Validation of the Hadron Mass Quantization from Experimental Hadronic Regge Trajectories

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Abstract: This contribution provides a validation to the earlier proposal of hadron mass quantization in the units of 70 MeV mass quanta. The linear experimental Hadronic Regge Trajectories constructed from the recent 2014 Particle Data Group Listings serve as a prominent tool in solving the Hadronic mass spectrum mystery. Application of the Barut's solution to relativistic Balmer formula helps in deriving quark masses for mesons and baryons. This astonishingly produces the quark masses very close the 70 MeV mass quanta, which turns out to be the mass quantum for buliding hadrons. The slight deviation from this mass quantization is also evidently explained.

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1 Introduction

The hadronic mass spectrum is a mystery. Hadrons are not point particles like leptons and they do possess an internal structure. Mass quantization is evident at atomic scale. This led to the proposal that even hadronic masses ought to be quantized as nature loves symmetry. Thereby, the general expectancy is that hadrons have rotational and other excited states which become evident through the discovery of large number of hadronic resonances. Malcolm Mac Gregor [1-3] did an inventive work in this field of mass quantization and has shown that in case of hadrons, there exists a mass band structure in the units of Q= 70 MeV. This quanta lead to a common shell structure separating

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hadron states. Gregor observed [2,3] that the hadron spectrum shows regularity with energy separations in the units of 70 MeV [4, 5]. He proposed that quarks are light and have small binding energies. These small binding energies of quarks leash two types of quanta. The first one is a 70 MeV spinless quanta Q, which reproduces the pion and the second one is a 330 MeV spin quanta S, which reproduces the nucleon. The second quanta is basically a derivate of the original 70 MeV mass quanta. This 70 MeV spinless mass quanta of Gregor [1-3] can be undoubtedly derived from the electron mass and the fine structure constant as

$$Q = m_e/\alpha = 70MeV \tag{1}$$

Also, the Nambu empirical mass formula:

$$m_n = (n/2)137m_e(\text{n is a positive integer}) \tag{2}$$

which can tabularize the particle spectrum, is candidly related to this particular quantum. In the present contribution, this famous 70 MeV mass quanta is derived from the experimental linear Regge trajectories derived from the Particle Data Group 2014 listings [6]. Regge trajectories basically classify dynamic hadrons of the same quantum numbers but different spins and masses. The idea behind the Regge theory for the purpose of classification of elementary particles is quite simple [7]. With the continuation of nuclear experiments, physicists began to find discover more and more hadrons. These hadrons projected out to be copies of the original hadrons, but with higher spins and masses. It was viable to plot some of them on a spin (J) versus $(mass)^2$ (M^2) plot and an astonishing pattern was observed. Each set of particles with the same internal quantum numbers lied on straight lines and these lines were called Regge Trajectories. The Regge trajectories are parameterized by the equation:

$$M^2 = J/\alpha' + \beta \tag{3}$$

where α' is called the Regge slope and β is the Regge intercept. It turned out that the Regge trajectories were far more valuable than just another classification scheme of hadrons. They serve as a tool to study hadron dynamics. Evidently, with the availability of large hadronic data, new experimental Regge trajectories are being plotted. In the present contribution, application of the Barut's solution to relativistic Balmer formula [8,9] helps in deriving quark masses for mesons and baryons which are plotted on their respective Regge trajectories. This astoundingly produces the quark masses very close the 70 MeV mass quanta, which turns out to be the ultimate hadronic building block. This provides a validation to concept of hadron mass quantization [10] from the experimental hadronic Regge trajectories [11].

2 Hadron Spectrum and the 70 MeV mass quanta

The spinless quantum Q=70 MeV and the spin quantum S=330 MeV, both act as the adequate basis set for light hadrons, where the quark-antiquark binding energies specifically match the nucleon-antinucleon binding energies. Moreover, when the special relativity is employed to the rotating masses, it is revealed that S is itself composed of three quanta Q in a relativistically spinning configuration, so that Q emerges out as the ultimate hadronic mass quantum. The relativistically spinning mass is 3/2 times the 70 MeV mass quanta.

$$Q = 70MeV \Rightarrow Q_s = (3/2)Q = 105MeV \tag{4}$$

where Qs is termed as the relativistically spinning mass quanta. Therefore,

$$3Q_s \approx 330MeV = (\text{S quanta}) \tag{5}$$

For the mesonic sector, the lowest mass particle in the pion, whose mass is approximately 140 MeV/c^2, which can be viewed as a composite mass of two 70 MeV/c^2 mass quanta. We know that $\pi = Q\overline{Q}$, where Q and $\overline{Q}$ are 70 MeV light mass quanta. In the similar fashion, K meson is 7Q that is seven times the mass quantum Q. The other mesonic resonances which are exact multiple of 70 are:- 1)$\rho(770)$ 2)$a_0(980)$ 3)$\pi_2(2100)$ 4)$a_6(2450)$ 5)$K_1(1400)$ 6)$K_2(1820)$ etc. Also from the level spacing observed in the case of baryons [2], it appears that excitations occur in the units of Q=70 MeV, eg 3 3Q=210 MeV, 4 4Q=280 MeV etc. It may be so that the particles and resonances which lie on the baryonic RT, may not be exact multiples of Q. For some baryons, following combinations of S and Q account for their masses.

$\Sigma = 330 \oplus \overline{330} \oplus 330 \oplus (4 \otimes \overline{70})$

$\Lambda = 330 \oplus \overline{330} \oplus 330 \oplus (3 \otimes \overline{70})$

$\Omega = 330 \oplus \overline{330} \oplus 330 \oplus (3 \otimes \overline{70}) \oplus (4 \otimes \overline{70}) \oplus (4 \otimes \overline{70})$

$\Xi = 330 \oplus \overline{330} \oplus 330 \oplus (3 \otimes \overline{70}) \oplus (3 \otimes \overline{70})$

$N = 330 \oplus \overline{330} \oplus 330$

The subsistence of the 70 MeV boson was also stemmed from the mass of the classical Dirac Magnetic Monopole [12]. Further, the sequel [13] of the previous reference provides testimony of this quanta from the findings of the Particle Data Group listings [14]. Furthermore, according to Gregor, all particles more massive than the electron can be constructed from a single mass quantum Q.

3 Hadron Spectrum and the 70 MeV mass quanta

In case of mesons, the pion is the lowest mass meson and thus the starting member of the meson RT. The pion is composed of two 70 MeV/c^2 mass quanta. From Barut's solution to a relativistic Balmer mass formula [8,9],

$$M^2 = (m_1 + m_2)^2 + 2m_1m_2\frac{J}{\alpha} \tag{6}$$

If m_1=m_2=m, then

$$slope = \frac{2m^2}{\alpha} \tag{7}$$

and

$$m = \sqrt{\frac{slope\alpha}{2}} \tag{8}$$

similarly

$$slope(baryons) = [(m_1 m_2 m_3)/(m_1 + m_2 + m_3)]\alpha_s \tag{9}$$

where m_1, m_2 and m_3 are individual quark masses and α_s is the strength of the coupling constant. If m_1=m_2=m_3=m, then

$$slope(baryons) = \frac{m^2}{3}\alpha_s \tag{10}$$

Taking the nucleon to be the starter of the series for the baryon RT, it is obvious that $m_1 = m_2 = m_3 = u = 315MeV$ (Since u and d are approximately equal in mass). The particle mass is taken to be 97 percent of the total mass, the rest being the contribution of the binding energy. Therefore, the quark mass will be

$$0.97 \times 315 = 305.6MeV \tag{11}$$

$$\alpha_s = 137/4 = 34.25 \tag{12}$$

This value of coupling constant is taken from the work of Sawada [15], who proposed that coupling parameter takes this particular value when Coulombic interaction of the magnetic monopole [16] is taken into account. This analysis of slope is done with the hypotheses of idealized mass quanta in the units of spinless, 70 MeV and spin1/2, 315 MeV mass quantas. An accordance of these values with experimental data would provide validation to this concept of hadron mass quantization. In all, 9 Figures have been plotted for a series of mesonic and baryonic resonances. Figures I-V have been plotted for series of mesonic resonances, while Figures VI-IX correspond to series of baryonic resonances. The slopes have been calculated for these trajectories and their quark masses have been calculated subsequently. Table I displays the quark mass for each trajectory along with the deviation from standard value of 70 MeV and 305.6 MeV respectively for mesonic and baryonic RTs. From the table it is evident that the deviation from the standard values is very low and profoundly the experimental quark masses match very distinctly with the theoretical postulation of the 70 MeV mass quanta. The reason for slight deviation from the exact quantized masses has been explained in the next section.

Table I: Calculated values of quark masses for several series of Hadronic RTs

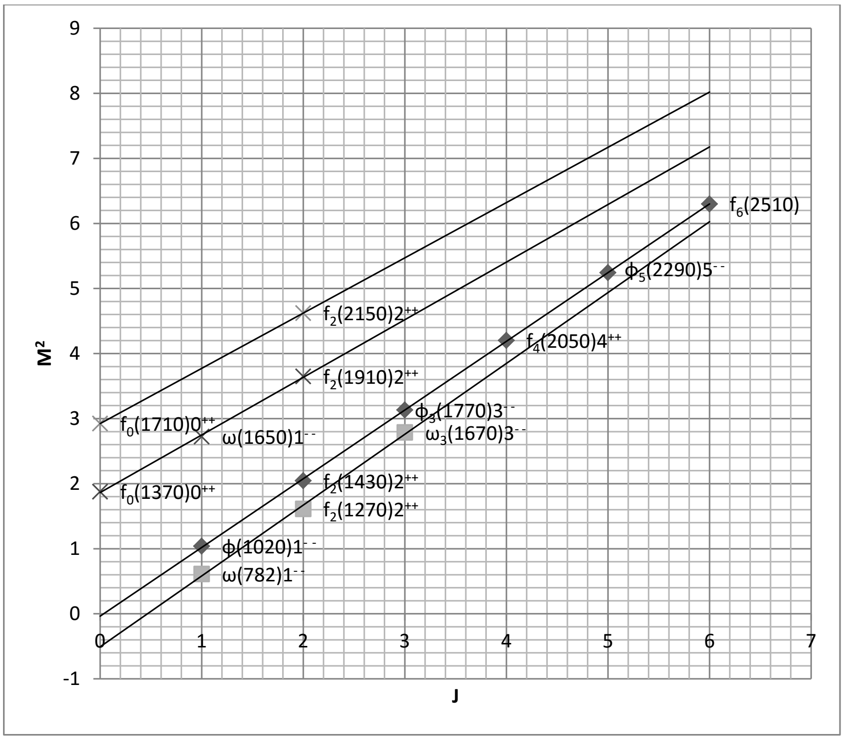
S. No		Hadrons		Slope	Quark mass	Deviation from standard value
Fig I	**MESONS**	Unflavored u-d mesons	1	0.849	55MeV	15MeV
			2	0.885	56MeV	14MeV
			3	1.056	62MeV	8MeV
			4	1.088	63MeV	7MeV
Fig II		Unflavored u-d mesons	1	0.726	51MeV	19MeV
			2	0.928	58MeV	12MeV
			3	1.227	66MeV	4MeV
Fig III		Unflavored u-d mesons	1	0.697	50MeV	20MeV
			2	0.989	60MeV	10MeV
Fig IV		Unflavored u-d mesons	1	0.909	57MeV	13MeV
			2	0.935	58MeV	12MeV
Fig V		K mesons	1	0.856	55MeV	15MeV
			2	1.145	64MeV	6MeV
			3	1.187	65MeV	5MeV
Fig VI	**BARYONS**	N baryons	1	1.118	310MeV	4.4MeV
Fig VII		Δ baryons	1	0.937	286MeV	19.6MeV
			2	1.026	299MeV	6.6MeV
			3	1.081	307MeV	1.4MeV
			4	1.196	323MeV	17.4MeV
Fig VIII		Λ baryons	1	0.831	269MeV	36.6MeV
			2	0.882	277MeV	28.6MeV
			3	1.073	306MeV	0.4MeV
Fig IX		Σ baryons	3	0.81	266MeV	39.6MeV
			4	1.101	309MeV	3.4MeV

4 Factors contributing to deviation from 70 MeV mass quanta

Several features manifestly cause the departure of masses of hadrons from the integral multiples of 70 MeV mass quanta. Foremost, the Coulomb corrections and spins are protuberant contributory factors along with magnetic moment ratios and charge splitting. Another conscientious factor could be the strangeness quantum number. Constituent quark binding energies is also one of the factors steering to variation in mass. Additionally, the constituent quark binding energies also relegates the masses of hadrons by nearly about 3 to 4 percent [2,17]. The relativistically spinning configuration of rotating masses also leads to digression of masses of hadrons from the integral multiples of seventy.

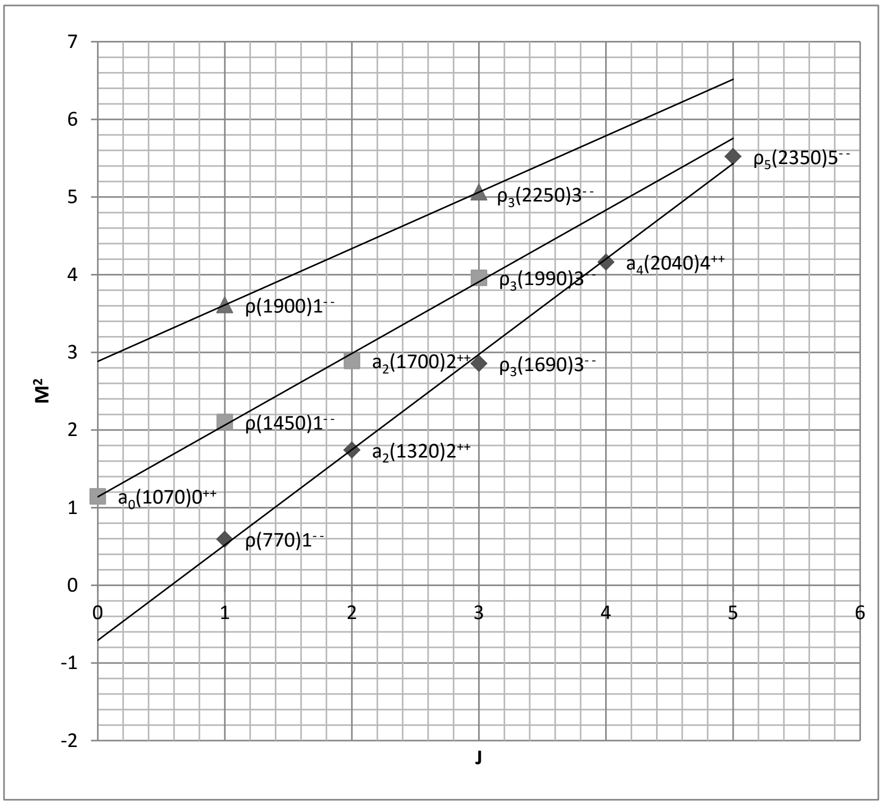
5 Conclusions

Quark mass quantization is a revolutionary notion and the present contribution provides an authentication to it. A restored estimation of the hadron dynamics is feasible once quantization becomes evident. The derivation of quark masses from Barut's solution to relativistic Balmer formula provides a foundation for calculating quark masses for mesons and baryons from the experimental hadronic RTs for Particle Data Group 2014 listings. Resplendently, the quark masses calculated from this approach are very intimate to the earlier proposed 70 MeV mass quanta. This in itself is validation to the proposal of quantized hadronic masses, which will ultimately prove to be very useful in unraveling the ambiguities of hadron dynamics.



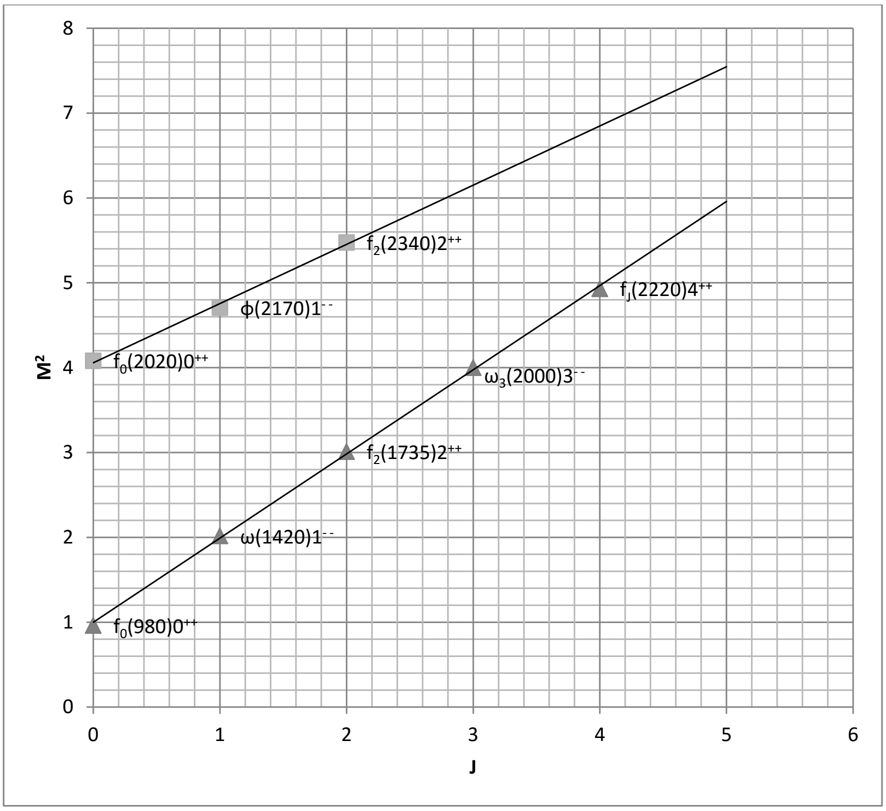
$M^2 = 0.849J + 2.924$
$M^2 = 0.885J + 1.863$
$M^2 = 1.056J - 0.035$
$M^2 = 1.088J - 0.506$

Figures I:-Regge trajectories for four separate series of unflavored (u-d) mesons along with their straight line equations and the string tensions.



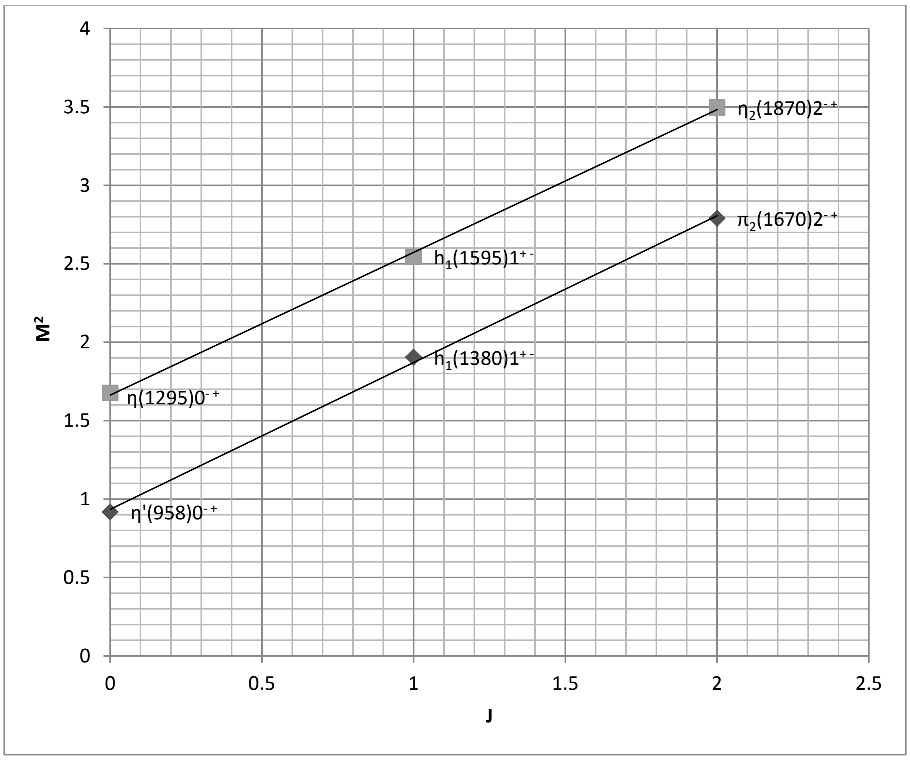
$M^2 = 0.726J + 2.883$
$M^2 = 0.928J + 1.126$
$M^2 = 1.227J - 0.708$

Figure II :- RTs for three separate series of unflavored (u-d) mesons along with their straight line equations and the string tensions.



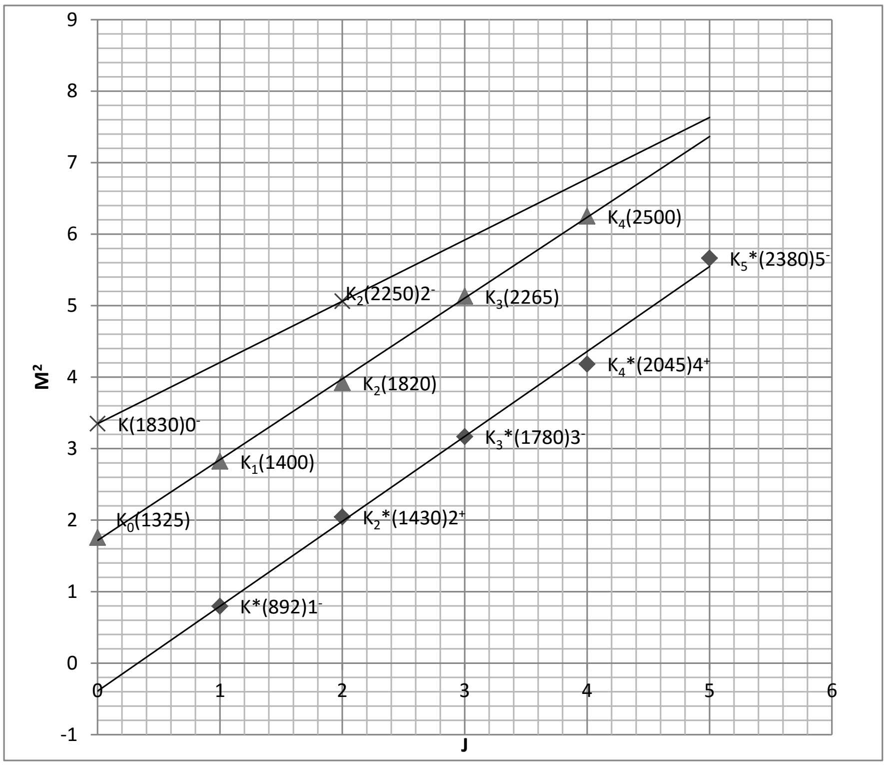
$M^2 = 0.697J + 4.057$
$M^2 = 0.987J + 0.989$

Figure III:- RTs for two separate series of unflavored (u-d) mesons along with their straight line equations and the string tensions.



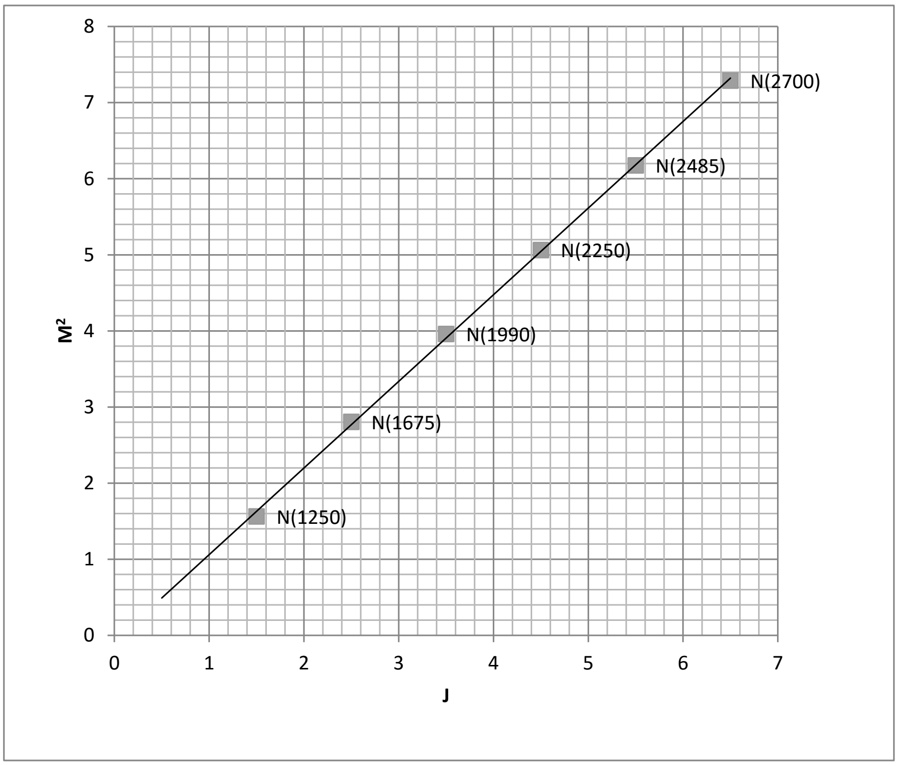
$M^2 = 0.909J + 1.662$
$M^2 = 0.935J + 0.934$

Figure IV- RTs for two separate series of unflavored (u-d) mesons along with their straight line equations and the string tensions.



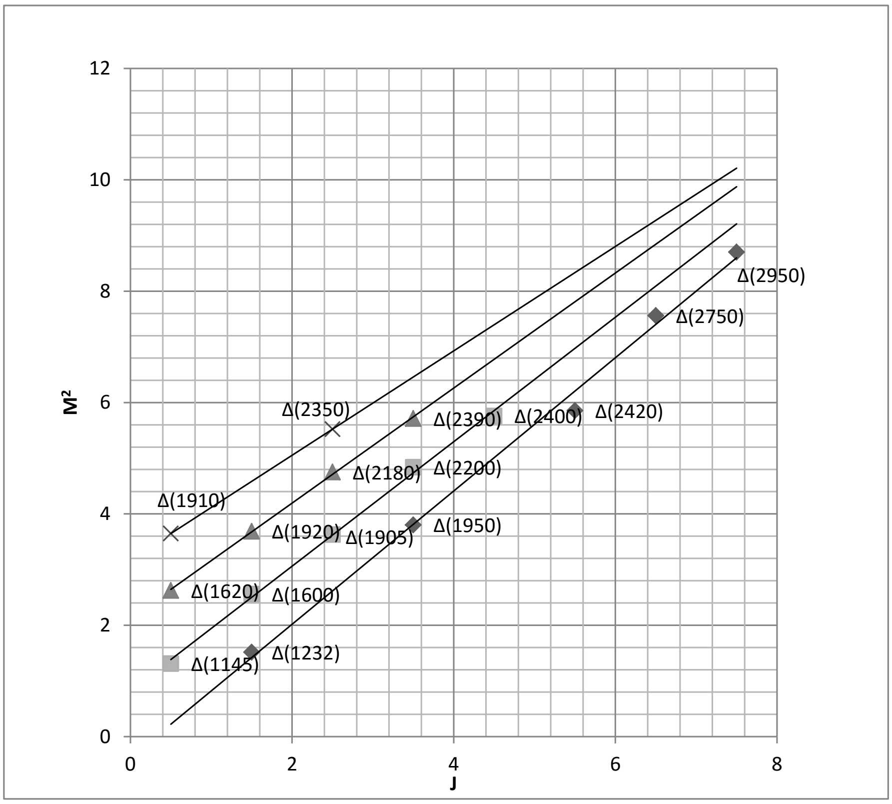
$M^2 = 0.856J + 3.348$
$M^2 = 1.145J + 1.657$
$M^2 = 1.187J - 0.391$

Figure V:- RTs for three separate series of K-mesons along with their straight line equations and string tensions.



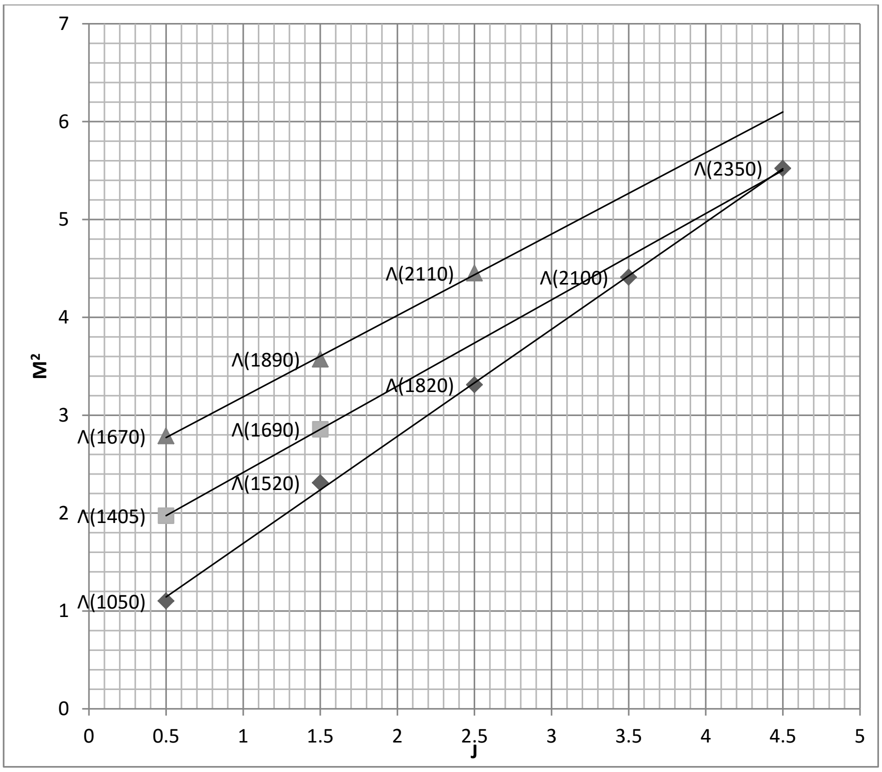
$M^2 = 1.118J + 0.025$

Figure VI:- RTs for one series of N-baryons along with their with straight line equations and string tensions.



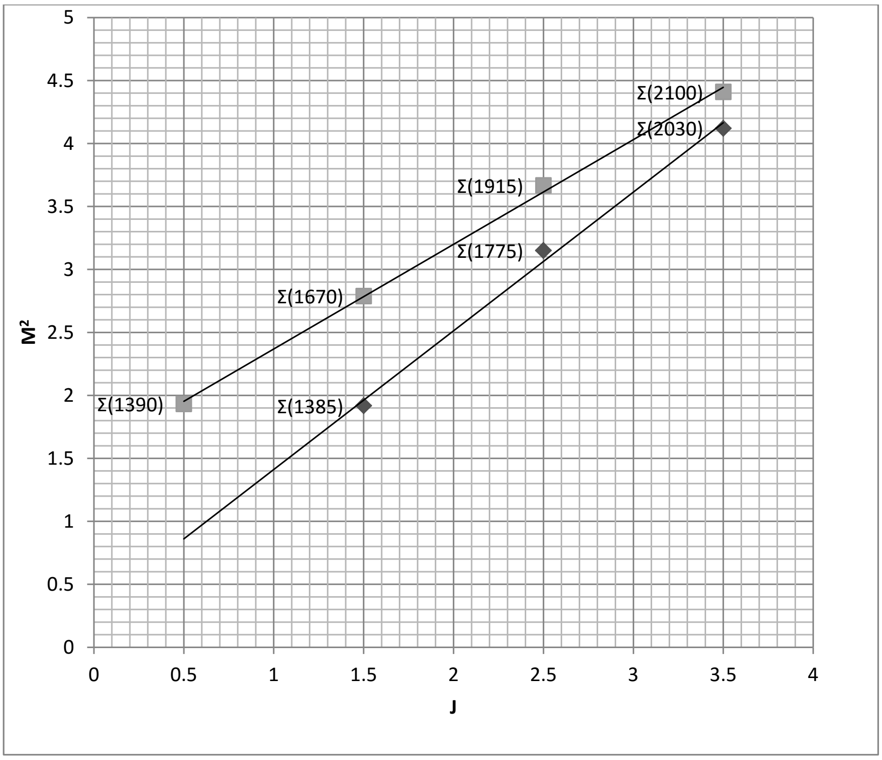
$M^2 = 0.937J + 3.179$
$M^2 = 1.026J + 2.125$
$M^2 = 1.081J + 0.954$
$M^2 = 1.196J - 0.374$

Figure VII :- RTs for four separate series of Δ baryons along with their straight line equations and string tensions.



$M^2 = 0.831J + 2.357$
$M^2 = 0.882J + 1.533$
$M^2 = 1.073J + 0.668$

Figure VIII :- RTs for three separate series of Λ baryons along with their straight line equations and string tensions.



$M^2 = 0.81J + 1.595$
$M^2 = 1.101J + 0.309$

Figure IX :- RTs for two separate series of Σ baryons along with their straight line equations and string tensions.

References

[1] M.H.Mac Gregor, Nuovo Cim. A, **103**, 983 (1990).

[2] M.H.Mac Gregor, Phys. Rev., **D9**, 1259-1329 (1974).

[3] M.H.Mac Gregor, Phys. Rev., **D10**, 850-883 (1974).

[4] N. Hothi, S. Bisht, Hadronic Journal, **38**, 465-476(2015).

[5] L.Chiatti, Phys. Essays, **27**, 1,143-145 (2014).

[6] K.A.Olive et al.Particle Data Group, Chin. Phys. CPhys. Essays, **38**, 090001 (2014).

[7] G. D. Coughlan, J. E. Dodd,The Ideas of Particle Physics, **Second Edition**, Cambridge University Press, London (1991).

[8] A.O.Barut, Atoms with magnetic charges as models of hadrons, in Topic in Modern Physics, edited by Wesley.E..Brittin and Halis Odabasi, Boulder **: Colorado Associated Univesity Press,** Colorado (1971).

[9] A.O.Barut, Phys. Rev., **D3**, 1747 (1971).

[10] N. Hothi, S. Bisht, Indian Jour. of Phys., **85**, 12, 1833-1842 (2011).

[11] S. Bisht, N. Hothi and G. Bhakuni, Electronic Jour. of Theo. Phys., **24**, 299 (2011).

[12] D. Akers, hep-ph/0303139 (2003)

[13] D. Akers, hep-ph/0303261 (2003).

[14] K. Hagiwara et al.Particle Data Group, Phys. Rev., **D66**, 010001-1 (2002).

[15] T.Sawada, hep-ph/0004080 (2000).

[16] R. Mignani and E. Recami, Phys. Letters, **B62**, 41-43 (1976).

[17] D. Akers, hep-ph/0309075 (2003).