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Higher Order Perturbative QCD and Non-perturbative Higher Twist Correctionto F_2^{NS} Structure Function

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Abstract: We have determined the F_2^{NS} structure function by means solving the DGLAP evolution equations with perturbative Quantum Chromodynamics(pQCD) corrections upto next-next-to-leading order(NNLO) using a Q^2 dependent regge ansatz as the initial input. In addition we have extracted the higher twist contribution to the structure function F_2^{NS} in NNLO perturbative orders and then incorporated them to our results. Our NNLO results along with higher twist corrections are observed to be compatible with corresponding experimental data and other phenomenological analysis.

Keywords: Quantum Chromodynamics; Regge Theory; F_2^{NS} Structure Function; Higher Twist Correction
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1 Introduction

F_2^{NS}, the non-singlet part of F_2 structure function is regarded as one of the most important observable in order to investigate Quantum Chromodynamics(QCD) as the underlying theory of strong interaction and therefore it has been an object of intensive investigation both theoretically and experimentally in recent years(see for example [1] and references therein). In QCD, the structure functions are governed by a set of integro-differential equation, known as the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations [2], which is a renormalisation

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group equation for the distribution of quarks and gluons inside hadrons. Although QCD predicts the Q^2 dependence of structure functions in accord with the DGLAP equations but they have limitations on absolute prediction of structure functions. DGLAP equations have the capability to predict only the evolution of structure functions with Q^2, once an initial distribution is given, they cannot predict the initial values from which the evolution starts. Further, due to its complicated mathematical structure, an exact analytic determination of the structure functions is currently out of reach and one needs to apply approximated methods to arrive on predictions from the DGLAP equation. Accordingly several approximate numerical as well as semi-analytical methods for the solution of DGLAP equation have been discussed considerably over the past years [3–10] for several QCD observable.

In our previous paper Ref. [10], in order to determine the GLS sum rule we obtained the small-x behaviour of xF_3 structure function by means of solving the DGLAP evolution equation using the Q^2 dependent Regge behaved ansatz, $xF_3(x,t) = Ax^{(1-bt)}$ as initial input with pQCD corrections upto next-next-to-leading order(NNLO). In this paper we extend the similar formalism to account for the F_2^{NS} structure function.

The Q^2 dependency of the F_2^{NS} structure function is governed by the DGLAP equation

$$\frac{\partial F_2^{NS}(x,Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{d\omega}{\omega} F_2^{NS}(\frac{x}{\omega},Q^2)P(\omega). \tag{1}$$

Here, $P(\omega)$ is the splitting function associated with the $F_2^{NS}(x,Q^2)$ structure function which is defined up to NNLO by [11]

$$P(\omega) = \frac{\alpha(Q^2)}{2\pi}P^0(\omega) + \Big(\frac{\alpha(Q^2)}{2\pi}\Big)^2 P^1(\omega) + \Big(\frac{\alpha(Q^2)}{2\pi}\Big)^3 P^2(\omega) \tag{2}$$

Here, $P^0(\omega)$, $P^1(\omega)$ and $P^2(\omega)$ are the corresponding LO, NLO and NNLO corrections to the splitting functions. The DGLAP equation is valid to all orders in the strong coupling constant $\frac{\alpha(Q^2)}{2\pi}$. One important point to be noted is that both the splitting function and strong coupling constant for F_2^{NS} in LO, NLO and NNLO has the form similar to xF_3 structure function and hence they are predicted with the same DGLAP equation. Thus in accord with Ref. [10], with the ansatz of $F_2^{NS}(x,t) = Ax^{(1-bt)}$ type as initial input the DGLAP evolution equations in LO, NLO and NNLO can be solved to have,

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{LO} P(x_0,t)dt\right].\left(\frac{x}{x_0}\right)^{(1-bt)}, \tag{3}$$

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} P(x_0,t)dt + \int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO} Q(x_0,t)dt\right]\left(\frac{x}{x_0}\right)^{(1-bt)} \tag{4}$$

and

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t_0)\exp\left[\int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} P(x_0,t)dt + \int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} Q(x_0,t)dt + \int_{t_0}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} R(x_0,t)dt\right]\left(\frac{x}{x_0}\right)^{(1-bt)} \quad (5)$$

respectively. Here

$$P(x,t) = \frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega}\left\{\frac{1+\omega^2}{\omega}\omega^{-(1-b_i^{NS}t)} - 2\right\}, \quad (6)$$

$$Q(x,t) = \int_x^1 \frac{d\omega}{\omega}P^{(1)}(\omega)\omega^{-(1-bt)}, \quad (7)$$

,

$$R(x,t) = \int_x^1 \frac{d\omega}{\omega}P^{(2)}(\omega)\omega^{-(1-bt)} \quad (8)$$

in which the two loop and three loop correction terms to the splitting functions for non-singlet structure functions are given by [11]

$$P^{(1)}(\omega) = C_F^2\left[P_F(\omega) - P_A(\omega) + \delta(1-\omega)\{\frac{3}{8} - \frac{1}{2}\pi^2 + \zeta(3) - 8\tilde{S}(\infty)\}\right] + \frac{1}{2}C_F C_A\left[P_G(\omega) + P_A(\omega) + \delta(1-\omega)\left\{\frac{17}{12} + \frac{11}{9}\pi^2 - \zeta(3) + 8\tilde{S}(\infty)\right\}\right] + C_F T_R N_F\left[P_{N_F}(\omega) - \delta(1-\omega)\left\{\frac{1}{6} + \frac{2}{9}\pi^2\right\}\right] \quad (9)$$

and

$$P^{(2)}(\omega) = N_F\left[-183.187D_0 - 173.927\delta(1-\omega) - \frac{5120}{81}L_1 - 197.0 + 381.1\omega + 72.94\omega^2 + 44.79\omega^3 - 1.497\omega L_0^3 - 56.66L_0L_1 - 152.6L_0 - \frac{2608}{81}L_0^2 - \frac{64}{27}L_0^3\right] + N_F^2\frac{64}{81}\left[-D_0 - \left(\frac{51}{16} + 3\zeta_3 - 5\zeta_2\right)\delta(1-\omega) + \frac{\omega}{1-\omega}L_0\left(\frac{3}{2}+5\right) + 1 + (1-\omega)\left(6 + \frac{11}{2}L_0 + \frac{3}{4}L_0^2\right)\right]. \quad (10)$$

with

$$P_F(\omega) = -2\frac{1+\omega^2}{1-\omega}\ln\omega\ln(1-\omega) - \left(\frac{3}{1-\omega}+2\omega\right)\ln\omega - \frac{1}{2}(1+\omega)\ln^2\omega \\ -5(1-\omega), \quad (11)$$

$$P_G(\omega) = \frac{1+\omega^2}{(1-\omega)_+}\left[\ln^2\omega + \frac{11}{3}\ln\omega + \frac{67}{9} - \frac{1}{3}\pi^2\right] + 2(1+\omega)\ln\omega \\ +\frac{40}{3}(1-\omega), \quad (12)$$

$$P_{N_F}(\omega) = \frac{2}{3}\left[\frac{1+\omega^2}{(1-\omega)_+}(-\ln\omega - \frac{5}{3}) - 2(1-\omega)\right], \quad (13)$$

$$P_A(\omega) = 2\frac{1+\omega^2}{1+\omega}\int_{\omega/(1+\omega)}^{1/(1+\omega)}\frac{dz}{z}\ln\frac{1-z}{z} + 2(1+\omega)\ln\omega + 4(1-\omega). \quad (14)$$

Here the following abbreviations are used,

$$D_0 = \frac{1}{(1-\omega)_+}, \quad L_1 = \ln(1-\omega), \quad L_0 = \ln\omega. \quad (15)$$

The results for $F_2^{NS}(x,Q^2)$ structure function in accord with eq. (3), (4) and (5) are depicted in Fig. 1 against Q^2 in comparison with NMC [12] data and NNPDF [13] parametrization results. Our results are evolved with respect to the input point $F(x_0,t_0) = 0.010348$ and with the best fitted value for the parameter $b = 0.118$. Here, our LO, NLO and NNLO results are represented by the dotted, dashed and solid curves respectively. The solid circles are used to represent the NMC data point and they are along with vertical upper and lower error bars for total uncertainties of statistical and systematic errors. As far this figure is concerned, we observe a very good consistency between theoretical and experimental as well as parametrization results within the kinematical region $x \leq 0.035$ and $Q^2 \leq 20GeV^2$ of our consideration, especially, if the NNLO results are concerned.

Thus we see that higher order pQCD corrections have a significant contribution towards the precise predictions of the structure functions. However recent analysis indicates that precise prediction of structure functions demand to incorporate several non-perturbative effects, in addition to pQCD corrections. There are several non-perturbative effects such as higher twist effects, nuclear corrections, target mass corrections(TMC) etc., to be included into the joint QCD analysis of structure functions and sum rules. However in accord with [14] the contribution due to TMC within the region of our consideration is neglected. In this paper we present an analysis of the NNLO results of F_2^{NS} structure function taking into account the Higher twist corrections.

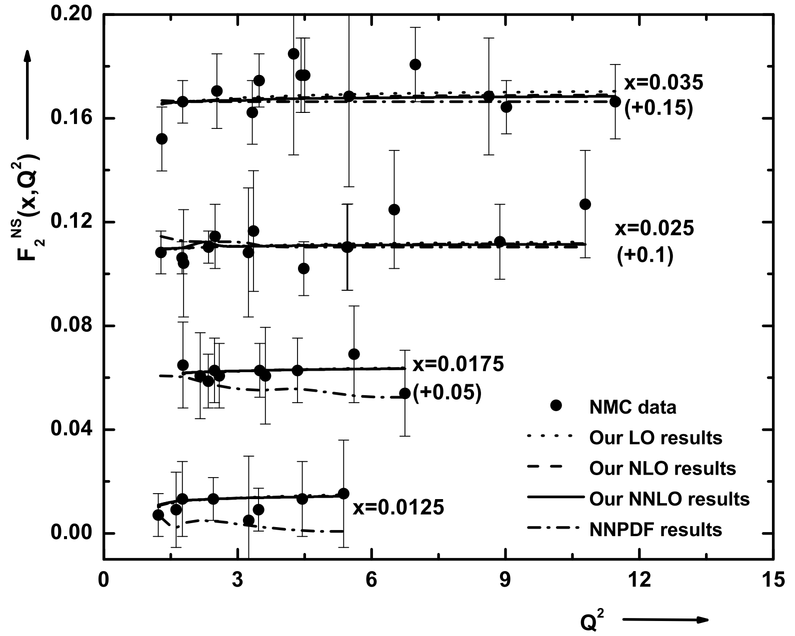

Fig. 1 Q^2(in the unit of GeV^2) evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (3)-(5) in comparison with NMC [12] and NNPDF [13]results. For clarity, the points are offset by the amount given in parenthesis. (Q^2's are taken in the unit of GeV^2).

2 Higher Twist Corrections on F_2^{NS} structure function

The behaviour of the deep inelastic structure functions can be analyzed with the perturbative QCD. A method used for this analysis is the operator product expansion method(OPE) [15]. The OPE is successful in describing the contributions from different quark-gluon operators to hadronic tensor and helps in ordering them according to their twist. In accord with OPE, the DIS structure functions and sum rules consist of two parts, the leading twist(LT) and the higher twist(HT) contributions:

$$F(x, Q^2) = F^{LT}(x, Q^2) + \frac{H_i(x, Q^2)}{Q^2}, \tag{16}$$

where i labels the type of the structure function ($F = F_2, F_3, g_1$). The leading twist term is associated with the single particle properties of quarks and gluons inside the nucleon and is responsible for the scaling of DIS structure function via perturbative QCD $\alpha_s(Q^2)$ corrections. The higher twist terms reflect instead the strength of multi-parton interactions (qq and qg). Since such interactions spoil factorization one has to consider their impact on the parton distribution functions extracted in the analysis of low-Q^2 data. Because of the non-perturbative origin it is difficult to quantify the magnitude and shape of the higher twist terms from first principles and current models can only provide a qualitative description for such contributions, which must then be determined phenomenologically from data.

The higher twist terms are governed by the terms contributing at different orders of $1/Q^2$:

$$\frac{H_i(x,Q^2)}{Q^2} = \frac{h_1(x)}{Q^2} + \frac{h_2(x)}{Q^4} + ..., \tag{17}$$

the leading term in this expansion is known as twist-two, the sub-leading ones twist-three, etcetera. The higher twist terms are suppressed by terms of order $1/Q^2$, $1/Q^4$..., respectively.

The currently available experimental data on deep inelastic structure functions covers a large kinematical regime with high precision measurements. This provides an interesting challenge for theoretical physics when it comes to describing this data in the low-Q^2 domain. pQCD predictions, even with higher order corrections up to NNLO and NNNLO observed to be not sufficient for a precise description of deep inelastic structure function data, which in turn reveals that the discrepancy among data and pQCD predictions are not primarily the sub-leading terms in powers of α_s, but corrections which are proportional to the reciprocal value of the photon virtuality Q^2, viz. higher-twist terms.

The usual approach in available analyses who attempt in extracting the leading twist PDFs is either to parametrize the higher twist contributions by a phenomenological form and fit the parameters to the experimental data [16, 17], or to extract the Q^2 dependence by fitting it in individual bins in x [18–22]. Such an approach effectively includes contributions from multiparton correlations along with other power corrections that are not yet part of the theoretical treatment of DIS at low Q^2. These include $O(1/Q2)$ contributions such as jet mass corrections [23] and soft gluon resummation [24], as well as contributions which are of higher order in α_s but whose logarithmic Q^2 behavior mimics terms $\propto \frac{1}{Q^2}$ at low virtuality [22, 25].

In the following we present a simple model in order to extract the higher twist contribution to F_2^{NS} structure function and comment on the phenomenological implications of our results.

In order to estimate the higher twist contribution to the F_2^{NS} structure function, we have performed an analysis based on a simple model. Here the first higher twist term is extracted and to do so we have parameterised the non-singlet structure functions as

$$\left(F_2^{NS}(x_i,Q^2)\right)^{data} = \left(F_2^{NS}(x_i,Q^2)\right)^{LT} + \frac{h_1(x_i)}{Q^2}. \tag{18}$$

Here leading twist(LT) term corresponds to the pQCD contribution to structure functions and the constants $h_1(x_i)$ (one per x - bin) parameterize the x dependence of higher twist contributions. For the leading twist term, we have utilised the results for the non-singlet structure functions obtained in our previous paper. Incorporating our results for non-singlet structure functions in NNLO as the LT terms we have extracted the difference, $\left(F_2^{NS}(x_i, Q^2)\right)^{data} - \left(F_2^{NS}(x_i, Q^2)\right)^{LT}$ from their corresponding experimental data and then fitted with $h_1(x_i)/Q^2$. From the best fitting values, we have determined the higher twist contribution terms h_i per x-bin. In this analysis we have performed our fitting analysis within the kinematical region $0.0125 \leq x \leq 0.5$ and $1 \leq Q^2 \leq 20GeV^2$.

The parametrization for the F_2^{NS} structure function incorporating higher twist contributions in terms of the parameter $h_1(x_i)$ is fitted to the NMC data for the x-bins $x_i = 0.0125, 0.0175, 0.025, 0.035$. Here we have used the NNLO results (5) for the term $(F_2^{NS}(x_i, Q^2))^{LT}$. Best fitted values of h_1 at different values of x for the F_2^{NS} structure functions are presented in Table 1 and Fig. 2 along with the $\frac{\chi^2}{d.o.f.}$ value.

x_i	h_1^{NNLO}
0.0125	-0.00397 ± 0.0025
0.0175	-0.00283 ± 0.0029
0.025	-0.0045 ± 0.0026
0.035	-0.0022 ± 0.0052
$\frac{\chi^2}{d.o.f.}$	0.85

Table 1 Higher Twist corrections to F_2^{NS} structure functions at NNLO.

In Fig. 2 we have presented the best fitting results of (18) for F_2^{NS} in comparison with NMC experimental data. Here both the NNLO results, with HT and without HT are shown. Significant higher twist contribution to F_2^{NS} structure function is observed in the low-x, low-Q^2 region. We observe that our expressions along with the HT corrections provide better description of NMC data than without HT within our kinematical region of consideration.

3 Conclusion

Some analytical expressions are obtained to describe small x behaviour of $F_2^{NS}(x, Q^2)$ structure function by means of solving DGLAP equations up to NNLO using a Regge behaved ansatz with Q^2 dependent intercept as the initial input. The solutions are analysed phenomenologically and observed that there is a good agreement between our results with those of other experimental measurements. Further the solutions are used to extract the higher twist contributions in accord with a simple model. Higher twist corrected results are also compared with the available experimental

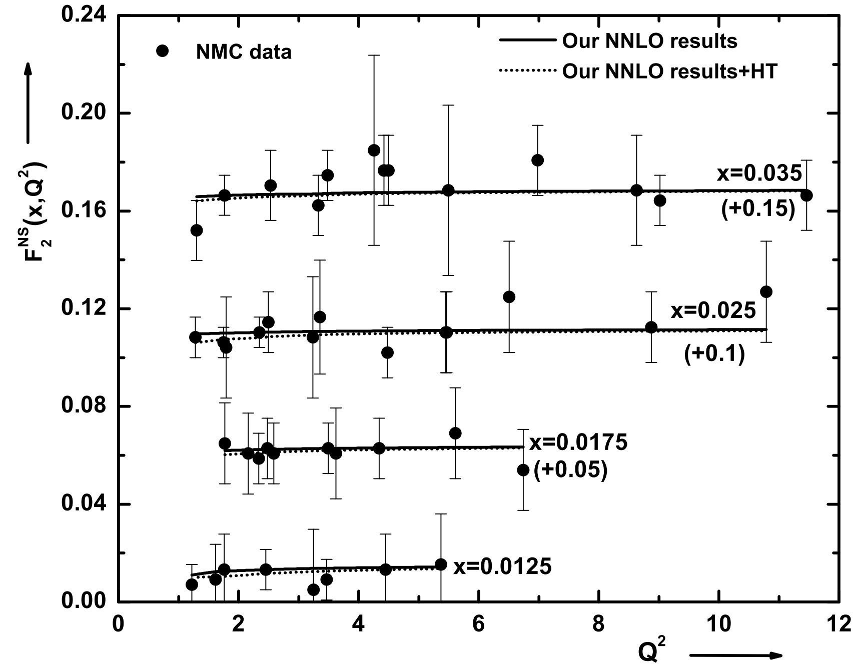

Fig. 2 Higher twist corrections to F_2^{NS} structure function at NNLO. (Q^2's are taken in the unit of GeV^2).

data. From the phenomenological analysis we have the following observations:

i. The Regge inspired ansatz $F_2^{NS} = Ax^{1-bt}$ in accord with DGLAP equations provides a very good description of the small-x behaviour of $F_2^{NS}(x, Q^2)$ structure function, which are consistent with other results taken from Ref. [12, 13], which signifies that the model is applicable in describing the small-x behaviour of $F_2^{NS}(x, Q^2)$ structure function although it being simple. Moreover, in this method we do not require the knowledge of initial distributions of structure functions at all values of x from 0 to 1. Here, we just require one input point at any fixed x and Q^2 and with respect to that point both the x and Q^2 evolution of structure functions can be obtained.

ii. Considering the NNLO solution of DGLAP evolution as the leading twist term we have estimated the higher twist contribution with a simple model. Inclusion of these higher twist terms to NNLO results provides a better description of the NMC [12] results.

Based on all these observation we conclude with that the simple but efficient Q^2 dependent Regge ansatz for $F_2^{NS}(x, Q^2)$ is capable of evolving successfully the $F_2^{NS}(x, Q^2)$ structure function in accord with DGLAP equation at small-x and along with the higher twist corrections provide well description of the experimental data and parametrization.

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