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Isotropic Robertson-Walker Universe with Van der Waals Equation of State in Brans-Dicke Theory of Gravitation

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Abstract: In this paper, we study an isotropic Robertson-Walker space-time metric to describe relativistic dark energy model universe with a modified Van der Waals equation of state. Exact solutions of Einstein field equations are obtained with variable cosmological constant of the form $\Lambda = \zeta \frac{\ddot{R}}{R}$, where ζ is a constant. Some physical and dynamical parameters are studied.

Keywords: Brans-Dicke Theory; Cosmological Constant; Dark Energy; Van der Waals EoS
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1 Introduction

The cosmological and astronomical data obtained from the Supernovae Ia (SNeIa), the cosmic microwave background (CMB), the Large Scale Structures (LSS) and X-ray experiments support the discovery of accelerated expansion of the present day universe [1–12]. Accelerated expansion of universe may be described by dark energy which has positive energy density and negative pressure [13,14]. The Brans-Dicke (B-D) theory [15] of gravitation introduces a scalar field ϕ which has the dimensions inverse of the gravitational constant and interacts equally with all forms of matter.The Brans-Dicke (B-D) theory has attained significant attention in recent years due to its vast applications in modern cosmology. There has been a lots of important works in different types of expanding cosmological models of the universe [16–23] with Brans-Dicke theory. The presence of dark energy is believed to be the reason behind the expansion of universe. Different authors like Al-Rawaft and Taha [24], Al-Rawaft [25], Overduin and Cooperstock [26] studied

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about cosmological models with variable cosmological constant of the form $\Lambda = \zeta \frac{\ddot{R}}{R}$, where ζ is a constant. Following the same decay law, Arbab [27, 28] investigated cosmic acceleration with positive cosmological constant. Also, Khadekar et al. [29] investigated well known astrophysical phenomenona of a cosmological model with cosmological term of the form $\Lambda \propto \frac{\ddot{R}}{R}$. Thirukkanesh and Ragel [30] discussed about anisotropic spheres with Van der Waals-type equation of state. In this paper, we discussed about isotropic cosmological model with Van der Waals equation of state in Brans-Dicke theory of gravitation considering Robertson-Walker metric.

2 Metric and Solutions of Field Equations

Here, we consider the spherically symmetric Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\Phi^2)\right] \tag{1}$$

where k is the curvature index which can take values $-1, 0, 1$.
The Brans-Dicke (B-D) theory of gravity is described by the action

$$S = \int d^4x\sqrt{|g|}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega}{\phi}g^{sl}\phi_{,l}\phi_{,s}\right) + L_m\right] \tag{2}$$

where R represents the curvature scalar associated with the 4D metric g_{ij}; g is the determinant of g_{ij}; ϕ is a scalar field; ω is a dimensionless coupling constant; L_m is the Lagrangian of the ordinary matter component.
The Einstein field equations in the most general form are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{\kappa}{\phi}T_{ij} - \frac{\omega}{\phi^2}[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,s}\phi_{,s}] - \frac{1}{\phi}(\phi_{,ij} - g_{ij}\Box\phi) \tag{3}$$

$$(3 + 2\omega)\Box\phi = \kappa T \tag{4}$$

where $\kappa = 8\pi$, T is the trace of T_{ij}, Λ is the cosmological constant, R_{ij} is Ricci-tensor, g_{ij} is metric tensor, $\Box\phi = \phi^{;s}_{;s}$, $\Box$ is the Laplace-Beltrami operator and $\phi_{,i}$ is the partial differentiation with respect to x^i coordinate.
The energy-momentum tensor for the perfect fluid distribution is

$$T_{ij} = (p + \rho)u_i u_j - pg_{ij} \tag{5}$$

with u_i is four velocity vector satisfying $g^{ij}u_i u_j = 1$, p is the pressure and ρ is the energy density. A comma (,) or semicolon (;) followed by a subscript denotes partial differentiation or a covariant differentiation respectively. The velocity of light is taken to be unity.
Now for the metric (1), (3) and (4) gives

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda = -\frac{\kappa p}{\phi} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} \tag{6}$$

$$3\left(\frac{k}{R^2}+\frac{\dot{R}^2}{R^2}\right)-\Lambda=\frac{\kappa\rho}{\phi}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}-3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} \tag{7}$$

$$(3+2\omega)\left[\ddot{\phi}+3\frac{\dot{R}}{R}\dot{\phi}\right]=\kappa(\rho-3p) \tag{8}$$

The energy momentum equation $T^{ij}_{;j}=0$ leads to the form

$$\dot{\rho}+3H(\rho+p)=0 \tag{9}$$

We assume a modified Van der Waals [30] equation of state

$$p=\alpha\rho^2+\frac{\beta\rho}{1+\gamma\rho}-A \tag{10}$$

where α,β,γ and A are real constants.
Also, we assume [26] cosmological constant in the form

$$\Lambda=\zeta\frac{\ddot{R}}{R} \tag{11}$$

From equations (9) and (10), we get

$$\rho=\frac{XR^6+R^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}{R^6} \tag{12}$$

and,

$$p=\alpha\left[\frac{XR^6+R^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}{R^6}\right]^2+\frac{\beta\left[\frac{XR^6+R^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}{R^6}\right]}{1+\gamma\left[\frac{XR^6+R^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}{R^6}\right]}-A \tag{13}$$

where X is the root of $\alpha\gamma Z^3+(\alpha+\gamma)Z^2+(1+\beta-A\gamma)Z-A=0$.
From equations (6), (7), (8) and (11), we get

$$3\frac{k}{R^2}+3\frac{\dot{R}^2}{R^2}+(3-2\zeta)\frac{\ddot{R}}{R}=\omega\left[3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}+\frac{\ddot{\phi}}{\phi}-\frac{1}{2}(\frac{\dot{\phi}}{\phi})^2\right] \tag{14}$$

where a dot (.) denotes differentiation with respect to time t.
The gravitational variable [31] is given by

$$G=\left(\frac{4+2\omega}{3+2\omega}\right)\frac{1}{\phi} \tag{15}$$

2.1 Case I: $k = 0$ and $\zeta > 3$.

From (14) we get

$$R = R_0 \left[c_1 t + c_2\right]^{\frac{(2\zeta-3)}{2(\zeta-3)}} \tag{16}$$

and

$$\phi = \phi_0 \left[c_1 t + c_2\right]^{-\left(\frac{4\zeta-3}{\zeta-3}\right)} \tag{17}$$

where $R_0 = 2^{\frac{(2\zeta-3)}{2(\zeta-3)}} \left\{\frac{(\zeta-3)}{(2\zeta-3)}\right\}^{\frac{(2\zeta-3)}{2(\zeta-3)}}$, $\phi_0 = \left[\frac{(\zeta-3)}{R_0^3 c_1 (4\zeta-3)}\right]^2$.
The gravitational variable is given by

$$G = \left(\frac{4+2\omega}{3+2\omega}\right) \frac{1}{\phi_0 \left[c_1 t + c_2\right]^{-\left(\frac{4\zeta-3}{\zeta-3}\right)}} \tag{18}$$

Using (16), equations (13) and (14) becomes

$$\rho = \frac{X R_0^6 \left(c_1 t + c_2\right)^{\frac{3(2\zeta-3)}{(\zeta-3)}} + \left\{R_0 \left(c_1 t + c_2\right)^{\frac{(2\zeta-3)}{2(\zeta-3)}}\right\}^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}{R_0^6 \left(c_1 t + c_2\right)^{\frac{3(2\zeta-3)}{(\zeta-3)}}} \tag{19}$$

and,

$$\begin{aligned} p = \alpha & \left[\frac{X R_0^6 \left(c_1 t + c_2\right)^{\frac{3(2\zeta-3)}{(\zeta-3)}} + \left\{R_0 \left(c_1 t + c_2\right)^{\frac{(2\zeta-3)}{2(\zeta-3)}}\right\}^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}{R_0^6 \left(c_1 t + c_2\right)^{\frac{3(2\zeta-3)}{(\zeta-3)}}}\right]^2 + \\ \beta & \left[\frac{X R_0^6 \left(c_1 t + c_2\right)^{\frac{3(2\zeta-3)}{(\zeta-3)}} + \left\{R_0 \left(c_1 t + c_2\right)^{\frac{(2\zeta-3)}{2(\zeta-3)}}\right\}^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}{(1+\gamma X) R_0^6 \left(c_1 t + c_2\right)^{\frac{3(2\zeta-3)}{(\zeta-3)}} + \left\{R_0 \left(c_1 t + c_2\right)^{\frac{(2\zeta-3)}{2(\zeta-3)}}\right\}^{-3\left(\frac{3X^2\alpha\gamma+2X\alpha+\beta-1-A\gamma}{X\gamma+1}\right)}}\right] - A \end{aligned} \tag{20}$$

Spatial volume is given by

$$V = V_0 \left[c_1 t + c_2\right]^{\frac{3(2\zeta-3)}{2(\zeta-3)}} \tag{21}$$

where $V_0 = R_0^3$.
Hubble's parameter is given by

$$H = \frac{(2\zeta-3)}{2(\zeta-3)} \frac{c_1}{(c_1 t + c_2)} \tag{22}$$

Scalar expansion is given by

$$\Theta = \frac{(2\zeta-3)}{2(\zeta-3)} \frac{3c_1}{(c_1 t + c_2)} \tag{23}$$

Deceleration parameter is given by

$$q = -\left(\frac{3}{2\zeta-3}\right) \tag{24}$$

The anisotropy parameter is given by

$$\Delta = 0 \tag{25}$$

Shear scalar is given by

$$\sigma = 0 \tag{26}$$

Cosmological constant is given by

$$\Lambda = \zeta \frac{3(2\zeta - 3)}{\{2(\zeta - 3)\}^2} \left[\frac{c_1}{(c_1 t + c_2)} \right]^2 \tag{27}$$

2.2 Case II:$k = 0$ and $\zeta = 3$.

From (14), we get

$$R = e^{\mu \frac{t}{t_0}} \tag{28}$$

and

$$\phi = \phi_0 e^{-6\mu \frac{t}{t_0}} \tag{29}$$

where $\phi_0 = \frac{t_0^2}{36\mu^2}$, μ is a constant, t_0 is the age of the universe at present epoch. The gravitational variable is given by

$$G = \left(\frac{4 + 2\omega}{3 + 2\omega} \right) \frac{1}{\phi_0 e^{-6\mu \frac{t}{t_0}}} \tag{30}$$

Using (28), equations (13) and (14) becomes

$$\rho = \frac{X e^{6\mu \frac{t}{t_0}} + e^{-3\mu \frac{t}{t_0} \left(\frac{3X^2\alpha\gamma + 2X\alpha + \beta - 1 - A\gamma}{X\gamma + 1} \right)}}{e^{6\mu \frac{t}{t_0}}} \tag{31}$$

and,

$$\begin{aligned} p = \alpha & \left[\frac{X e^{6\mu \frac{t}{t_0}} + e^{-3\mu \frac{t}{t_0} \left(\frac{3X^2\alpha\gamma + 2X\alpha + \beta - 1 - A\gamma}{X\gamma + 1} \right)}}{e^{6\mu \frac{t}{t_0}}} \right]^2 + \\ \beta & \left[\frac{X e^{6\mu \frac{t}{t_0}} + e^{-3\mu \frac{t}{t_0} \left(\frac{3X^2\alpha\gamma + 2X\alpha + \beta - 1 - A\gamma}{X\gamma + 1} \right)}}{(1 + \gamma X) e^{6\mu \frac{t}{t_0}} + e^{-3\mu \frac{t}{t_0} \left(\frac{3X^2\alpha\gamma + 2X\alpha + \beta - 1 - A\gamma}{X\gamma + 1} \right)}} \right] - A \end{aligned} \tag{32}$$

Spatial volume is given by

$$V = e^{3\mu \frac{t}{t_0}} \tag{33}$$

Hubble's parameter is given by

$$H = \frac{\mu}{t_0} \tag{34}$$

Scalar expansion is given by

$$\Theta = \frac{3\mu}{t_0} \tag{35}$$

Deceleration parameter is given by

$$q = -1 \tag{36}$$

The anisotropy parameter is given by

$$\Delta = 0 \tag{37}$$

Shear scalar is given by

$$\sigma = 0 \tag{38}$$

Cosmological constant is given by

$$\Lambda = 3\left(\frac{\mu}{t_0}\right)^2 \tag{39}$$

3 Conclusion

Here, we get the following results:
Case I: In this case, scale factor, spacial volume, gravitational variable increases as time increases. On the other hand, scalar field, hubble's parameter, scalar expansion and cosmological constant decreases as time increases and becomes zero as time tends to infinity. The deceleration parameter is less than zero which gives us an accelerating universe. Also anisotropy parameter as well as shear scalar are zero giving us an isotropic and shear free model. Again for $\alpha > 0, \beta > 0, \gamma > 0, A > 0$, both energy density and pressure are positive , also pressure is greater than energy density. But for $\alpha < 0, \beta < 0, \gamma > 0, A > 0$, energy density is positive and pressure is negative giving us a dark energy model universe with accelerated expansion of the universe.
Case II: In this case, scale factor, spacial volume, gravitational variable increases as time increases. On the other hand scalar field decreases as time increases and becomes zero as time tends to infinity. Hubble's parameter, scalar expansion and cosmological constant are small and positive constant. Here also, the deceleration parameter is less than zero which gives us an accelerating universe. The anisotropy parameter as well as shear scalar are zero giving us an isotropic and shear free model. Again for $\alpha > 0, \beta > 0, \gamma > 0, A > 0$, both energy density and pressure are positive , also pressure is greater than energy density. but for $\alpha < 0, \beta < 0, \gamma > 0, A > 0$, energy density is positive and pressure is negative giving us a dark energy model universe with accelerated expansion of the universe.

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