EJTP **13, No. 36** (2016) 1–12 *Electronic Journal of Theoretical Physics*

The GIMP Nature of Dark Matter

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Received 14 August 2016, Accepted 25 September 2015, Published 10 November 2016

Abstract: We conjecture that dark matter consists of purely gravitational field singularities of spacetime. These do not couple to any of the standard elementary particles via the gauge fields of strong, electromagnetic, or weak interactions, but are subject only to gravitational interactions. Thus, instead of searching in vane for *WIMPs* by means of ordinary particle-physics experiments, one should try to detect GIMPs (Gravitationally Interacting Massive Particles). Their purely gravitational nature explains their invisibility in ordinary elementary-particle experiments.

Keywords: Dark Matter; Dark Energy; Gravitational Fields; Gauge Fields; Weakly Interacting Massive Particles (WIMP)
PACS (2010): 98.80.Cq; 98.80. Hw; 04.20.Jb; 04.50+h

Almost hundred years ago, in 1919, Einstein published a remarkable paper entitled "Do Gravitational Fields Play a Significant Role in the Composition of Material Elementary Particles?" [1]. He thus asked this question long before any of the numerous elementary particles was discovered. After their discovery, physicists thought for a long time that the answer is negative. However, as we understand increasingly well the structure of the universe, the answer is rather an "almost yes". Moreover, if we restrict his title to the dark-matter part of the universe, which makes up about one third of the universe, the answer seems to be "yes".

Indeed, in 1933, Fritz Zwicky [2] plotted the orbital velocities of stars in the galaxy as a function of their distance from the center and encountered an unexpected surprise: the velocities do not decrease with distance in a way expected from the visible masses. This made him postulate the existence of *dark matter*. In fact, the observed velocity curves ask for large amounts of invisible matter in each galaxy. The presently best theoretical fits to the data [3] are shown in Figs. 1 and 2.

If a Friedmann model [4] is used to explain the evolution of the universe, one needs

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a large percentage of dark matter, roughly 27% of the mass energy of the universe. If dark matter is added to the so-called *dark energy*, which accounts for roughly 70% of the energy, one finds that the visible matter is practically negligible (see Fig. 3). This is the reason for ignoring visible atoms completely in the most extensive computer simulations of the evolution of cosmic structures [5], the so-called *Millennium Simulation*.

In Fig. 4 we show the decomposition of matter when the universe became first transparent to light. The largest chunk is that of dark matter. There are many speculations as to what particles it may consist of. It is the purpose of this note to give the simplest possible explanation [6]. We argue that it is not made of any standard elementary particles. Instead it consists of singular worldlines and worldsurfaces in the solutions of *Einstein's vacuum field equation* $G_{\mu\nu} = 0$. Their energy quanta appear as massive particles. If this is done à la Feynman via functional integrals, the Einstein-Hilbert action which governs spacetime also governs the fluctuations of these singular configurations of worldlines and worldsurfaces.

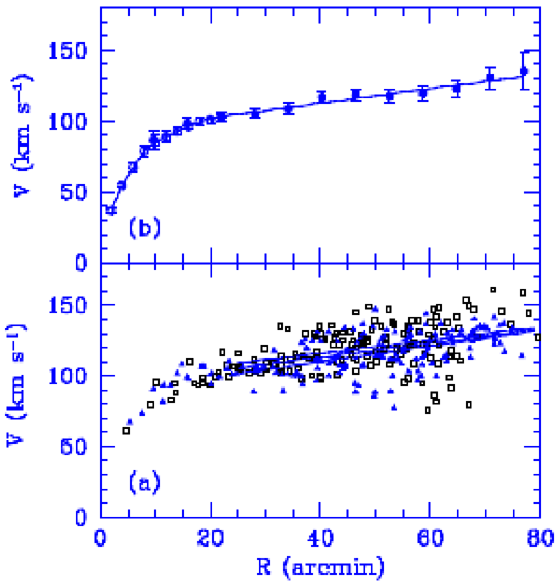

Figure 1 (a) Velocity of stars in our Galaxy as a function of the distance from the center. Filled triangles refer to the northern half, open squares to the southern half of the galaxy. Straight lines are the best linear fits for $R \geq 25$ arcmin to all data shown, once to the northern and once to the southern data set. (b) The straight line is the best linear fit to all unbinned data for $R > 25$ arcmin for the M33 rotation curve: Filled circles are from data displayed in (a), open circles follow Newton's theory, for comparison.

Let us remember that all static electric fields in nature may be considered as originating from nontrivial solutions of homogeneous Poisson equation for the electric potential $\phi(x)$ as a function of $x = (t, \mathbf{x})$:

$$\Delta\phi(x) \equiv \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}\, \phi(x) = 0. \tag{1}$$

The simplest of them has the form e/r, where $r = |\mathbf{x}|$. It is attributed to pointlike electric charges, whose size e can be extracted from the pole strength of the singularity of the electric field $\mathbf{E}$ which points radially outward and has a strength e/r^2. This becomes

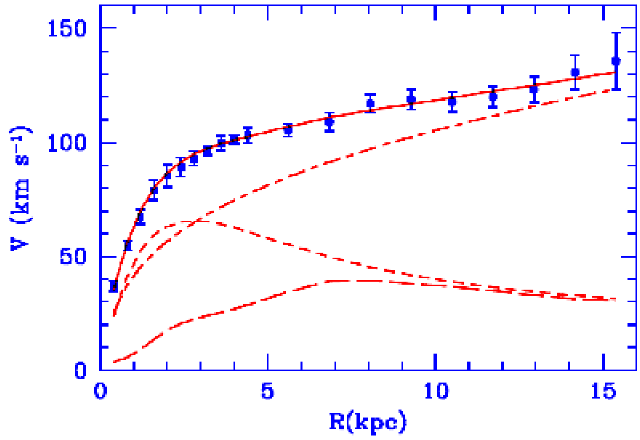

Figure 2 Velocity curve (points) of the galaxy M33 and comparison with a best fit model calculation (continuous line). Also shown is the halo contribution (dashed-dotted line), the stellar disk (short dashed line), and the gas contribution (long dashed line).

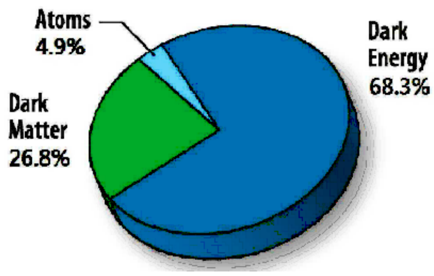

Figure 3 Various types of energy in the universe, (Credit NASA/WMAP Science Team).

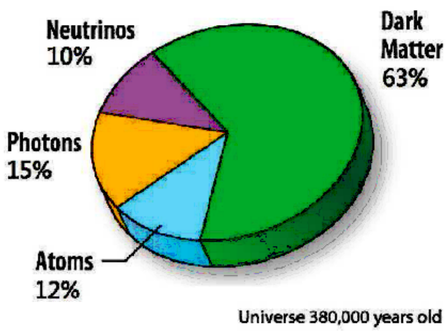

Figure 4 Various gravitational sources when the Universe became transparent, Credit: same as previous figure).

explicitly visible by performing an area integral over the **E** field around the singularity. We apply the famous Gauss integral theorem,

$$\int_V d^3x\, \boldsymbol{\nabla} \cdot \mathbf{E} = \int_A d^2\mathbf{a} \cdot \mathbf{E}, \tag{2}$$

and use the fact that the area integral is equal to the volume integral over $\boldsymbol{\nabla} \cdot \mathbf{E} = -\Delta\phi(x)$. Inserting this we see that the field which solves the homogeneous Poisson equation can have a nonzero volume integral (2): $\int_V d^3x\, \Delta\phi(x) = -4\pi e$. This fact can be formulated in a local way with the help of a Dirac delta-function $\delta^{(3)}(\mathbf{x})$ as

$$\Delta\phi(x) = -4\pi e\, \delta^{(3)}(\mathbf{x}). \tag{3}$$

In the sequel, it will be useful to re-express the Gauss theorem (2) in a one-dimensional form as

$$\int^R dr\partial_r r^2 E_r(r) = R^2 E_r(R). \tag{4}$$

This is valid for all R, in particular for small R, where $E_r(R) = 4\pi e/R^2$. Then we can express the combination of Eqs. (2) and (4) in radial form as

$$\int d^3x\, \boldsymbol{\nabla}\cdot\mathbf{E} = -4\pi\int^R dr\, r^2\boldsymbol{\nabla}\cdot\boldsymbol{\nabla} e/r = 4\pi e\int^R dr\, \partial_r\delta(r). \tag{5}$$

This implies that we can find the electric charge e from the integral over the radial Poisson equation:

$$e\int^R dr\, \partial_r\delta(r) = e. \tag{6}$$

The integrand displays once more the homogeneous Maxwell equation in the presence of a pointlike singularity $\boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi e\, \delta^{(3)}(\mathbf{x})$ in a radial form.

For gravitational objects, the situation is quite similar. The Einstein equation in the vacuum, $G_{\mu\nu} = 0$, possesses simple nontrivial solutions in the form of the Schwarzschild metric defined by

$$ds^2 = B(r)c^2dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{7}$$

with $B(r) = 1 - r_S/r$, $A(r) = 1/B(r)$, where $r_S \equiv 2G_{\mathrm{N}}M/c^2$ is the Schwarzschild radius and G_{N} Newton's gravitational constant. Its Einstein tensor has the component

$$G_t{}^t = A'/A^2r - (1-A)/Ar^2, \tag{8}$$

which vanishes in the vacuum.

Let us now allow pointlike singularity in spacetime and calculate the volume integral $\int_V d^3x\,\sqrt{-g}G_t{}^t$. Inserting (8) we find $\int_V d^3x\sqrt{B/A}[A'/Ar-(1-A)/r^2]$. If this is evaluated with the gravitational singularities in the same way as in the electromagnetic case in Eqs. (2)–(5), we find that it is equal to $\int^R dr\partial_r(r - r/A) = (2G_{\mathrm{N}}/c)M\int^R dr\partial_r\delta(r) = (2G_{\mathrm{N}}/c)M$. Thus we obtain the nonzero integral

$$\int_V d^3x\sqrt{-g}\, G_t{}^t = \kappa c M, \tag{9}$$

where κ is defined in terms of the Planck length l_{P}, as

$$\kappa \equiv 8\pi l_{\mathrm{P}}^2/\hbar = 8\pi G_{\mathrm{N}}/c^3. \tag{10}$$

From (9) we identify the mass of the object as being M.

If the mass point moves through spacetime along a trajectory parametrized by $\tau \equiv \int dt\sqrt{(dx/dt)^2}/c^2$ as $x^\mu(\tau)$, it has an energy-momentum tensor concentrated on a worldline

$$\overset{\mathrm{m}}{T}{}^{\mu\nu}(x) = Mc\int_{-\infty}^{\infty} d\tau\, \dot{x}^\mu(\tau)\dot{x}^\nu(\tau)\delta^{(4)}(x - x(\tau)), \tag{11}$$

where a dot denotes the τ-derivative.

We may integrate the associated solution of the homogeneous Einstein equation $G_{\mu\nu} = 0$ over spacetime, and find that its Einstein-Hilbert action

$$\mathcal{A}_{\rm EH} = -\frac{1}{2\kappa}\int d^4x\sqrt{-g}R \tag{12}$$

vanishes. The situation is quite different, however, if we allow spacetime to be perforated by singularities. For line-like singularities, the integral (12) in the Einstein-Hilbert action looks as if it contains a δ-function-like source obeying $G^{\mu\nu} = \kappa \overset{\rm m}{T}_{\mu}{}^{\mu}(x)$. If we insert here the equation of motion of a point particle $\dot{x}^2(\tau) = 1$, we arrive from the field equation $R = -G = -\kappa \overset{\rm m}{T}_{\mu}{}^{\mu}(x) = -Mc\int d\tau\, \dot{x}(\tau)^2\delta^{(4)}(x - x(\tau))$ at an action which is proportional to the classical action of a point-like particle:

$$\mathcal{A}_{\rm EH}^{\rm worldline} \propto -Mc\int d\tau. \tag{13}$$

A slight modification of this, which is the same at the classical level but different for fluctuating orbits, describes also the quantum physics of a spin-0 particle via a path integral over all orbits[2]. Thus Einstein's action for a singular world line in spacetime can be used to define also the quantum physics of a spin-0 point particle.

In addition to pointlike singularities, the homogeneous Einstein equation will possess also surface-like singularities in spacetime. These may be parametrized by $x^{\mu}(\sigma,\tau)$, and their energy-momentum tensor has the form

$$T^{\mu\nu}(x) \propto \int_{-\infty}^{\infty} d\sigma d\tau (\dot{x}^{\mu}\dot{x}^{\nu} - x'^{\mu}x'^{\nu})\delta^{(4)}(x - x(\sigma,\tau)), \tag{14}$$

where a prime denotes a σ-derivative. In the associated vanishing Einstein tensor, the δ-function on the surface manifests itself in the nonzero spacetime integral [9]

$$\int d^4x\sqrt{-g}\, G_{\mu}{}^{\mu} \propto \int d^2a \equiv \int_A d\sigma d\tau\sqrt{(\dot{x}x')^2 - \dot{x}'^2 x'^2}. \tag{15}$$

By analogy with the line-like case we obtain, for such a singular field configuration, that the Einstein-Hilbert action (12) gives no longer zero, but reduces to a worldsurface integral

$$\mathcal{A}_{\rm EH}^{\rm worldsurface} \propto -\frac{1}{2\kappa}\int_A d^2a = -\frac{\hbar}{16\pi l_{\rm P}^2}\int_A d^2a. \tag{16}$$

The prefactor on the right-hand side has been expressed in terms of the Planck length $l_{\rm P}$.

The important observation is now that, apart from a numerical proportionality factor of order unity, the right-hand side of Eq. (16) is precisely the Nambu-Goto action [10, 11] of a bosonic closed string in the true physical spacetime dimension four

$$\mathcal{A}_{\rm NG} = -\frac{\hbar}{2\pi l_{\rm s}^2}\int_A d^2a. \tag{17}$$

[2] See the discussion in Section 19.1 of the textbook [8], in particular Eq. (19.10).

In this formula, l_s is the so-called string length parameter l_s. It measures the string tension, and corresponds in spacetime to a certain surface tension of the worldsurface. This length scale can be related to the rather universal slope parameter $\alpha' = dl/dm^2$ of the Regge-trajectories [12]. These are found in plots of the angular momenta against the squares of the meson masses m^2. The relation between the string length parameter l_s and the slope parameter of the Regge theory is $l_s = \hbar c\sqrt{\alpha'}$. Note that now there is no extra mass parameter M, this being in contrast to the situation in world lines. The masses of elementary particles come from the eigenmodes of string vibrations.

The original string model was proposed to describe color-electric flux tubes and their Regge trajectories whose slopes α' lie around 1 GeV^{-2}. However, since the tubes are really fat objects, as fat as pions, only very long flux tubes are approximately line-like. Short tubes degenerate into spherical "MIT-bags" [13]. The flux-tube role of strings was therefore abandoned, and the action (17) was re-interpreted in a completely different fashion, as describing the fundamental particles of nature, assuming l_S to be of the order of l_P. Then the spin-2 particles of (17) would interact like gravitons and define Quantum Gravity. However, the ensuing "new string theory" [14] has been criticized by many authors [15]. One of its most embarrassing failures is that it has not produced any experimentally observable results. The particle spectra of its solutions have not matched the existing particle spectra.

The arisal of the string action proposed here has a chance of curing this problem. If "strings" describe "dark matter", there would be no need to reproduce other observed particle spectra. Instead, their celebrated virtue of extracting the interaction between gravitons from the properties of their spin-2 quanta can be used to fix the proportionality factor between the Einstein action (16) and the string action (17).

It must be kept in mind that, just as $-Mc\int d\tau$ had to be modified for fluctuating paths, also the Nambu-Goto action (17) needs a modification for fluctuating worldsurfaces. That was found by Polyakov [16] when studying the consequences of conformal symmetry. He replaced (17) by a new action that is equal to (17) at the classical level but contains, in $D \neq 26$ dimensions, another spin-0 field with a Liouville action.

Since the singularities of Einstein's fields possess only gravitational interactions, their identification with "dark matter" seems very natural. All visible matter consists of singular solutions of the Maxwell equations as well as of the field equations of the standard model. A grand-canonical ensemble of these and the singular solutions without matter sources explain the most important part of all matter in the Friedmann model of cosmological evolution.

But the main contribution to the energy comes from the above singularities of Einstein's equation. Soon after the universe was created, the temperature was so high that the configurational entropy of the surfaces overwhelmed completely the impeding Boltzmann factors. Spacetime was filled with these surfaces in the same way as superfluid helium is filled with worldsurfaces of vortex lines.

Vortex lines in superfluid helium are known to attract material particles such as frozen helium. This phenomenon provides us with an important tool to visualize vortex lines and tangles thereof [17]. In spacetime, we expect that any stable neutral particle will be attracted by its singularities. These could be the elusive objects which many experimental particle physicists have been looking for in elaborate searches of *WIMPs* [18]. If the conjecture of this paper is correct, they should rather be serching for *GIMPs* (Gravitationally Interacting Massive Particles).

In helium above the temperature of the superfluid phase transition, these lie so densely packed that the superfluid behaves like a normal fluid [19, 20]. The Einstein-Hilbert action of such a singularity-filled turbulent geometry behaves like the action of a grand-canonical ensemble of world surfaces of a bosonic closed-string model.

Note once more that here these are two-dimensional objects living in four spacetime dimensions. There is definitely a need to understand their spectrum by studying the associated Polyakov action. To be applicable in four physical dimensions one should not circumvent the accompanying Liouville field. Or one must find a way to take into account the fluctuations of the gravitational field around the field near the singular surface.

It should be realized that, in the immediate neighborhood of line- and surface-like singularities, the curvature will be so high that Einstein's linear approximation $-(1/2\kappa)R$ to the Lagrangian must break down. It will have to be corrected by some nonlinear function of R. This starts out like Einstein's, but continues differently, similar to the action that was suggested some time ago in 2002 [21]. Since then, many modifications of this idea have been investigated [22].

After the big bang, the universe expanded and cooled down, so that large singular surfaces shrank by emitting gravitational radiation. Their density decreased, and some phase transition made the cosmos homogeneous and isotropic on the large scale. But it remained filled with gravitational radiation and small singular surfaces that had shrunk until their sizes reached the levels stabilized by quantum physics. The statistical mechanics of this cosmos can be described by analogy with a spacetime filled with superfluid helium. The specific heat of that is governed by the zero-mass phonons and by rotons [23]. Recall that in this way Landau discovered the fundamental excitations called rotons, whose existence was deduced by him from the temperature behavior of the specific heat. In the universe, the role of rotons is played by the smallest surface-like singularities of the homogeneous Einstein equation. They must be there to satisfy the cosmological requirement of dark matter.

The situation can also be illustrated by a further analogy with many-body systems. The defects in a crystal, whose "atoms" have a lattice spacing l_{P}, simulate precisely the mathematics of a Riemann-Cartan spacetime, in which disclinations and dislocations define curvature and torsion [19, 20, 24]. Thus we may imagine a model of the universe as a "floppy world crystal" [25], a liquid-crystal-like phase [26] in which a first melting transition has led to correct gravitational $1/r$-interactions between disclinations. The initial hot universe was filled with defects and thus it was in the "world-liquid"-phase of the "world crystal". After cooling down to the present liquid-crystal state, there remained plenty of residual defects around, which form our *dark matter* [27, 28].

In the process of cooling down over a long time, the dark matter fraction can have decreased so that the expansion of the universe could have become faster and faster over the millennia. Thus it is well possible that the baby universe had so much dark-matter content that it practically did not expand at all for a long time. It would have been closer to the *steady-state universe* advocated in 1931 by Einstein and in the 1940's by Hoyle, Bondi, and Gold [29]. This would relieve us from the absurd-sounding assumption that the entire universe came once out of a tiny beginning in which all matter of the world was compressed into a sphere of the order of a Planck radius.

We know that the cosmos is now filled with a cosmic microwave background (CMB) of photons of roughly 2.725 Kelvin, the remnants of the big bang. They contribute a constant $\Omega_{\rm rad}h^2 = (2.47 \pm 0.01) \times 10^{-5}$ to the Friedmann equation of motion, where $h = 0.72 \pm 0.03$ is the Hubble parameter, defined in terms of the Hubble constant H by $h \equiv H/(100\ \text{km/Mpc sec})$. The symbol Ω denotes the energy density divided by the so-called critical density $\rho_{\rm c} \equiv 3H^2/8\pi G_{\rm N} = 1.88 \times 10^{-26} h^2\,\text{kg/m}^3$ [30]. The baryon density contributes $\Omega_{\rm rad}h^2 = 0.0227 \pm 0.0006$, or 720 times as much, whereas the dark matter contributes $\Omega_{\rm dark}h^2 = 0.104 \pm 0.006$, or 4210 as much. Let us assume, for a moment, that all massive strings are frozen out and that only the subsequently emitted gravitons form a thermal background [31]. Then the energy of massless states is proportional to T^4, and the temperature of this background would be $T_{\rm DMB} \approx 4210^{1/4} \approx 8T_{\rm CMB} \approx 22$K. We expect the presence of other singular solutions of Einstein's equation to change this result.

There is an alternative way of deriving the above-described properties of the fluctuating singular surfaces of Einstein's theory. One may rewrite Einstein's theory as a gauge theory [19, 20], and formulate it on a spacetime lattice [32]. Then the singular surfaces are built explicitly from plaquettes, as in lattice gauge theories of asymptotically-free nonabelian gauge theories [33]. In the abelian case, the surfaces are composed as shown in Ref. [34]. For the nonabelian case, see [35]. An equivalent derivation could also be given in the framework of *loop gravity* [36]. But that would require a separate study.

Summarizing we have seen that the Einstein-Hilbert action governs not only the classical physics of gravitational fields but also, via the fluctuations of its line- and surface-like singularities, the quantum physics of dark matter. A string-like action, derived from it for the fluctuating surface-like singularities, contains interacting quanta of spin-2 which define a finite Quantum Gravity.

References

[1] A. Einstein, Academy of Science meeting of 1919 in Berlin
`echo.mpiwg-berlin.mpg.de/ECHOdocuView?url=/permanent/echo/einstein/sitzungsberichte/69YV118A/index.meta` .
See also the paper by the physics historian
T. Sauer, Ann. Phys. (Berlin) **524**, A135A138 (2012).

[2] F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln". Helv. Phys. Acta. **6**, 110 (1933);
See also his textbook *Morphological Astronomy*, Springer, Berlin, 1957.
Note that Zwicky triggered the launching of the satellite 'Artificial Planet No. Zero'. It was 12 days after the Sputnik went in orbit in October 1957. In contrast to Sputnik,

his satellite remained indefinitely in orbit, whereas Sputnik eventually fall back to earth.

[3] E. Corbelli and P. Salucci, R. Astro. Soc. **311**, 441 (2000) (arxiv:9909252v1).
See also the improved data in
E. Corbelli, D. Thilker, S. Zibetti, C. Giovanardi, and P. Salucci, A&A (Astronomy and Astrophysics) **572**, 23 (2014) (arxiv.org:1409.2665v2).

[4] A discussion of the Friedmann universe is contained in Subsection 19.2.3 of the textbook [8].

[5] V. Springel, S.D.M. White, A. Jenkins, C.S. Frenk, N. Yoshida, Liang Gao, J. Navarro, R. Thacker, D. Croton, J. Helly, J.A. Peacock, S. Cole, P. Thomas, H. Couchman, A. Evrard, J. Colberg, F. Pearce, Nature **435**, 629 (2005).

[6] H. Kleinert, EJTP **8**, 27 (2011), (`http://www.ejtp.com/articles/ejtpv9i26p27.pdf`). See also Chapter 31 in [7].

[7] H. Kleinert, *Particles and Quantum Fields*, World Scientific, Singapore, 2016 (klnrt.de/b6).

[8] H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, 5th ed., World Scientific, 2009, p. 1–1579 (`http://klnrt.de/b5`).

[9] For this take the delta function on the surface as defined by
P.A.M. Dirac, Phys. Rev. **74**, 817 (1948) [see his Eq. (15)],
and make use of the distributional form of Gauss's integral theorem as formulated in
H. Kleinert, Int. J. Mod. Phys. A **7**, 4693 (1992) (`http://klnrt.de/203.pdf`),
or in the textbook [20] on p. 253.

[10] Y. Nambu, in Proc. Int. Conf. on Symmetries and Quark Models, Wayne State University 1969 (Gordon and Breach 1970) p. 269.

[11] T. Goto, Progr. Theor. Phys. **46**, 1560 (1971).

[12] P.D.B. Collins, *An Introduction to Regge Theory and High-Energy Physics*, Cambridge University Press, Cambridge, U.K., 1977.

[13] L. Vepstas and A.D. Jackson, Physics Reports **187**, 109 (1990).

[14] M. Green, J. Schwarz, and E. Witten, *Superstring Theory*, Cambridge University Press, Cambridge, U.K., 1987.

[15] See the list of critics in the Wikipedia article on "string theory". In particular
B. Schroer, *String theory deconstructed*, (`arXiv:hep-th/0611132`).

[16] A.M. Polyakov, *Gauge Fields and Strings*, Harwood Academic Publishers, New York, 1987.

[17] D.R. Poole, C.F.Barenghi, Y.A.Sergeev, and W.F. Vinen, Phys. Rev. B **71**, 064514 (2005);
R.E. Packard, Physica **109&110B**, 1474 (1982).

[18] T.J. Sumner, *Experimental Searches for Dark Matter*,
(http://relativity.livingreviews.org/Articles/lrr-2002-4);
LUX collaboration, (arXiv:1605.03844, arXiv:1602.03489,
arXiv:1512.03506).

[19] H. Kleinert, *Gauge Fields in Condensed Matter*, Vol. I, *Superflow and Vortex Lines*, World Scientific, Singapore, 1989, pp. 1–744 (`http://klnrt.de/b1`).

[20] H. Kleinert, *Multivalued Fields*, World Scientific, Singapore, 2008, pp. 1–497 (`http://klnrt.de/b11`).

[21] H. Kleinert and H.-J. Schmidt, Gen. Rel. Grav. **34**, 1295 (2002) (`klnrt.de/311`).

[22] See the review by
T.P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010) (arxiv:0805.1726).

[23] S. Balibar, *Rotons, Superfluidity, and Helium Crystals*, AIP lecture 2016 (`http://www.lps.ens.fr/balibar/LT24.pdf`).

[24] B.A. Bilby, R. Bullough, and E. Smith, Proc. Roy. Soc. London, A **231**, 263 (1955);
K. Kondo, in *Proceedings of the II Japan National Congress on Applied Mechanics*, Tokyo, 1952, publ. in *RAAG Memoirs of the Unified Study of Basic Problems in Engineering and Science by Means of Geometry* , Vol. 3, 148, ed. K. Kondo, Gakujutsu Bunken Fukyu-Kai, 1962.

[25] H. Kleinert, Ann. d. Physik, **44**, 117 (1987) (`http://klnrt.de/172`).

[26] H. Kleinert and J. Zaanen, Phys. Lett. A **324**, 361 (2004) (`http://klnrt.de/346`).

[27] Note the parallels with the work of Kerson Huang and collaborators in
K. Huang, H.-B. Low, R.-S. Tung, (arXiv:1106.5282v2), (aXiv:1106.5283v2). However, their turbulent "baby universe" is filled with tangles of vortex lines of some scalar field theory, whereas our spacetime contains only singularities of Einstein's homogeneous field equation. A bridge may be found by recalling that the textbook [19] explains how tangles of line-like defects can be described by a complex *disorder field theory*, whose Feynman diagrams are direct pictures of the worldlines. Thus, if Huang et al. would interpret their scalar field as a disorder field of the purely geometric objects of the theory presented here, the parallels would be closer. Note that in two papers written with K. Halperin [28], Huang manages to make his scalar field theory asymptotically free in the ultraviolet regime (though at the unpleasant cost of a sharp cutoff introducing forces of infinitely short range). This property allows him to deduce an effective *dark energy* in the baby universe. With our purely geometric tangles, such an effect may be reached using a lattice gauge formulation of Einstein's theory (see the textbooks [19] and [20]).

[28] K. Halpern and K. Huang, Phys. Rev. Lett., **74**, 3526 (1995); Phys. Rev. **53**, 3252 (1996).

[29] See the article `https://de.wikipedia.org/wiki/Steady-State-Theory`, and the papers
C. O'Raifeartaigh, B. McCann, W. Nahm, S. Mitton, (`arXiv:1402.0132`);
C. O'Raifeartaigh, S. Mitton, (`arXiv:1506.01651`).

[30] K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[31] We do this although the weakness of gravitational interactions may be an obstacle to thermal equilibration. See
S. Khlebnikov and I. Tkachev, Phys. Rev. D **56**, 653 (1997);
J. Garcia-Bellido, D.G. Figueroa, A. Sastre, Phys. Rev. D **77**, 043517 (2008) (`arXiv:0707.0839`).

[32] She-Sheng Xue, Phys. Lett. B **682**, 300 (2009); Phys. Rev. D **82**, 064039 (2010).

[33] K. Wilson, Physical Review D **10**, 2445 (1974).

[34] H. Kleinert and W. Miller, Phys. Rev. D **38**, 1239 (1988).

[35] J.M. Drouffe and C. Itzykson, Phys. Rep. **38**, 133 (1975).

[36] A. Ashtekar, Phys. Rev. Lett. **57**, 2244 (1986);
C. Rovelli, *Quantum Gravity*, Cambridge University Press (2004)
(http://www.cpt.univ-mrs.fr/∼rovelli/book.pdf);
L. Smolin, *Three Roads to Quantum Gravity*, Basic Books, London, 2003.