward on the way projected in the non-relativistic scheme, we have to turn a set of real Born fluxes into a set of complex wave functions. To do this, we have to formulate some relativistic analogue of the non-relativistic Schrödinger theorem (see p. 2.3). In order to do it, in accordance with formulas (7) and (8), we must learn to take the square root of 4-vector Born flux. This problem could be considered unsolvable if it had not been solved by P.A.M. Dirac already in 1928[19].

Following Dirac, we state that there is a Lorentz vector in the form of a set of four 4×4 matrices $\Gamma^{\nu}_{\alpha\beta}$ (ν is Lorentz index, α and β are the matrix indices $\alpha,\, \beta\, =\, \overline{1,\, 4}$) which allows to associate the four complex four-component wave functions $\psi^{\nu}_{\alpha\beta}$ with the four real 4- vectors of Born fluxes $\rho^{\nu}_{\alpha\beta}$ so that

$$\underset{ab}{\psi^{*}_{\alpha}}\,\Gamma^{\nu}_{\alpha\beta}\,\underset{ab}{\psi_{\beta}} = \underset{ab}{\rho^{\nu}}. \tag{27}$$

In the formula (27) the asterisk (*) is a sign of complex conjugation, α and β are the matrix indices (summation from 1 to 4 is made for repeated matrix indices); a and b are dyadic indices that do not have a vector character (dyadic indices are just the tags of the Dirac space, with repetition of dyadic indices, summation for them is not made).

Four components $\underset{ab}{\psi^{*}_{\alpha}}$ at fixed dyadic indices a and b are called ***Dirac spinor in representation*** $\langle a|b\rangle$, and thus the indices α and β in matrices $\Gamma^{\nu}_{\alpha\beta}$ can be named not matrix, but spinor ones. However, instead of the established resounding term "spinor" we will use, albeit awkward, the term "Dirac vector" and we will call the indices α and β "Dirac vector indices". Classical relativistic physics [1], [2], [3] is formulated in terms of Lorentz (and Yang-Mills) vectors while quantum relativistic physics in terms of Dirac vectors.

[19] Perhaps, modern history of science underestimates this part of P.Diracs biography. This achievement, in a sense, puts this physicist, and, perhaps, only him alone, on the level with Isaac Newton: Dirac had to invent a new mathematical apparatus (spinors) in order to express the new physics – just like Isaac Newton had to invent the derivative and integral to express the laws of mechanics discovered by him. Neither Maxwell nor Schrödinger had to invent the partial differential equations – they had already existed. And even Einstein did not have to invent the Riemann geometry and tensor analysis: they were in the depths of mathematics as a finished product. Werner Heisenberg re-invented matrices, but this invention reflected not only his brilliant abilities, but also insufficient level of his personal mathematical training: matrices had already existed in mathematics.

Matrices $\Gamma^{\nu}_{\alpha\beta}$ in (27) have the form:

$$\Gamma^0_{\alpha\beta} = \delta_{\alpha\beta},$$
$$\hat{\Gamma}^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix},$$
$$\hat{\Gamma}^2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}, \tag{28}$$
$$\hat{\Gamma}^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Usually, in the notation of Dirac algebraic relations (27), instead of matrices (28), Dirac matrices $\hat{\gamma}^{\nu}$ are used, such ones that:

$$\hat{\Gamma}^{\nu} = \hat{\gamma}^0 \hat{\gamma}^{\nu}. \tag{29}$$

In some loose way it is possible to express the essence of equalities (27) as follows: at extracting the square root of a real Lorentz vector we get a complex Dirac vector.
Relations (27) are the relativistic version of relations (11) and they do not determine Dirac vector ψ_α on their own. It is obvious that the four complex functions ψ_α can not be determined by four real equations (27). To determine ψ_α, besides (27), we need a relativistic version of the Fourier-relation (9), (10).

4.3 Relativistic Fourier-transform

The Fourier transform in non-relativistic quantum theory (9) connects the two components of Schrödinger Fourier-doublet, each of which is determined in one semi-component of a non-relativistic dyad $\{\mathbf{x}|\mathbf{k}\}$. While constructing the relativistic Fourier transform (it will be denoted with symbol RFT), we have to take into account that Dirac vectors ψ_α have two dyadic indices. For mathematical representation of Fourier-procedures with this "doubling of dyadic indices", we will introduce the negation symbol of dyadic index

$\overline{a}$ as follows:

$$\overline{a} = \begin{cases} 2, \text{ if } & a = 1, \\ 1, \text{ if } & a = 2. \end{cases} \tag{30}$$

The transition from $\underset{ab}{\psi}$ to $\underset{\overline{a}\overline{b}}{\psi}$ will be called a dyadic conjugation.

Formal notation RFT, which replaces the formal non-relativistic notation (9), must look as follows:

$$\underset{ab}{\psi_\alpha} \overset{\text{RFT}}{=} \underset{\overline{a}\overline{b}}{\psi_\alpha}. \tag{31}$$

Relation (31) means that $\underset{ab}{\psi}$ is a Fourier-transform of dyad-conjugated vector $\underset{\overline{a}\overline{b}}{\psi}$. If we manage to give a clear mathematical sense to formal symbol RFT, the integral equation (31) together with algebraic Dirac equations (27), will allow to explicitly determine Dirac vectors $\underset{ab}{\psi_\alpha}$ by the observable Born fluxes $\underset{ab}{\rho^\nu}$. It is obvious that in quantum electrodynamics, we get not one Fourier-doublet of Schrödinger wave function, but two independent Fourier-doublets of Dirac vectors: Fourier-doublet, diagonal by dyadic indices, and Fourier-doublet, non-diagonal by dyadic indices. Accordingly, in contrast to non-relativistic theory, all information on the quantum-electro-dynamic states is contained not in one half of the Fourier-doublet, but in two independent halves of the two independent Fourier-doublets of the Dirac vectors, for example, $\underset{11}{\psi_\alpha}$ and $\underset{12}{\psi_\alpha}$. Accordingly, mathematics of quantum electrodynamics requires construction of two independent complex wave equations, unlike mathematics of non-relativistic quantum mechanics which requires one independent complex wave equation.

Constructing the relativistic analogue of non-relativistic Fourier-transform (10), we will open a formal notation RFT (31) as follows:

$$\begin{aligned} \underset{ab}{\psi_\alpha} &= \frac{1}{(2\pi)^{7/2}} \iint e^{i\phi} \underset{\overline{a}\overline{b}}{\psi_\alpha} \, \mathrm{d}\underset{\overline{a}}{\sigma} \, \mathrm{d}\underset{\overline{b}}{\Omega}, \\ \underset{\overline{a}\overline{b}}{\psi_\alpha} &= \frac{1}{(2\pi)^{7/2}} \iint e^{-i\phi} \underset{ab}{\psi_\alpha} \, \mathrm{d}\underset{a}{\sigma} \, \mathrm{d}\underset{b}{\Omega}. \end{aligned} \tag{32}$$

Integrals in (32) are taken over the entire three-dimensional hyper-surface $\underset{x}{\sigma}$ in the coordinate continuum (or, correspondingly, $\underset{k}{\sigma}$ in the momentum continuum) – this is a three-fold integral; as well as over the entire four-dimensional current continuum J (or, correspondingly, continuum of potentials A) – this is a four-fold integral. The degree of the Fourier multiplier $\sqrt{2\pi}$ corresponds to seven-fold integration in (32).

Symbol ϕ in (32) denotes relativistic Fourier-phase, which should be constructed so that it was a reasonable relativistic generalization of non-relativistic Fourier phase in (10). This non-relativistic Fourier phase is the inner product $\mathbf{x} \cdot \mathbf{k}$ of the two semi-components of non-relativistic space-momentum dyad $\{\mathbf{x}|\mathbf{k}\}$.

Regardless of the particular type of phase ϕ, RFT (32) contains a serious vulnerability: integrals in (32) are taken over arbitrarily chosen hyper-surfaces $\underset{x}{\sigma}$ and $\underset{k}{\sigma}$. In contrast to non-relativistic situation, we can not connect these two hyper-surfaces in two different four-dimensional continuums. In non-relativistic quantum mechanics, for example, we integrated over the entire 3-space of momentums ***at the same instant of time*** t at which the wave function in the coordinate space was determined. We can not transfer this construction of absolute time to relativistic scheme. Of course, we have to suppose that all the measuring equipment, by means of which we detect ***something*** in the space of currents, potentials, coordinates and momentums, is fixed in some locally Galilean frame of reference in some vicinity of each point of the space-time continuum, but the author has to admit that he does not know how to derive a mathematically faultless coupling between $\underset{x}{\sigma}$ and $\underset{k}{\sigma}$ from this physically faultless statements. We will use the above cited "rule of Leonardo" and will not detail this important part of relativistic picture.

4.4 Relativistic Fourier-phase

Non-relativistic formula for the Fourier phase $\phi = \mathbf{x} \cdot \mathbf{k}$ allows to suggests that the relativistic Fourier-phase should be a linear combination of two scalar products of dyadic semi-components of the both quantum-electro-dynamical dyads $\{x|k\}$ and $\{\mathrm{J}|\mathrm{A}\}$. However, these two scalar products enter into the phase with different weight:

$$\psi = x^{\mu} k_{\mu} + \alpha \mathrm{J}^{\mu} \mathrm{A}_{\mu}, \tag{33}$$

where α is some phenomenological constant of quantum electrodynamics, which we will unhesitatingly identify with the constant of fine structure.

Here it is appropriate to remind the reader that in the theory appears a fundamental length r_0 [1], which we accept as a length unit[20]. The velocity of light is also accepted as a unit. The quantity of electron charge is accepted as a unit. Accordingly, all the quantities appearing in the theory are dimensionless.

As it has been noted in [1], the relativistic theory of matter can not be constructed without considering the Riemann curvature of space. It means that one more dimensionless constant, proportional to gravitational constant G, appears in the theory. In another way this can be expressed as follows: the theory must include the Planck length r_p and, correspondingly, the dimensionless constant r_0/r_p. This constant and constant α, appearing in (33), should of course, be connected with each other. However, at present, embryonic stage of the theory construction, this connection, as well as the other important details of the picture, remains non-detailed.

If the expression of relativistic Fourier-phase, introduced by formula (33) is correct, the true physical meanings of the constant of fine structure α is that this quantity is a measure of mixing σ-variables and Ω-variables, the measure of their relative weight.

In the world with $\alpha << 1$, the "observer" and his "equipment" are often found in an empty space, the space without currents. In such world, the concept of free point particles with the interpretation of "interaction" as collision of particles (not without difficulties

[20] Constant r_0 was estimated in [2] $r_0 \propto 10^{-26}$ cm.

and divergences) is useful.
In the world with $\alpha >> 1$, the "observer"[21] and his "equipment" are often found in spaces occupied by currents: the subject of study would rather be the properties and movement of "voids", "bubbles" which are not occupied by currents. Observers like us feel comfortable only in the world with $\alpha << 1$. But we do not know the principle by which God fixes a specific value $\alpha \cong {}^{1}/_{137}$.

4.5 Fourier-phase in the Riemannian Space

The two terms in relativistic Fourier-phase (33) have different geometrical importance. The second term, proportional to scalar product $\mathrm{J}^{\nu}\mathrm{A}_{\nu}$, does not change its type in the curved Riemannian space – only for lowering/raising of Lorentz indices there should be used the components of non-Galilean metric tensor $\mathrm{g}_{\mu\nu}$, determined locally, at the same point of the space-time Riemann continuum, where the measurements in J-space or A-space were made.
However, the first term in (33) makes no sense at all in the Riemann continuum: coordinates x^{μ} do not form a vector (vectors are only coordinate differentials). With some psychological effort, we probably have to say the same about components k^{μ}: they do not form a vector. It is hard to imagine how x and k form a dyad of equal semi-components if space curvature k does not correspond to the Riemann curvature of 4- continuum x. However, it is even more difficult to imagine a curved momentum space: in modern theoretical physics, apparently, there is no idea that can suggest how to curve momentum continuum.
Let us rewrite expression (33) in the form:

$$\phi = \eta + \alpha \mathrm{J}^{\mu}\mathrm{A}_{\mu}, \tag{34}$$

$$\text{where } \eta \overset{\text{(PE)}}{=} x^{\mu}k_{\mu}. \tag{35}$$

Symbolic notation (35) means that this equality is valid only within the framework of pseudo-Euclidean geometry of four-dimensional continuums x and k.
How should we rearrange the expression for phase η in the Riemann continuum? The expression for η which we suggest below (36), is rather a gesture of despair than a display of physical intuition.
Let us identify space x and space k. Of course, we can not identify the measurement procedure in these spaces – they are inter-complementary by Niels Bohr. But let us try to interpret x^{ν} and k^{ν} as coordinates of two points in one four-dimensional continuum. As part of this (which is not easy to imagine, and which is impossible to accept), we can choose some arbitrary point O as the origin (common for x and k) and determine the lengths of the three geodesic lines C_{ox}, C_{ok}, C_{xk}:

[21] In such world there hardly can be "observers" like us. Cosmologist A.L. Zel'manov once said that ***we are the witnesses of the processes of a certain type, because other processes take place without witnesses***.

- $s_{ox} = \int\limits_{C_{ox}} \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$ – the length of geodesic line C_{ox} connecting point O with point x;
- $s_{ok} = \int\limits_{C_{ok}} \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$ – the length of geodesic line C_{ok} connecting point O with point k;
- $s_{xk} = \int\limits_{C_{xk}} \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$ – the length of geodesic line C_{xk} connecting point x with point k.

Knowing the lengths of the three geodesic lines, we can construct phase η as follows:

$$\eta = \frac{1}{2}\left(s_{ox}^2 + s_{ok}^2 - s_{xk}^2\right). \tag{36}$$

Expression (36)[22] coincides with (35) for the pseudo-Euclidean geometry with using the Minkowski coordinates; expression (36) will be also valid in arbitrary curvilinear coordinates introduced in the pseudo-Euclidean continuum, and, finally, we can ***postulate*** (36) for the Riemann continuum, if we simultaneously require the metric tensor to obey the Einstein equations (so far disregarding the fact that we have not been able to construct a quantum-electro-dynamic energy-momentum tensor and, therefore, can not even enter the explicit form of the Einstein equations).

The idea of identification of spaces x and k, is, undoubtedly, disgusting, and evidently contradicts the initial interpretation of x and k as "semi-components of dyad $\{x|k\}$ complementary by Bohr". Perhaps, among the readers of the article there will be found a mathematician who, in contrast to the author, knows the subject of the Fourier-transform in a larger scope than the classical book by Ian Sneddon can provide [5]. The reader, who is able to provide a more convincing mathematical interpretation for phase η in the Riemann continuum. But we, due to the lack of an alternative, will consider formula (36) acceptable for relativistic quantum theory.

4.6 Some Concluding Remarks on Dirac Mathematics of Quantum Electrodynamics

- By analogy with the non-relativistic quantum theory, we can assert that the system of equations (27) and (32) that determines Dirac vectors by Born fluxes, is solvable not for any arbitrary specified quadruple of Born fluxes, but only for such quadruples that satisfy some a priori solvability conditions. These conditions are the relativistic generalization of non-relativistic Heisenberg inequalities (6). Taking into account the fact that in the relativistic scheme there are two independent Fourier-doublets of the Dirac vectors, there must be more relativistic solvability conditions than in the non-relativistic scheme. And if we take into account the Riemann complexity of the form of the relativistic Fourier-phase (36), the very possibility of entering the explicit form of relativistic version of the Heisenberg postulate (6) becomes unobvi-

[22] The reader will be right if he suspects a school "cosine theorem" in (36).

ous[23].

The very fact of existence of the Fourier-coupling (32) between Dirac vectors in the dyadically-coupled representations (the coupling between $\underset{ab}{\psi}$ and $\underset{\bar{a}\bar{b}}{\psi}$) means, in particular, that there is no quantum-electro-dynamic state which could be interpreted as a free electromagnetic field, i.e., the ensemble of photons. If wave function $\underset{22}{\psi}$ is not identically zero, $\underset{11}{\psi}$ can not be identically zero either: if there are photons, somewhere there are sources of these photons. This is quite natural, but also paradoxical result which is poorly consistent with the usual research methods in theoretical physics.

- In classical electrodynamics of continual currents [1], the a priori condition of space-likeliness is imposed on currents J^ν:

$$\mathrm{J}^\nu \mathrm{J}_\nu \leq 0. \tag{37}$$

Hardly can this condition be appropriate in quantum electrodynamics as a hard a priori constraint of the Dirac space geometry. However, condition (37) will actually control the arrangement of quantum-electrodynamic states, if Dirac vector $\underset{11}{\psi_\alpha}$ contains a multiplier of the form $\exp(-\lambda \mathrm{J}_\nu \mathrm{J}^\nu)$ with $\lambda > 0$. Under satisfying this condition, components $\underset{11}{\psi_\alpha}$ will decrease exponentially in the area of positive values of the pseudo-Euclidean square of the current module. Availability of such exponential multiplier can be provided if we interpret the current equation of the classical version of the theory [1]:

$$\mathrm{J}^\nu + \mathrm{A}^\nu = 0,$$

as operator relation:

$$\hat{\mathrm{J}}^{\,\nu} + \hat{\mathrm{A}}^{\,\nu} = 0, \tag{38}$$

and suggest that

$$\hat{\mathrm{A}}^{\,\nu} = \frac{\partial}{\partial \mathrm{J}_\nu} \text{ in J-representation.} \tag{39}$$

The equation for Dirac vector $\underset{11}{\psi_\alpha}$ follows from (31) and (39) :

$$\frac{\partial \underset{11}{\psi_\alpha}}{\partial \mathrm{J}_\nu} + \mathrm{J}^\nu \underset{11}{\psi_\alpha} = 0$$

with solution

$$\underset{11}{\psi_\alpha} = C_\alpha(x) e^{-\frac{1}{2}\mathrm{J}^\nu \mathrm{J}_\nu}, \tag{40}$$

where $C_\alpha(x)$ is the function that depends on space coordinates.

Undoubtedly, relation (40) seems too simple and poor to contain the whole quantum electrodynamics. Besides, operator relation (39) is incompatible with the form of

[23] The author uses these remarks to disguise his mathematical weakness, his inability to construct a relativistic analogue of Weyl theorem. Undoubtedly, among the readers of this article there may be found a more successful mathematician able to formulate Weyl theorem for quantum electro-dynamics.

Fourier-phase (33): the lack of imaginary unit i in (39) requires its availability before the second term in (33). This, in its turn, changes the interpretation of equations (32): RTF becomes the Fourier transform only within the first dyad $\{x|k\}$.
The situation becomes even more complicated if we remember that in the classical version of electrodynamics of continual currents [1], constructed here, there is one more current module restriction:

$$\mathrm{J}^{\nu}\mathrm{J}_{\nu} \geq -\mathrm{j}_0^2, \tag{41}$$

where j_0 is some (unknown) fundamental constant.
Representation of restriction (41) with use of the corresponding exponential multiplier in Dirac vector ψ_α requires a more complicated expression for potential operator $\hat{\mathrm{A}}^{\nu}$ than (39), and, therefore, a more complicated expressions for Fourier phase than (33).
By the end of this observation we find ourselves in front of the ruins of the newly constructed quantum relativistic scheme: it is badly compared with classical electrodynamics of continuous currents [1].
What can be considered reliable in the constructed scheme?
Probably, the concept of the Dirac space and Dirac vectors itself, connected with the Born fluxes by equations (27) and with each other – by relativistic Fourier transform (32) with ***non-specified*** relativistic Fourier phase ϕ.
Probably, the idea of partition of Fourier phase into two terms, each of which is connected with one dyad, is reliable. But neither of the terms in the notation of specific expression for phase ϕ (34) is reliable. The first term in form (36) is doubtful because of rather arbitrary method of accounting the Riemann curvature. The second term does not allow to account for current module restrictions that we substantiated in the classical version of the theory [1].

- The Dirac equation in its usual form:

$$i\,\gamma^{\mu}_{\alpha\beta}\,\partial_\mu\psi_\beta = m\,\psi_\alpha + e\mathrm{A}_\mu\,\gamma^{\mu}_{\alpha\beta}\,\psi_\beta, \tag{42}$$

hardly can be directly applied to the constructed here relativistic quantum scheme. It contains particle mass m and particle charge e, i.e. the integral characteristics of some stationary quantum state, while their appearance in local relation (42) seems irrelevant. Besides, this equation does not contain any dyadic indices and does not allow determining which of the two independent Dirac vectors it could be related with.
Instead of mass m in the first term on the right side of (42) there should appear a multiplier connected with the energy-momentum operator $\hat{\mathrm{T}}_{\mu\nu}$.[24]
Its Lorentz indices can be "extinguished" by the product of Dirac matrices $\gamma^{\mu}_{\beta\gamma}\,\gamma^{\nu}_{\gamma\delta}$. Free Dirac indices of this product can be extinguished by the "plate" of two Dirac vectors $\psi^*_\beta \dots \psi_\delta$. To keep some memory of the "linearity and autonomy" of the

[24] We will afford to neglect the fact that we do not have an expression for this operator yet.

wave equations, we have to assume that this non-linear "plate" is formed of the vector which is dyad-conjugated by one of the dyadic indices to vector ψ appearing in the equation. In addition, this multiplier with bilinear "plate" must imply the integration by the semi-component of the dyad {current|potential}, which is coupled with the semi-component entering the argument of vector ψ. As a result, the Dirac equation can be re-written in the following form:

$$i\,\gamma^{\mu}_{\alpha\beta}\,\partial_{\mu}\underset{1a}{\psi_{\beta}} = \underset{1a}{\psi_{\alpha}}\left(\int \underset{1\overline{a}}{\psi^{*}_{\beta}}\,\gamma^{\mu}_{\beta\gamma}\,\hat{\mathrm{T}}_{\mu\nu}\,\gamma^{\nu}_{\gamma\delta}\,\underset{1\overline{a}}{\psi_{\delta}}\,\mathrm{d}\underset{\overline{a}}{\Omega}\right) + \ldots. \tag{43}$$

In equation (43) the last term on the right side of (42), in which we have to do similar manipulations to remove the symbol of electric charge e from the equation notation, is not written out. The coefficient with $\underset{1a}{\psi_{\alpha}}$ on the right side of (43) is the coordinate function x. Therefore, relation (43) is linear by $\underset{1a}{\psi_{\alpha}}$, but integrally the quantum state is described nonlinearly, since the coefficient with $\underset{1a}{\psi_{\alpha}}$ is functionally dependent on the second Dirac vector.

Instead of charge e in the last term on the right side of (42), we probably have to write the current operator $\hat{\mathrm{J}}_{\nu}$. Its Lorentz index can be balanced by the Dirac matrix $\gamma^{\nu}_{\gamma\delta}$. Free Dirac indices of this matrix can be balanced by the plate of the two Dirac vectors. To ensure the "linearity" of the Dirac equation, we should provide for a transition to a coupled dyadic index and integration by the corresponding semi-component of the dyad {current|potential}. As a result, we form up the Dirac equation (42) as follows:

$$i\,\gamma^{\mu}_{\alpha\beta}\partial_{\mu}\underset{1a}{\psi_{\beta}} = \underset{1a}{\psi_{\alpha}}\left(\int \underset{1\overline{a}}{\psi^{*}_{\beta}}\gamma^{\mu}_{\beta\gamma}\hat{\mathrm{T}}_{\mu\nu}\,\gamma^{\nu}_{\gamma\delta}\underset{1\overline{a}}{\psi_{\delta}}\mathrm{d}\underset{\overline{a}}{\Omega}\right) + \left(\hat{\mathrm{A}}_{\mu}\gamma^{\mu}_{\alpha\beta}\underset{1a}{\psi_{\beta}}\right)\left(\int \underset{1\overline{a}}{\psi^{*}_{\gamma}}\hat{\mathrm{J}}_{\nu}\gamma^{\nu}_{\gamma\delta}\underset{1\overline{a}}{\psi_{\delta}}\mathrm{d}\underset{\overline{a}}{\Omega}\right). \tag{44}$$

We have fixed in (44) the first dyadic index equal to 1 taking into consideration the form of the left-side of equation (44), which contains the operator of differentiation by x_{μ}. Under transition to the momentum representation on the left side of (44), we should write $\gamma^{\mu}_{\alpha\beta}\,k_{\mu}\underset{2a}{\psi_{\beta}}$.

Equation (44) is linear relative to the Dirac vector $\underset{1a}{\psi}$, but in general the problem of quantum electrodynamics is non-linear and even considerably nonlinear (does not allow linearization). By cubic nonlinearities (44), it resembles a "fundamental field equation" investigated in the later works of Werner Heisenberg [6]. But the equation of W. Heisenberg did not contain any integration operations. Unlike Albert Einstein, who worked on the ***classical*** unified field theory, Werner Heisenberg was looking for a quantum version of the field theory, but his thinking, as well as Albert Einstein's thinking, was clearly monadic: he believed that the field must be described by ***one*** wave function (Weyl two-component spinor or Dirac four-component spinor). Our description of quantum electrodynamics requires the use of two independent Dirac vectors, while multi-sector quantum relativistic states, located not only in the singlet (Maxwellian) sector, but also in the triplet and octuplet Yang-Mills sectors of physics, require even a larger set of the Dirac vectors.

If we fix the free dyadic index a in (44) supposing that $a = 2$, the operator of potential $\hat{\mathrm{A}}_\mu$ on the right side of (44) will just be reduced to a simple multiplication by potential A_μ.

If we assume that dyadic index in (44) can take on both values $a = 1$ and $a = 2$, equations (44) form a complete system of two (vector) Dirac equations for two independent Dirac vectors $\underset{11}{\psi}$ and $\underset{12}{\psi}$. But in this case there arises a question: where and how should Maxwell equations show their worth? Maxwellian "trace" should certainly manifest itself in the formation of operator $\hat{\mathrm{T}}_{\mu\nu}$ which contains the quantum version of electromagnetic field tensor – but is it sufficient for (44) to be accepted as a complete description of quantum electrodynamics?

Of course, all these hard doubts under the construction of quantum electrodynamics equations are generated by the fact that we are trying to construct ***equations of the theory per se*** with the lack of any general guiding physical principle. In classical physics, this principle is the principle of least action, and the accompanying aesthetic requirements of simplicity and symmetry claimed to the Lagrangian. Apparently, there is no such principle in the relativistic quantum theory, and the equations of the theory should be just discerned[25].

- If in the classical expression for electromagnetic field tensor

$$F_{\mu\nu} = \partial_\mu \mathrm{A}_\nu - \partial_\nu \mathrm{A}_\mu,$$

we make a change of $\partial_\mu \to i\,\hat{k}_\mu$, but will interpret potential A_μ, as well as momentum k_μ as an operator acting on the Dirac vectors, field operator $\hat{F}_{\mu\nu}$ can be presented in the form:

$$\hat{F}_{\mu\nu} = i\,\left(\hat{k}_\mu\,\hat{\mathrm{A}}_\nu - \hat{k}_\nu\,\hat{\mathrm{A}}_\mu\right). \tag{45}$$

Operators $\hat{k}_\mu$ and $\hat{\mathrm{A}}_\nu$ act on different groups of the Dirac vector arguments and must commute.

If, within the same approach, we treat the Maxwell equations as operator relations:

$$\partial_\mu F^{\mu\nu} = 4\pi\,\mathrm{J}^\nu \to i\,\hat{k}_\mu \hat{F}^{\,\mu\nu} = 4\pi\,\hat{\mathrm{J}}^{\,\nu},$$

and accept that in A-representation $\hat{\mathrm{J}}^{\,\nu} = \dfrac{\partial}{\partial\,\mathrm{A}_\nu}$, instead of Maxwell equations, we

[25] The reader must have noticed that we tacitly ignore the Feynman approach to the derivation of relations of quantum physics. This approach is based on the integration by all the field realizations of exponent of the classical action functional (and in the non-relativistic situation – on integrating by all the classical trajectories of the particles). Such course of actions creates the illusion of reality of existence of classical fields as both continuous and differentiable time and place functions (but in the non-relativistic problem – the illusion of existence of smooth trajectories of point particles). Support of such illusion seems unacceptable from the philosophical point of view – despite the technical success of the Feynman approach. Besides, some mathematicians believe that in the pseudo-Euclidean continuum of relativistic theory it is absolutely impossible to give any sense to Feynman integrals by field realizations.

can enter the equation for the second Dirac vector:

$$\gamma^{\nu}_{\alpha\beta}\frac{\partial}{\partial \mathrm{A}_{\nu}}\left(\underset{22}{\psi_{\beta}}\right) = -\frac{1}{4\pi}k_{\mu}k^{\mu}\mathrm{A}_{\nu}\gamma^{\nu}_{\alpha\beta}\underset{22}{\psi_{\beta}}. \tag{46}$$

In notation (46) the operator relation is accepted:

$$\hat{k}_{\mu}\,\hat{\mathrm{A}}^{\,\mu} = 0, \tag{47}$$

which replaces the classical condition imposed on the divergence of potential A_{ν}. Operator relation (47) looks rather strange and, of course, makes sense only under the integral sign by σ- and Ω-variables in the "plates" of ψ-vectors.
Equation (46) should be taken as postulated, without any reference to equation (47). We will interpret the classical expression for electrodynamic energy-momentum tensor $\mathrm{T}^{\mu\nu}$ [1] as an operator acting on the Dirac vectors:

$$\begin{aligned}
&\mathrm{T}^{\mu\nu} \;\to\; \hat{\mathrm{T}}^{\,\mu\nu} = \hat{\mathrm{T}}^{\,\mu\nu}_{\,cur} + \hat{\mathrm{T}}^{\,\mu\nu}_{\,f};\\
&\hat{\mathrm{T}}^{\,\mu\nu}_{\,cur} = \hat{\mathrm{J}}^{\,\mu}\hat{\mathrm{J}}^{\,\nu} - \frac{1}{2}\mathrm{g}^{\mu\nu}\hat{\mathrm{J}}_{\lambda}\hat{\mathrm{J}}^{\,\lambda};\\
&\hat{\mathrm{T}}^{\,\mu\nu}_{\,f} = \frac{1}{2}\left(-\hat{F}^{\,\mu\lambda}\hat{F}^{\,\nu}_{\,\lambda} + \frac{1}{4}\mathrm{g}^{\mu\nu}\hat{F}^{\,\lambda\eta}\hat{F}_{\lambda\eta}\right).
\end{aligned} \tag{48}$$

Relation (48) implies that the electrodynamic energy-momentum operator consists of the energy-momentum operator of currents $\hat{\mathrm{T}}^{\,\mu\nu}_{\,cur}$ and the energy-momentum operator of free field $\hat{\mathrm{T}}^{\,\mu\nu}_{\,f}$.
Supposing that in (44) $a = 2$, we transform (44) into the equation for vector $\underset{12}{\psi_{\alpha}}$, which coefficients on the right side of (44) depend on the integrals of current space containing the operators acting on vector $\underset{11}{\psi_{\alpha}}$ under the integral sign. This vector is a relativistic Fourier-transform of vector $\underset{22}{\psi_{\alpha}}$. Vector $\underset{22}{\psi_{\alpha}}$ obeys the linear and autonomous equation (46).
While constructing the right side of the Einstein gravitational equations, we have to use operator (48) instead of the classical energy-momentum tensor, perhaps, in the following form:

$$\mathrm{T}^{\mu\nu} = \int \underset{11}{\psi^{*}_{\alpha}}\hat{\mathrm{T}}^{\,\mu\nu}_{\,cur}\underset{11}{\psi_{\alpha}}\mathrm{d}\underset{\mathrm{J}}{\Omega} + \int \underset{12}{\psi^{*}_{\alpha}}\hat{\mathrm{T}}^{\,\mu\nu}_{\,f}\underset{12}{\psi_{\alpha}}\mathrm{d}\underset{\mathrm{A}}{\Omega}. \tag{49}$$

The account of the Riemann curvature of the space / time coordinates (and, hence, the momentum space) makes the geometric meaning of momentum operator k_{ν} which appears in (45) unclear and, accordingly, devalues the formulated mathematical construction.
So, what remains after all these futile attempts to construct a system of equations of the relativistic quantum theory?[26]

[26] Albert Einstein wrote to Maurice Solovine that with his persistent desire to understand the basic principles, "the most part of his time was wasted on futile efforts" ([7], a letter from 30. .1924). The failure, of course, is an inalienable part of the attempts to "understand the basic principles".

There remains some contour, outline of the non-embodied intention. And according to this contour, the quantum electrodynamics equations should consist of three groups:

(1) The equation growing from the Dirac equation (42) for one of the two independent Dirac vectors. Information on the second vector is included into this equation in the form of coefficients containing $\underset{\mathrm{J}}{\Omega}$ or $\underset{\mathrm{A}}{\Omega}$ integrals of the quadratic forms that contain the second Dirac vector.

(2) The equation for the second Dirac vector growing from the Maxwell equations. Perhaps, this equation should also be quasi-linear and contain information on the first Dirac vector in coefficients represented by $\underset{\mathrm{J}}{\Omega}$ or $\underset{\mathrm{A}}{\Omega}$ integrals. We have constructed the equation (46) as linear, and this linearity violates the "Dirac equality" of vectors $\underset{12}{\psi}$ and $\underset{22}{\psi}$, which causes some distrust to this equation.

(3) Einstein's classical gravity equations, the right side of which represents $\underset{\mathrm{J}}{\Omega}$ or $\underset{\mathrm{A}}{\Omega}$ integrals of quadratic forms that contain Dirac vectors (49).

Operator relations, replacing the current equations of classical electrodynamics with continual currents [1], must be added to these equations.

What might be considered the criterion of such program success?

- The existence of a variety of stationary solutions that correspond to massive leptons [1].
- The correct result of mass calculation of the muon and the triton.
- Predicting of the masses of more massive leptons (if there are more than three states) in the spectrum of stationary states.

5 Dirac Space and Dirac Vectors in Quantum Singlet-triplet (Electroweak) Theory

In the quantum singlet-triplet theory, the two dyads of quantum electrodynamics – "space/momentum" $\{x|k\}$ and {singlet current | singlet potential} $\{\mathrm{J}|\mathrm{W}\}$ – are complemented with the third dyad – {triplet current | triplet potential} $\{\mathbf{J}|\mathbf{W}\}$. Correspondingly, all of the observable physical quantities, for example, the Born fluxes, get the third dyadic index $a = 1$ or $a = 2$. For the three dyads we obtain eight variants for possible measurement procedures or, respectively, eight variants of the Dirac space with the arguments of the first or second semi-components of each dyad:

$$\begin{aligned} \langle 1|1|1\rangle &\quad\longleftrightarrow\quad \langle 2|2|2\rangle, \\ \langle 1|1|2\rangle &\quad\longleftrightarrow\quad \langle 2|2|1\rangle, \\ \langle 1|2|1\rangle &\quad\longleftrightarrow\quad \langle 2|1|2\rangle, \\ \langle 1|2|2\rangle &\quad\longleftrightarrow\quad \langle 2|1|1\rangle. \end{aligned} \tag{50}$$

The set of dyadic indices in the right column (50) is a dyadic negation of the set of indices of the left column. Dirac vectors with a set of indices of the left column (50) are the dyad-conjugated Dirac vectors with a set of indices on the right column. Dyad-conjugated

Dirac vectors of the left and right columns should be connected with the singlet- triplet relativistic Fourier-transform.

Of course, as in quantum electrodynamics, the Dirac vectors as ***constructible*** quantities, are preceded by the Born densities as observable quantities. For example, let us assume that $\underset{111}{N}$ is the total number of all measurements within the framework of the set of measurement procedures $\langle 1|1|1\rangle$, made over some infinite three-dimensional hypersurface σ (with the time-like unit vector n^{μ} at each point) and that $\Delta \underset{111}{N}$ is the number of measurements which reveal ***something*** in a small but finite 3-volume $\Delta\sigma$ of surface σ, and ***simultaneously***, in a small but finite 4-zone of singlet 4-current $\Delta\underset{\mathrm{S}}{\Omega}$, and ***simultaneously*** in a small but finite 12-dimensional volume $\Delta\underset{\mathrm{T}}{\Omega}$ of the triplet current space. The Born density $\underset{111}{\xi}$ is determined by the relation similar to (25):

$$\lim_{\underset{111}{N}\to\infty}\left(\frac{\Delta \underset{111}{N}}{\underset{111}{N}}\right)=\int\limits_{\Delta\sigma}\int\limits_{\Delta\underset{\mathrm{S}}{\Omega}}\int\limits_{\Delta\underset{\mathrm{T}}{\Omega}}\underset{111}{\xi}\,\mathrm{d}\sigma\mathrm{d}\underset{\mathrm{S}}{\Omega}\mathrm{d}\underset{\mathrm{T}}{\Omega}. \tag{51}$$

The integral in (51) is a 19-fold.

As in quantum electrodynamics, we postulate the fact of existence of the Born flux $\underset{111}{\rho^{\nu}}$ with the formula similar to (26):

$$\underset{111}{\xi}=\underset{111}{\rho^{\nu}}\,n_{\nu}. \tag{52}$$

Similarly, within the framework of the eight possible variants for singlet-triplet states of the measurement procedures, we will construct all eight Lorentz vectors of Born fluxes $\underset{abc}{\rho^{\nu}}$ (a, b, c = 1or2). Each of the eight Born fluxes is the function of its intrinsic set of arguments, i.e. defined in its intrinsic version of Dirac space[27]. Dirac space of the singlet-triplet theory is twenty-dimensional: four dimensions are generated by the first and the second (singlet) dyads each, and 4×3 dimensions are generated by the third triplet-dyad.

Dirac algebraic equations (27), connecting the Dirac vectors with the Born densities, take the third dyadic index, but preserve their form:

$$\underset{abc}{\psi^{*}_{\alpha}}\,\Gamma^{\nu}_{\alpha\beta}\,\underset{abc}{\psi_{\beta}}=\underset{abc}{\rho}{}^{\nu}. \tag{53}$$

Equations (53) are not enough to determine the set of Dirac vectors $\underset{abc}{\psi}$ and, as above, in quantum electrodynamics, we prescribe the presence of Fourier-coupling between the dyad-coupled Dirac vectors:

$$\underset{abc}{\psi_{\alpha}}\overset{RFT(ST)}{=}\underset{\bar{a}\bar{b}\bar{c}}{\psi_{\alpha}}. \tag{54}$$

[27] Neutrino problems, requiring isotropization of singlet current J^{ν} or one of the three Yang-Mills currents $\mathbf{J}^{\nu}$, are easily and elegantly solved in the classical field theory with continual currents [1], [2]. In the quantum version of the theory, neutrino situation is connected with the appearance of singularity of one or two Born fluxes on cone $\mathrm{J}^{\nu}\mathrm{J}_{\nu}=0$ or, for example, on cone $\overset{3}{\mathrm{J}}{}^{\nu}\,\overset{3}{\mathrm{J}}_{\nu}=0$ in current spaces. The correct construction of the Dirac vectors in such singular situation is not an easy task. Therefore, the quantum description of neutrino seems a more difficult problem than its classical description, presented in [1], [2].

Symbolic notation (54), as well as its singlet analogue (31), implies the relativistic Fourier transform, which should be structured for quantum version of the singlet-triplet theory. By analogy with singlet formula (32), we can expand the formal notation (54) as follows:

$$\begin{aligned} \underset{abc}{\psi_\alpha} &= \frac{1}{(2\pi)^{19/2}} \iiint e^{i\phi} \underset{\bar{a}\bar{b}\bar{c}}{\psi_\alpha} \, \underset{\bar{a}}{\mathrm{d}\sigma} \, \underset{\bar{b}}{\mathrm{d}\Omega} \, \underset{\bar{c}}{\mathrm{d}\Omega}, \\ \underset{\bar{a}\bar{b}\bar{c}}{\psi_\alpha} &= \frac{1}{(2\pi)^{19/2}} \iiint e^{-i\phi} \underset{abc}{\psi_\alpha} \, \underset{a}{\mathrm{d}\sigma} \, \underset{b}{\mathrm{d}\Omega} \, \underset{c}{\mathrm{d}\Omega}. \end{aligned} \tag{55}$$

Relativistic singlet-triplet Fourier-phase will be constructed by analogy with the singlet formulas (33) – (36):

$$\phi = \eta + \alpha \left(\frac{1}{p_S} \mathrm{J}^\nu \mathrm{W}_\nu + \frac{1}{p_T} \mathbf{J}^\nu \cdot \mathbf{W}_\nu \right), \tag{56}$$

where p_S and p_T are Weinberg parameters [2].

Relation (56) means that the Fourier-phase ϕ is formed by a linear combination of scalar products of two semi-components of each field dyad. The relative weights of these products are borrowed from the interaction Lagrangian of the classical singlet- triplet theory [2].

The space-momentum part of the Fourier-phase η can be entered either in a usual form (35) – if we tend to ignore the problems of the Riemannian curvature, or in a hypothetical form (36) which allows to account these Riemann problems in some form.

As in quantum electrodynamics, the Fourier-phase formula (56) is open to criticism (does not account the constraints on currents and the couplings between currents [2]; contains an arbitrary construction (36) for phase η, etc.). However, now we can not suggest any other formula for the Fourier-phase.

Integral equations (54) together with algebraic Dirac equations (53) allow to construct eight complex Dirac vectors[28] by eight of the observable real Born fluxes, but this eight of the Dirac vectors forms four Fourier-doublets, and, therefore, for a complete description of the singlet-triplet state it is sufficient to know only four Dirac vectors – one vector of each of the Fourier-doublet.

Consequently, the quantum theory of the singlet-triplet states should contain four fundamental equations controlling these four independent Dirac vectors (plus geometrical Einstein equations that control metric tensor).

One of these fundamental equations arises from the Dirac equation (entered with regard for the triplet sector). The possible way to construct this equation has been shown above in the discussion of fundamental equations of quantum electro-dynamics. Two more equations arise from Maxwell and Yang-Mills classical field equations.

One more equation, which does not have any source in prior physics, remains "unsupported". Perhaps, we have to dare to use the Dirac equation twice, for both Fourier vectors.

However, we have already gone too far into the undeveloped and unfriendly quantum

[28] Under satisfying the solvability conditions which are the singlet-triplet relativistic generalizations of the Heisenberg inequalities. There must be more of these conditions than in quantum electrodynamics.

territory. It is time to build a fort, fortify our positions and wait for the saving cavalry to come.

6 Dirac Space and Dirac Vectors in Quantum Singlet-triplet-octuplet Theory (The Standard Model)

In the quantum version of the singlet-triplet-octuplet theory (STO-theory, or the Standard Model) the three dyads of the singlet-triplet theory are complemented with the fourth dyad "octuplet current/octuplet potential" $\{\boldsymbol{\mathfrak{J}}|\boldsymbol{\mathfrak{W}}\}$. Accordingly, all of the observable physical quantities, for example, the Born fluxes, get the fourth dyadic index $a = 1$ or $a = 2$. For the four dyads there appear 16 different possible measurement procedures or, respectively, 16 variants of the Dirac space. These 16 variants can be arranged into eight dyad-conjugated pairs:

$$\begin{array}{lcl}
\langle 1|1|1|1\rangle & \longleftrightarrow & \langle 2|2|2|2\rangle, \\
\langle 1|1|1|2\rangle & \longleftrightarrow & \langle 2|2|2|1\rangle, \\
\langle 1|1|2|1\rangle & \longleftrightarrow & \langle 2|2|1|2\rangle, \\
\langle 1|1|2|2\rangle & \longleftrightarrow & \langle 2|2|1|1\rangle, \\
\langle 1|2|1|1\rangle & \longleftrightarrow & \langle 2|1|2|2\rangle, \\
\langle 1|2|2|1\rangle & \longleftrightarrow & \langle 2|1|2|1\rangle, \\
\langle 1|2|2|1\rangle & \longleftrightarrow & \langle 2|1|1|2\rangle, \\
\langle 1|2|2|2\rangle & \longleftrightarrow & \langle 2|1|1|1\rangle.
\end{array} \tag{57}$$

The set of dyadic indices in the right column is the dyadic negation of the set of indices of the left column. The Dirac vectors with a set of indices of the left and right columns are dyad-conjugated and, therefore, must be connected by relativistic (STO)-Fourier-transform.

As well as in quantum electrodynamics or the quantum singlet-triplet theory, the Dirac vectors (as constructible quantities) are preceded by the Born-densities (as observable quantities). Without deciphering the notations which are already obvious to the reader, we can rewrite formula (51) of the ST-theory for the STO-theory.

$$\lim_{\underset{1111}{N}\to\infty}\left(\frac{\Delta \underset{1111}{N}}{\underset{1111}{N}}\right) = \int\!\!\int\!\!\int\!\!\int_{\Delta\sigma\Delta\underset{\mathrm{S}}{\Omega}\Delta\underset{\mathrm{T}}{\Omega}\Delta\underset{\mathrm{O}}{\Omega}} \underset{1111}{\xi}\ \mathrm{d}\sigma\mathrm{d}\underset{\mathrm{S}}{\Omega}\mathrm{d}\underset{\mathrm{T}}{\Omega}\mathrm{d}\underset{\mathrm{O}}{\Omega}. \tag{58}$$

The integral in (58) is 51-fold: the integration by the 32-dimensional space of octuplet vectors has been added to integral dimension (51).

Using the same formula (52), which was used for the ST-theory, we are constructing Born flux $\underset{1111}{\rho^{\nu}}$ from Born density $\underset{1111}{\xi}$:

$$\underset{1111}{\xi} = \underset{1111}{\rho^{\nu}}\ n_{\nu}. \tag{59}$$

By changing the dyadic indices by the formulas similar to (58) and (59), we construct all 16 Lorentz vectors of Born fluxes $\underset{abcd}{\rho^{\nu}}$ (a, b, c, d = 1 or 2) within the framework

of the sixteen variants of measurement procedures possible in the STO- states. Each of the Born fluxes is a function of its intrinsic set of arguments, i.e. each vector is determined in its intrinsic version of the Dirac space. The Dirac space for STO-state is 52-dimensional: four dimensions are generated by the first and second dyad each (space-momentum and singlet), 12 dimensions are generated by the third dyad (triplet) and 32 dimensions correspond to the fourth dyad (octuplet).
Dirac algebraic equations (27) or (53), take the fourth dyadic index, but do not change their form:

$$\underset{abcd}{\psi^*_\alpha}\, \Gamma^\nu_{\alpha\beta}\, \underset{abcd}{\psi_\beta} = \underset{abcd}{\rho}{}^{\nu}. \tag{60}$$

In addition to these equations, which are not sufficient to determine the sixteen complex four-component Dirac vectors ψ by the sixteen real four-component Lorentz vectors ρ, we prescribe the existence of the Fourier-coupling between the dyad-conjugated Dirac vectors:

$$\underset{abcd}{\psi_\alpha} \overset{RFT(STO)}{=} \underset{\overline{a}\overline{b}\overline{c}\overline{d}}{\psi_\alpha}. \tag{61}$$

Symbolic notation (61), as well as its electrodynamic analogue (31) and its ST-analogue (54), implies the relativistic Fourier-transform, which must be reasonably constructed for quantum version of the STO- theory.
By analogy with electrodynamic formula (32), and ST-formula (55), we can open the formal notation (61) as follows:

$$\begin{aligned} \underset{abcd}{\psi_\alpha} &= \frac{1}{(2\pi)^{51/2}} \iiiint e^{i\phi} \underset{\overline{a}\overline{b}\overline{c}\overline{d}}{\psi_\alpha}\, \mathrm{d}\underset{\overline{a}}{\sigma}\, \mathrm{d}\underset{\overline{b}}{\Omega}\, \mathrm{d}\underset{\overline{c}}{\Omega}\, \mathrm{d}\underset{\overline{d}}{\Omega}, \\ \underset{\overline{a}\overline{b}\overline{c}\overline{d}}{\psi_\alpha} &= \frac{1}{(2\pi)^{51/2}} \iiiint e^{-i\phi} \underset{abcd}{\psi_\alpha}\, \mathrm{d}\underset{a}{\sigma}\, \mathrm{d}\underset{b}{\Omega}\, \mathrm{d}\underset{c}{\Omega}\, \mathrm{d}\underset{d}{\Omega}. \end{aligned} \tag{62}$$

Relativistic Fourier phase for STO-theory will be constructed by analogy with (56):

$$\phi = \eta + \alpha \left(\frac{1}{p_S} \mathrm{J}^\nu \mathrm{W}_\nu + \frac{1}{p_T} \mathbf{J}^\nu \cdot \mathbf{W}_\nu + \frac{1}{p_O} \mathfrak{J}^\nu \cdot \mathfrak{W}^\nu \right), \tag{63}$$

(p_S, p_T, p_O are Weinberg parameters [3]).
Formula (63), as well as its analogues (56) and (34), is probably incomplete: it does not account the couplings between currents and restrictions to current modules[29].
Integral equations (62) together with the Dirac algebraic equations (60) allow to construct sixteen complex Dirac vectors by the sixteen observable real Born fluxes. However, these sixteen Dirac vectors form eight Fourier-doublets and, consequently, for complete description of the STO-state it is sufficient to know only eight Dirac vectors – one vector for each Fourier-doublet. Therefore, the quantum version of the theory of STO-states should contain eight independent fundamental equations that control this eight independent Dirac

[29] Within the framework of the constructed here apparatus of quantum physics of currents and potentials there is just nowhere else to introduce these restrictions and couplings – if they are, in general, important: besides relativistic Fourier-phase there is no other "free space" for their accounting in the theory. The only alternative is the direct restriction on the geometry of Dirac spaces.

vectors (plus geometric Einstein equation controlling the metric tensor).
One of these equations grows from the Dirac equation which must be written now with the account of the octuplet sector. Three more equations are based on the three classical field equations (Maxwell equations in the singlet sector and Yang- Mills equations in the triplet and octuplet sector). Half of the required fundamental equations of the STO-theory "hang" without any classical basis. We have three more current equations of the classical field theory with continuum currents in reserve – for the singlet ,triplet and octuplet sectors (see [1], [2], [3]). However, in quantum theory, these equations are most likely to become operator relations similar to (38), rather than the missing fundamental equations for the Dirac vectors. Apparently, the Dirac equation should appear in the theory more than once while controlling the behavior of several Dirac vectors.
But if we take into account that beyond the STO-theory there is nothing at all (this is the whole known physics), the theory must also contain as one of the solutions and the Big Bang model (the birth of the Universe ***from nothing***) – and, inevitably, in thinking about this theory there may appear a Pascal feeling of "looking into the abyss".

7 Conclusion

This article presents a perplexing unfinished project of construction the quantum version of physics of currents and potentials[30]. If historical analogy with the era of construction non-relativistic quantum mechanics is permissible, the project has been presented to the reader in the early "de Broglie" stage, in the stage of an inspiring but vague idea, long before the "Schrödinger" stage where appears a clear and complete mathematical construction. However, we do have some advantages over de Broglie's era: we already know Born's statistical interpretation of the Dirac wave functions, we have Bohr's idea of complementarity and we even know the dimension of the mathematical spaces in which not yet written wave equations of the theory should work.
But we have no idea how massive particles in the form of the intrinsic states of the Dirac vectors can appear within the framework of this concept...
The most radical statement which can be formulated on the basis of this article is as follows:
In contrast to non-relativistic quantum theory, quantum relativistic states can not be described by one wave function (one Dirac vector); the number of wave functions is determined by sector multiplicity of the condition – two Dirac vectors for the singlet state (and for any one-sector state); four Dirac vectors for the ST-states (and for any two-sector states); eight Dirac vectors for three-sector STO-states. The number of fundamental wave equations of the theory is equal to the number of independent Dirac vectors. Some of the fundamental equations of the theory, similarly to the Dirac equation, have no "precursors" in classical physics.

[30] However, as Leonardo da Vinci once said: "La prima pittura fu sol di una linea". ("The first painting consisted just of a single line").

Development of the "Schrödinger" stage, where the fundamental wave equation should appear, turned out for the author to be the "work exceeding our powers and our hopes"[31]. We do not know whether this project, this concept of 52-dimensional Dirac capacitance for eight Dirac vectors, is the stone, the unshakable foundation upon which there may be erected a solid construction of theoretical physics, which is not shaken by regularizations, renormalizations, anomalies and Higgs fields. It is appropriate to repeat the words of Saint Augustine: "We will be searching as if we can find, and we will find if our search is endless".

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