Gravitational Lensing by Reissner-Nordstrom Extremal Black Hole

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Abstract: The gravitational lensing by a Reissner-Nordstrom (RN) extremal black hole is studied in the weak field limit. The equation for the deflection angle and from that the analytical expressions for the positions and magnification of the images are find out. We consider the black hole at the center of our galaxy and estimated the optical resolution needed to investigate its weak field behavior through its images.

Keywords: Gravitational Lensing; RN extremal Black hole
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1 Introduction

Gravitational lensing provided the first experimental verification of General Relativity (GR) [1] through observations of starlight bending around the Sun during the solar eclipse. In 1979, Walsh et.al reported the observation of two quasi stellar objects known as 0957 + 561 A and B separated in the sky by only 5.7 arc seconds, had nearly identical magnitudes, redshifts and detailed spectra. It was difficult to describe them as two distinct objects and thus suggested that light from a single source passes through the gravitational field of some large intervening object was arriving at the Earth from two different directions [2]. The phenomenon is now referred to as gravitational lensing and the object causing a detectable deflection is known as gravitational lens. Now the gravitational lensing is observed in many astrophysical contexts ranging from stars through galaxies and clusters of galaxies up to the large-scale structure of the universe [3].

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The full theory of gravitational lensing has been developed following the scheme of the weak field approximation. When the light goes very close to a heavy compact body like a black hole, an infinite series of images generate, which provide more information about the nature of the black hole's surrounding. In these cases the strong gravitational lensing can help us to explore the characteristics of the gravitational source. There has been significant theoretical effort over several decades to understand lensing in the strong deflection regime. Virbhadra and Ellis obtained the lens equation using an asymptotically flat background metric and analyzed the lensing by a Schwarzchild black hole in the center of the Galaxy using numerical methods [4]. Fritelli et al. [5] found an exact lens equation without any reference to a background metric and compared their results with those of Virbhadra and Ellis for the Schwarzchild black hole case. Eiroa, Romero and Torres [6] applied the same technique to a Reissner - Nordstrom black hole. More topics within the region of the strong field lensing can be listed as, gravitational source with naked sigularities [7], GMGHS charged black hole [8], a spining black hole [9] - [10] a braneworld black hole [11]- [12], an Einstein-Born-Infeld black hole [13], a black hole in Brans-Dicke theory [14], a black hole with Barriola-Vilenkin monopole [15]- [16] and the deformed Horava-Lifshitz black hole [17] etc.

The aim of this article is to review the theoretical aspects of gravitational lensing by RN extremal black holes. Section II describe the geometry of lensing. Strong field gravitational lensing by RN extremal black hole was explained in Section III. Application to a galactic black Hole is given in Section IV. Section V contains the conclusion.

2 Geometry Of Lensing

In a lensing system there are,

- Source : from where the light comes. The source can be a Quasar, a galaxy, etc.
- Lens : the one which deflect the path of light. It may be a galaxy, dark matter or a supermassive black hole, etc.
- Image : they are the result of gravitational lensing.

Fig.(1) gives a schematic diagram of the lensing situation and defines standard quantities: β the angular position of source, θ the angular position of an image, α the bending angle, D_{ol} the distance between observer and lens, D_{os} the distance between observer and source, and D_{ls} the distance between lens and source [4].

When source, lens and observer are all aligned we obtain the deformation of light from a source into a ring. If the alignment is perfect, ie, the lens is perfectly symmetric with respect to the line between the source and observer, We could see a ring of image and is called Einstein ring. From the Fig.(3) it is clear that angular separation of the source from the lens is zero.ie, $\beta = 0$. Here the bending of light is similar to the refraction of light at the interface of two media. However there

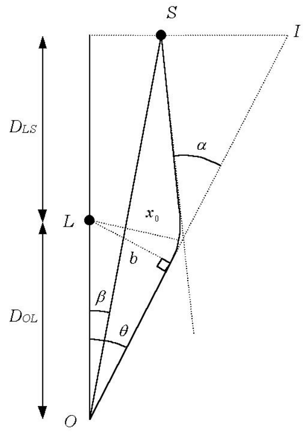

Fig. 1 Geometry of lensing

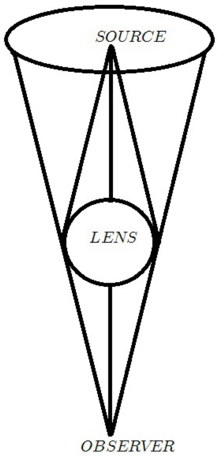

Fig. 2 Einstein Ring

are some basic difference between optical lensing and gravitational lensing. In the former, light rays from a point source get focussed at a point and we say that an image has been formed. In gravitational lensing, the null geodesics which connect the source and observer split up in such a manner that the observer receives light from different directions.

For a general spherically symmetric metric is given by,

$$ds^2 = A(x)dt^2 - B(x)dx^2 - x^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

the general formula for the deflection angle is given by [20],

$$\alpha = \frac{1 - y_0}{C_0 A_0'}[C_0' y_0 - C_0 A_0'], \tag{2}$$

where A(x) at x_0 (distance of closest approach) is A_0, C(x) at x_0 is C_0, $C'(x)$ at x_0 is C_0' and $A'(x)$ at x_0 is A_0'. A photon incoming from infinity with some impact parameter, will reach a minimum distanc and then emerge in another direction. That distance is called the distance of closest approach x_0

3 Gravitational Lensing by RN Extremal Black hole

The RN black hole's (event and inner) horizons in terms of the black hole parameters are given by $r_\pm = M \pm \sqrt{M^2 - Q^2}$, where M and Q, are respectively, mass and charge of black hole. In extreme case, these two horizons coincide, i.e. $r_\pm = M$. Then the metric corresponds to RN extremal black hole is given by,

$$ds^2 = (1 - \frac{M}{r})^2 dt^2 - \frac{1}{(1-\frac{M}{r})^2} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (3)$$

In this metric we use a change of variable,

$$x = \frac{r}{2M}. \quad (4)$$

Therefore the value of A_0, C_0, C_0' and A_0' will be obtained as,

$$A_0 = (1 - \frac{1}{2x_0})^2 = y_0, \quad (5)$$

$$B_0 = (1 - \frac{1}{2x_0})^{-2}, \quad (6)$$

$$C_0 = x_0^2, \quad (7)$$

Substituting Eqs.(5) to (7) in Eq.(2), we get the formula for deflection angle and is given by,

$$\alpha = \frac{1 - 4x_0}{2x_0}. \quad (8)$$

The general behavior of deflection angle as a function of the closest approach x_0 is shown in Fig.(3) and is as obtain in [18]. The thin lens approximation [19] can be

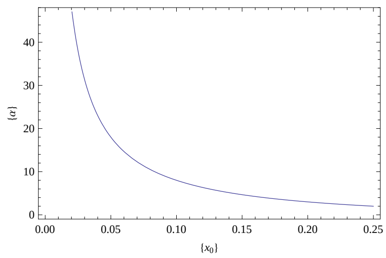

Fig. 3 Behavior of deflection angle as a function of the closest approach x_0

expressed as

$$\beta = \theta - \alpha \frac{D_{ls}}{D_{os}} \quad (9)$$

Substituting the expression for α in thin lens approximation, we get

$$2x_0\theta - 2\beta x_0 - \frac{D_{ls}}{D_{os}} + 4\frac{D_{ls}}{D_{os}} x_0 = 0. \quad (10)$$

From the lensing geometry we will get a relation between distance of closest approach and angular separation of image with lens θ and is given by,

$$x_0 = \frac{\theta D_{ol}}{2M}. \quad (11)$$

Substituting Eq.(11) in Eq.(10) we get a quadratic equation in θ (image position) and is given by,

$$A_1\theta^2 + A_2\theta + A_3 = 0, \quad (12)$$

where,

$$A_1 = \frac{D_{ol}}{M}, \quad (13)$$

$$A_2 = \frac{D_{ol}}{M}(2\frac{D_{ls}}{D_{os}} - \beta), \quad (14)$$

$$A_3 = -\frac{D_{ls}}{D_{os}}. \quad (15)$$

By solving the quadratic equation(12) we will get two roots. If they are real we will have two images. Its value depends on the parameters M, D_{ol}, D_{ls} and D_{os}. The first experimental observation regarding gravitational lensing was twin quasars [2]. Its actually the image of single quasar which is lensed by gravitational field. Several such observations are obtain latter. This explain our theoretical predictions on the image formations were correct.

3.1 Radius Of Photon Sphere

Photon sphere is a lowest possible orbit around a black hole, where the speed of light c is required to maintain the orbit. To find the deflection angle x_{ps}, we impose the condition $\alpha = 0$ for $x = x_{ps}$

$$\alpha = \frac{1}{2x_0} - 2. \quad (16)$$

Therefore radius of photon sphere is,

$$x_{ps} = \frac{1}{4}. \quad (17)$$

3.2 Magnification

Magnification [20] of lensed image at angular position θ is given by,

$$\mu = \mid \frac{d\theta}{d\beta}\frac{\theta}{\beta} \mid . \quad (18)$$

To obtain $\frac{d\theta}{d\beta}$, differentiate Eq.(10) with respect to θ,and obtained as,

$$2\frac{D_{ol}}{M}\theta + \frac{D_{ol}}{M}\left[2\frac{D_{ls}}{D_{os}} - \beta\right] - \theta\frac{D_{ol}}{M}\frac{d\beta}{d\theta} = 0. \quad (19)$$

Table 1 Positions of two Images

Source Position, β	First image, θ_1	Second Image, θ_2
10^{-4}	-0.9999	5.3379×10^{-15}
1	-7.30575×10^{-8}	7.3058×10^{-8}
2	-5.3374×10^{-15}	1
3	-2.6687×10^{-15}	2
4	-1.7791×10^{-15}	3
5	-1.3344×10^{-15}	4

Therefore,

$$\mu = \frac{\theta^2}{\beta[2\theta + 2\frac{D_{ls}}{D_{os}} - \beta]}. \tag{20}$$

4 Application To A Galactic Black Hole

To illustrate our results, we consider gravitational lensing by the super massive black hole at the center of our Galaxy . Here we are considering the week field lensing, that is the observer, source and lens are far away. The black hole has a mass of $M = 2.8 \times 10^6$ [21] and the distance $D_{0l} = 8.5kpc$. Therefore, the ratio of mass to distance $\frac{M}{D_{ol}} \approx 1.57 * 10^{-11}$. We consider a point source, with the lens situated half way between the source and the observer. That is $\frac{D_{ls}}{D_{os}} = \frac{1}{2}$. We allow the angular position of the source to change keeping D_{ls} fixed. From the Fig.(9)

$$D_{os} = \frac{1}{2} D_{os} + D_{ol}, \tag{21}$$

therefore,

$$D_{os} = 2D_{ol}. \tag{22}$$

The image position θ for the above case is obtained from the solution of Eq.(12). The image position for various values of source position are calculated and given in the Table.(1).

Magnification of the images for various values of source position are calculated using Eq.(20) and is given in Table.(2). From this we found that the magnification of one images falls extremely and the other image rises, as the source position increases from perfect alignment. We plot magnification (μ) vs image position θ (in arc seconds) for source position β (in micro arc seconds). The total magnification are positive for all images on the same side of the source and negative for all images on the opposite side of the source.

Table 2 Magnification of two Images

Source Position, β	Magnification of θ_1	Magnification of θ_2
10^{-4}	-9999	2.8496×10^{-25}
1	-3.6529×10^{-8}	3.6529×10^{-8}
2	-1.4244×10^{-29}	0.5000
3	-1.18699×10^{-30}	0.6667
4	-2.6378×10^{-31}	0.7500
5	-8.9025×10^{-32}	0.8000

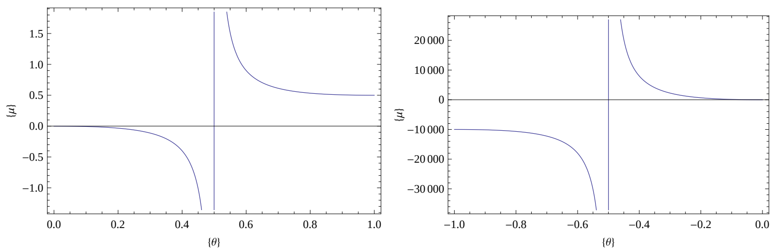

Fig. 4 μ vs θ for source position $\beta = 2\&10^{-4}$

5 Conclusion

The phenomenon of deflection of light due to a strong gravitational field is referred to as gravitational lensing and is one of the predictions of Albert Einstein's General theory of Relativity.

We have discussed gravitational lensing due to a Reissner-Nordstrom extremal black hole in the weak field limit. In this work we discussed the image formation by RN black hole. In particular, calculations are done for a super massive black hole with mass 2.8×10^6 times the mass of the sun, which may be an ideal model for the type of black hole in the center of our galaxy. The geometry of the lensing is studied using thin lens approximation, which leads to the possibility of two images depending mass and distance parameters. The position and magnification of images can be calculated. We found that the magnification of one image falls extremely and the other image rise as the source position increases from the perfect alignment. We also plotted magnification verses image position for different source positions. The total magnification are positive for all images on the same side of the source and negative for all images on the opposite side of the source.

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References

[1] A. Einstein, Science, 84, 506 (1936).

[2] D. Walsh, R. F. Carswell and R. J. Weymann, Nature 279, 381-384 (1979)

[3] P. Schneider, J. Ehlers, and E. E. Falco, Gravitational Lenses (Berlin: Springer, 1992).

[4] K. S. Virbhadra, G. F. R. Ellis, Phys.Rev. D 62, 084003 (2000). [arXiv:astro-ph/9904193].

[5] S. Frittelli, T. P. Kling, E. T. Newman, Phys. Rev. D 61, 064021 (2000).

[6] E.F. Eiroa, G.E. Romero, D.F. Torres, gr-qc/0203049.

[7] K. S. Virbhadra, G. F. R. Ellis, Phys. Rev. D65(2002)103004

[8] A. Bhadra, Phys. Rev. D67(2003)103009

[9] V. Bozza, Phys. Rev. D67(2003)103006

[10] V. Bozza, F. DeLuca, G. Scarpetta, M. Sereno, Phys. Rev. D72(2005)083003

[11] R. Whisker, Phys. Rev. D71(2005)064004

[12] E. F. Eiroa, Phys. Rev. D71(2005)083010

[13] E. F. Eiroa, Phys. Rev. D73(2006)043002

[14] K. Sarkar, A. Bhadra, Class. Quantum Grav. 23(2006)6101

[15] V. Perlick, Phys. Rev. D69(2004)064017

[16] H. Cheng, J. Man, arXiv: 1004.3423

[17] S. Chen, J. Jing, Phys. Rev. D80(2009)024036

[18] V. Bozza et.al arxiv.org/abs/gr-qc/0102068v1

[19] V. Bozza, S. Capozzielo, G. Iovane, G. Scarpetta, Gen. Rel. Grav. 33, 1535 (2001).

[20] V. Bozza et.al, arXiv:gr-qc/208075v3 (2002)

[21] D. Richstone et al., Nature, 395 A14 (1998).