EJTP **13, No. 35** (2016) 73–78 *Electronic Journal of Theoretical Physics*

Feynman Amplitude for Dirac and Majorana Neutrinos

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Received 17 August 2015, Accepted 5 January 2016, Published 25 May 2016

Abstract: Theoretically, neutrino can be assigned as a Dirac or a Majorana particle. Even though the experiments that have been performed so far, i.e. neutrinoless double beta decay to proof the nature of neutrino as a Majorana particle, gave negative result, we still have no strong argument to put neutrino as a Dirac particle . In this talk, we study and evaluate the Feynman amplitude for both Dirac and Majorana neutrinos for neutrino-electron elastic scattering and hoping that the scattering technique can be used as an alternative method to decide the neutrino nature whether neutrino is a Dirac or a Majorana particle. The results show that it is possible to distinguish neutrino nature whether neutrino is a Dirac or a Majorana particle from its own Feynman amplitude for low energy process for both charged and neutral current interactions. It is also apparent that the Feynman amplitude is different for neutrino-electron elastic scattering for charged and neutral current interactions.

Keywords: Feynman Amplitude; Charged Current Interaction; Neutral Current Interaction; Neutrino-electron Elastic Scattering
PACS (2010): 12.15.-y; 12.15.Ji; 13.85.Dz

1 Introduction

One of the long standing unsolved problem in neutrino physics is the question about the nature of neutrino whether it Dirac or Majorana particle. Theoretically, the nature of neutrino can be Dirac or Majorana particle. If neutrino to be Majorana particle, then the neutrinoless double beta decay should be detected in experiments that have been performed so far. But, there is no neutrinoless double beta decay experiment with positive result and for review of the neutrinoless double beta decay experiments can be

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read in Ref.[1]. To conclude neutrino is a Dirac particle, we should show that neutrino and its anti-neutrino have different helicity. As we know so far from experiments that we have only observed left-handed neutrino in nature. It is the motivation of this study to look another technique or method for probing the neutrino properties especially the type of neutrino particle whether it Dirac or Majorana. Zralek [2] suggested that the neutrino chirality and helicity can be used to distinguish whether neutrino is Dirac or Majorana particle by comparing Feynman amplitude (M) for neutrino process: $\nu_i \to \nu_j + \gamma$. Another technique to distinguish the nature of neutrino whether neutrino is Dirac or Majorana particle is the neutrinos with two-particle interferometry suggested by Gutierrez [3].

As we know, there are three kind of tools in physics that can provide us some information about the matter and its interactions in nature, i.e. scattering, decay, and bound state. The concept of scattering is widely used to obtain and to test some parameters in physics especially in nuclear and particle physics. From the scattering experiment we do not only obtain the information of the incident and scattered beam but also about the structure of target and interaction of the particle beam and target.

One of the important parameter in the scattering that can be used to study the interaction of the incident particle (or scattered) beams and target is the differential cross-section ($\frac{d\sigma}{d\Omega}$). In particle physics, the differential cross-section depend on the square of absolute value of Feynman amplitude which is called transition probability density. It is well known that Majorana neutrinos have a pure axial neutral current interaction while Dirac neutrinos have the standard vector-axial interaction. In spite of this crucial difference, usually Dirac neutrino processes differ from Majorana processes by a term proportional to the neutrino mass, resulting in almost unmeasurable observations of this difference [4].

In this talk, we study the Feynman amplitude for both Dirac and Majorana neutrinos that undergo elastic scattering with electron in center of mass system. In section 2 we derive Feynman amplitude in center of mass system for electron-neutrino ($\nu_e - e$) elastic scattering by considering charged and neutral current interaction for both Dirac and Majorana neutrinos and discuss some phenomenological implications. The section 3 is devoted for conclusions.

2 Feynman amplitude for neutrino-electron elastic scattering

The scattering process is very important subject both experimentally and theoretically for investigating the properties of fundamental particles and its interactions. One of the main parameter in particle scattering is the transition probability which is defined as the absolute square of Feynman amplitude that used to determine the differential cross-section of particle scattering. For example, the differential cross-section ($d\sigma/d\Omega$) for the particle $a + b \to 1 + 2$ that undergo scattering is given by:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} \left| M_{fi} \right|^2 , \tag{1}$$

where $p_a = -p_b = p_i^*$, $p_1 = -p_2 = p_f^*$, $s = (E_a + E_b)^2$, and M_{fi} is the Feynman amplitude.

One of the scattering processes which is very important because it can give us an understanding of the neutrino interactions with other fundamental particles is the neutrino scattering, i.e. $\nu_\mu + e^- \to \nu_e + \mu^-$. Theoretical studies of QED corrections to the scattering process: $\nu_e + e^- \to \nu_e + e^-$ were studied by Lee and Sirlin using the effective four fermion $V - A$ Lagrangian [5], and shortly afterward Ram [6] extended their calculations by including hard photon emission. Passera [7] investigated the contributions of the $O(\alpha)$ QED corrections to the spectrum energy of recoil electron for the process: $\nu_l + e^- \to \nu_e + l^- + \gamma$, where γ indicate the possible emission of photon and l^- is charge lepton: e^-, μ^-, τ^-. The cross-section for elastic scattering: $\nu_l + e^- \to \nu_e + l^-$ in the Born approximation and exactly fixed polarization states of the target and final states of the electrons, discussing their sensitivity on the incident anti-neutrino or neutrino flavor were investigated by Minkowski and Passera [8]. From the experimental side, the measurement of elastic scattering cross-section for: $\nu_\mu + e^- \to \nu_e + \mu^-$ were performed by Heisterberg *et al.* [9], Bergsma *et al.* [10], and Baker *et al.* [11].

To formulate the Feynman amplitude for neutrino-electron elastic scattering, first we consider the Feynman diagram for the electron-neutrino $(\nu - e)$ elastic scattering for charged and neutral current interactions as shown in Fig. 1. According to the Interme-

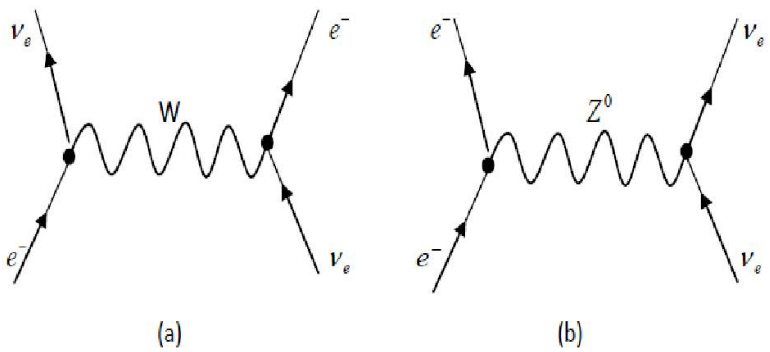

Fig. 1 Neutrino electron elastic scattering (a) charged current and (b) neutral current

diate Vector Boson (IVB) theory in the scheme of the Standard Model Particle Physics for electro-weak interactions, one should use the following Feynman rules:

- To each approriate vertex, one should use the factor:

$$(-ig/2\sqrt{2})\gamma^\alpha(1-\gamma_5), \tag{2}$$

for charged current interaction $(l - W - \nu$ vertex) and for neutral current:

$$\frac{-ig}{4\cos\theta_w}\gamma^\alpha(1-\gamma_5), \tag{3}$$

for $\nu - Z - \nu$ vertex, and:

$$\frac{ig}{4\cos\theta_w}\gamma^\alpha(1-4\sin^2\theta_w-\gamma_5, \tag{4}$$

for $l - Z - l$ vertex, where g is coupling constant, l is lepton, ν is neutrino, θ_w is the Weinberg angle, and γ^α is the Dirac matrices.

- To every propagator with mass M and spin-1, the factor to be used read:

$$-i(g_{\alpha\beta} - q_\alpha q_\beta)/(q^2 - M^2c^2), \tag{5}$$

- An incident (incoming) particle is denoted by $u(i)$ and the outgoing (scattered) particle is denoted by $\bar{u}(i)$.

Using the Feynman rules to the Feynman diagrams in Fig. 1, and remembering that Dirac particle is different to its own antiparticle ($\nu \neq \bar{\nu}$) whereas Majorana particle is the same with its own antiparticle ($\nu = \bar{\nu}$), then we have Feynman amplitude for the Dirac neutrino :

$$M_{cc}^D = \bar{u}(e)\left[\frac{-ig}{2\sqrt{2}}\gamma^\alpha(1-\gamma_5)\right]u(\nu_e)\left[\frac{-i(g_{\alpha\beta} - q_\alpha q_\beta/M_w^2)}{q^2 - M_w^2}\right]\bar{u}(\nu_e)\left[\frac{-ig}{2\sqrt{2}}\gamma^\beta(1-\gamma_5)\right]u(e)), \tag{6}$$

for charged current interaction of neutrino-electron elastic scattering, and

$$M_{nc}^D = \bar{u}(e)\left[\frac{ig}{4\cos\theta_W}\gamma^\alpha(1-4\sin^2\theta_w - \gamma_5)\right]u(e)\left[\frac{-i(g_{\alpha\beta} - q_\alpha q_\beta/M_z^2)}{q^2 - M_z^2}\right]\times \\ \bar{u}(\nu_e)\left[\frac{-ig}{4\cos\theta_W}\gamma^\beta(1-\gamma_5)\right]u(\nu_e)), \tag{7}$$

for neutral interaction of electron-neutrino elastic scattering, where q is the momentum transfer. If we assume that the $q^2 << M_w^2, M_z^2$ and the $q_\alpha q_\beta$ term arises from the mode of W or Z -boson propagator longitudinal polarization which contribute only $m_e m_\nu$ which is very small compare to M_w^2 and M_z^2, then Eqs. (6) and (8) read:

$$M_{cc}^D = \frac{-i}{8}\left(\frac{g}{M_w}\right)^2[\bar{u}(e)\gamma^\alpha(1-\gamma_5)u(\nu_e)][\bar{u}(\nu_e)\gamma^\beta(1-\gamma_5)u(e)], \tag{8}$$

for charged current interaction of neutrino-electron elastic scattering, and

$$M_{nc}^D = \frac{i}{16}\left(\frac{g}{M_z\cos\theta_w}\right)^2[\bar{u}(e)\gamma^\alpha(1-4\sin^2\theta_w - \gamma_5)u(e)][\bar{u}(\nu_e)\gamma^\beta(1-\gamma_5)u(\nu_e)], \tag{9}$$

for neutral current interaction.

For the Majorana neutrino case, since the neutrino is its own antiparticle (antineutrino), the term: $\bar{u}(\nu_e)\gamma^\beta(1-\gamma_5)u(e)$ in Eqs. (8) and (9) should be replaced by: $\bar{u}(\nu_e)\gamma^\beta(1-\gamma_5)u(e) - \bar{u}(\nu_e)\gamma^\beta(1+\gamma_5)u(e) = 2\bar{u}(\nu_e)\gamma^\beta u(e)$, then the Feynman amplitudes for the Majorana neutrino will be:

$$M_{cc}^M = \frac{i}{4}\left(\frac{g}{M_w}\right)^2[\bar{u}(e)\gamma^\alpha(1-\gamma_5)u(\nu_e)][\bar{u}(\nu_e)\gamma^\beta\gamma_5)u(e)], \tag{10}$$

and

$$M_{nc}^M = -\frac{i}{8}\left(\frac{g}{M_z\cos\theta_w}\right)^2[\bar{u}(e)\gamma^\alpha(1-4\sin^2\theta_w - \gamma_5)u(e)][\bar{u}(\nu_e)\gamma^\beta\gamma_5)u(e)], \tag{11}$$

for charged and neutral current interactions in neutrino-electron elastic scattering respectively.

From Eqs. (8)-(11), one can see the differences of the Feynman amplitude for neutrino-electron elastic scattering when we put neutrino to be a Dirac and a Majorana particles. It is also apparent that the Feynman amplitude is different for neutrino-electron elastic scattering for charged and neutral current interactions. As we stated in section 1 that Feynman amplitude (square of absolute value of Feynman amplitude) is an important parameter in differential cross-section, thus we can use the scattering process to determine the nature of neutrino whether neutrino is a Dirac or a Majorana particle. In this talk we only evaluate the Feynman amplitudes for low energy process by taking the approximation $q^2 << M_z^2, M_z^2$. For very high energy process (very large momentum transfer), indeed, the mass effects of W or Z boson propagator contributions to the Feynman amplitudes must can not be neglected.

3 Conclusion

We have studied and derived systematically the Feynman amplitudes for neutrino-electron elastic scattering in the frame of Standard Model Particle Physics for both Dirac and Majorana neutrinos for the cases charged current and neutral current interactions. We can distinguish neutrino nature whether neutrino is a Dirac or a Majorana particle from its own Feynman amplitude for low energy process for both charged and neutral current interactions. It is also apparent that the Feynman amplitude is different for neutrino-electron elastic scattering for charged and neutral current interactions.

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