Spatially Homogeneous Bianchi Type-I Perfect Fluid Cosmological Models in f(R) Gravity Theory

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Abstract: In this paper we have investigated a spatially homogeneous and anisotropic Bianchi type I model filled with perfect fluid in f(R) gravity theory. Exact solutions of the field equations are obtained by considering specific ansatz of the average scale factor, which correspond to accelerating models of the universe. The physical and kinematical features of the cosmological models are discussed.

Keywords: Bianchi type-I; Perfect Fluid; f(R) Gravity Theory
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1 Introduction

The astronomical observations of luminosity-distance and redshift relation of type Ia supernovae (Perlmutter et al. [1-2], Riess et al.[3]), Cosmic Microwave Background Radiation (Spergel et al. [4]) the galaxy power spectrum (Tegmark et al. [5]) etc. have confirmed that the expansion of the universe is accelerating. The appears to be in strong disagreement with the standard picture of a matter dominated universe. These observations can be accommodated theoretically by postulating that a certain exotic matter with negative pressure dominates the present epoch of our universe. Such exotic matter, called quintessence, behaves like vacuum field energy with repulsive character arising from the negative pressure. Initially some researchers attribute the observed acceleration to a positive breakdown of our understanding of the laws of gravitation, thus they attempted to modify the Friedmann equations (Freese and Lewies [6]; Freese [7]; Wang et al. [8], Zhu and Fujimoto [9], Dvali et al. [10-12]; Sahni et al. [13-14]). However, many more

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believe that the cosmic acceleration is driven by an exotic energy component, called dark energy(DE).

The simplest of the anisotropic models are Bianchi Type-I homogeneous models whose spatial sections are flat but the expansion or contraction rate is direction- dependent. For studying the possible effects of anisotropy in the early Universe on present day observations, a number of researchers, Misner[15]; Bertolami[16]; Bali and Upadhaya [17]; Bali and Anjali[18]; Bali et al.[19] have investigated Bianchi Type-I models in different physical contexts.

In view of the late time acceleration of the universe and the existence of the dark energy and dark matter, several modified theories of gravitation have been developed and extensively studied. Noteworthy amongst them is f(R) theory of gravity, formulated by Nojiri and Odintsov [20], which passes the Newton's law. This theory provides the very natural gravitational alternative for dark energy. Nojiri and Odintsov [21] has shown that the cosmic acceleration can be directly explained by taking any negative power of the curvature. This theory helps in modification of the model to achieve the consistency with the experimental tests of solar system. The f(R) theory of gravity is constructed by replacing the gravitational Lagrangian with a general function of Ricci scalar R. The f(R) gravity theory provides a very natural unification of the early time inflation and late-time acceleration. It describes the transition from deceleration to acceleration in the evolution of the universe (Nojiri and Odintsov [22-23]). This gravity theory has explained several features(Sotriou[23]; Santosh et al.[25]; Dev et al.[26]) including solar system test(Lecian and Montani[27]), Newtonian limit (Sotiriou[28]), gravitaional stability (Sotriou [29]) and singularity problem (Frolov [30]). Multamaki and Vilja [31, 32] investigated static spherically symmetric vacuum solution of field equations and non-vacuum solutions in presence of perfect fluid in f(R) gravity. Caramers and Bezerra [33] obtained spherically symmetric vacuum solutions in higher dimensions space. Sharif and Shamir [34] discussed exact vacuum solutions of field equations in f(R) gravity for anisotropic Bianchi type- I and V space times. Non-vacuum solutions in Bianchi type-I and type-V using perfect fluid in f(R) gravity have also been obtained by Sharif and Shamir [35]. Shamir [36] discussed the plane symmetric vacuum solutions of field equations in f(R) gravity for Bianchi type-III metric. Sharif and Kausar [37] presented non-vacuum solutions of Bianchi type- VI0 type with isotropic and anisotropic fluid in f(R) gravity. Sharif and Kausar [38] discussed a Bianchi type-III universe with anisotropic fluid. Further, Sharif and Kausar [39] presented dust filled static spherically symmetric solutions in f(R) gravity theory. Adhav [40] obtained exact solutions of field equations for a Bianchi type-I model filled with cosmic strings in the framework of f(R) gravity. Recently, Singh and Singh [41] investigated an anisotropic Bianchi type I cosmological model in f(R) gravity theory with perfect fluid and studied the stability condition of the functional form of f(R), and found that it is completely stable for describing the decelerating phase of the universe.

The paper is organized as follows: The metric and the field equations are presented in Sect.2. Section 3 deals with the exact solutions of the field equations and consequently two accelerated expanding cosmological models are derived with special forms of the average scale factor. The geometric and physical significance of the cosmological models are discussed. Concluding remarks are given in Sect. 4. The present models are consistent with the observations on the present-day universe.

2 Field Equations in f(R) Gravity Theory

The f(R) gravity theory is a subsequent generalization of general relativity. The gravitational action for f(R) gravity theory coupled with matter (in the units 16πG= c= 1) is given by (Singh and Singh [41]; Capozziello and Francaviglia[42]):

$$S = \int d^4x\sqrt{(-g)}\left[f(R) + L_m\right]. \tag{1}$$

Here f(R) is a general function of Ricci scalar and L_m is the matter Lagrangian. The corresponding field equations are obtained by varying the action with respect to $g_{\mu\nu}$. This action is just obtained by replacing R by f(R) in the standard Einstein- Hilbert action. The corresponding field equations are found by varying the action with respect to the metric as

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = T_{\mu\nu} \tag{2}$$

where

$$F(R) \equiv \frac{df(R)}{dR}, \ \ \Box \equiv \nabla^\mu\nabla_\nu. \tag{3}$$

Here ∇_μ denotes covariant differentiation and $T_{\mu\nu}$ is the standard matter energy -momentum tensor derived from the Lagrangian L_m. The other symbols have their usual meanings.

The energy -momentum tensor for a perfect fluid is given as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \tag{4}$$

where ρ is the energy density, p is the thermodynamical pressure of the fluid and u_μ is the four velocity vector satisfying $u^\mu u_\nu$ =1 .

The spatially homogeneous and anisotropic Bianchi type -I space -time is given by the metric

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \tag{5}$$

where A, B and C are the scale factors in an anisotropic background and are functions of time t only. The scalar curvature R for the metric (5) is given by

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC}\right] \tag{6}$$

where a dot denotes derivative with respect to t. Using comoving coordinates, the field equations (2) for the metric (5) and energy -momentum tensor (4) yield the following system of equations:

$$\ddot{F}+\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{F}-\left(\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}\right)F-\frac{1}{2}f=-p, \tag{7}$$

$$\ddot{F}+\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\dot{F}-\left(\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}\right)F-\frac{1}{2}f=-p, \tag{8}$$

$$\ddot{F}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{F}-\left(\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}\right)F-\frac{1}{2}f=-p, \tag{9}$$

$$\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{F}-\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)F-\frac{1}{2}f=\rho. \tag{10}$$

Thus, we get four nonlinear differential equations with six unknown, namely A, B, C, F, ρ and p. Here we obtain solutions of these equations using the approach of Saha [43] The average scale factor a(t) and the spatial volume V are defined by

$$V=a^3 \;\; =ABC. \tag{11}$$

For the metric (5), the average Hubble parameter H, which is the generalization of the Hubble parameter in isotropic case, is given by as

$$H=\frac{\dot{a}}{a} \;\; =\frac{1}{3}\left(H_1+H_2+H_3\right) \tag{12}$$

where $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$, $H_3=\frac{\dot{C}}{C}$ are directional Hubble parameters along x-,y- and z- axes . The expansion scalar θ, shear scalar σ^2 and anisotropy parameter A_m are given as

$$\theta=3H \;\; =\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{13}$$

$$\sigma^2 \;\; =\frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2\right]-\frac{\theta^2}{6}, \tag{14}$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{15}$$

where $\Delta H_i \;\; =H_i-H$, i=1, 2, 3.

An important observational quantity in cosmology is the deceleration parameter defined as

$$q=-\frac{a\ddot{a}}{\dot{a}^2}. \tag{16}$$

The sign of q indicates whether the model inflates or not. The positive sign corresponds to standard decelerating model whereas the negative sign indicates inflation.

3 Exact Solutions of Field Equations

Subtracting equation (8)-(7) , (8)-(9) and (10 -(9) and integrating the results, we obtain

$$\frac{B}{A} = d_1 \exp\left[c_1 \int \frac{dt}{a^3 F}\right], \tag{17}$$

$$\frac{C}{B} = d_2 \exp\left[c_2 \int \frac{dt}{a^3 F}\right], \tag{18}$$

$$\frac{A}{C} = d_3 \exp\left[c_3 \int \frac{dt}{a^3 F}\right] \tag{19}$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are constants of integration which satisfy

$$c_1 + c_2 + c_3 = 0, \;\; d_1 d_2 d_3 = 1. \tag{20}$$

From (17), (18) and (19), we readily obtain the quadrature solutions for scale factors A, B and C of the forms

$$A = ap_1 \exp\left[q_1 \int \frac{dt}{a^3 F}\right], \tag{21}$$

$$B = ap_2 \exp\left[q_2 \int \frac{dt}{a^3 F}\right], \tag{22}$$

$$C = ap_3 \exp\left[q_3 \int \frac{dt}{a^3 F}\right] \tag{23}$$

where

$$p_1 = \left(d_1^{-2} d_2^{-1}\right)^{\frac{1}{3}}, \;\; p_2 = \left(d_1 d_2^{-1}\right)^{\frac{1}{3}}, \;\; p_3 = \left(d_1 d_2^{2}\right)^{\frac{1}{3}} \tag{24}$$

and

$$q_1 = -\frac{(2c_1 + c_2)}{3}, \;\; q_2 = \frac{(c_1 - c_2)}{3}, \;\; q_3 = \frac{(c_1 + 2c_2)}{3}. \tag{25}$$

Note that p_1, p_2, p_3 and q_1, q_2, q_3 satisfy the relations

$$p_1 p_2 p_3 = 1, \;\; q_1 + q_2 + q_3 = 0. \tag{26}$$

In order to solve the integral (21)-(23), we assume the power law relation between the average scale factor ‘a’ and the scalar function F(R) as used by Johri and Desikan [44] in the context of FRW models in Brans-Dicke theory. In the context of f(R) gravity, Kutub Uddin et al. [45] has established the result F $\propto$ a^m, where m is an arbitrary integer .Therefore, we can take a power law relation between F and a(t) as

$$F = ka^{-2} \tag{27}$$

where k is constant. Here we take m=-2. Then, equations (21), (22) and (23) reduce to

$$A = ap_1 \exp\left[\frac{q_1}{k} \int \frac{dt}{a}\right], \tag{28}$$

$$B = ap_2 \exp\left[\frac{q_2}{k}\int\frac{dt}{a}\right], \tag{29}$$

$$C = ap_3 \exp\left[\frac{q_3}{k}\int\frac{dt}{a}\right]. \tag{30}$$

We can find the solution of these equations if a(t) is a known function of t. In the next sections, we use an appropriate forms of a(t), derived and used by Pradhan et al. [46],to obtain the scale functions A, B and C.

2.1 Model I

In this section, we follow Pradhan et al. [46] to present a special form of the average scale factor. We assume that the deceleration parameter q is time-dependent of the form

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = b(t) \tag{31}$$

when b(t) is a function of t. The motivation to choose such time-dependent q is due to the fact that the universe is in the state of accelerated expansion at present. Assuming b=b(a), the general solution of (31) is given by

$$\int\left[e^{\int \frac{b}{a}da}\right] = t + k \tag{32}$$

where k is an integration constant. In order to solve equation (32), we set

$$\int \frac{b}{a}da = \ln L(a) \tag{33}$$

without loss of any generality. From equation (32) and (33), we obtain

$$\int L(a)d(a) = t + k. \tag{34}$$

Of course L(a) in equation (34) is quite arbitrary. To obtain a physically viable model of the universe consistent with observations, we assume

$$L(a) = \frac{1}{\alpha\sqrt{(1+a^2)}} \tag{35}$$

where α is an arbitrary constant. With this choice of L(a), the exact solution of (34) is

$$a(t) = \sinh(\alpha T) \tag{36}$$

where T=t+k. We note that T=0 and T=∞ respectively correspond to the proper time t=-k and t=∞ . Recently Pradhan et al. [47] studied anisotropic fluid in Bianchi type-VI0 space-time with the scale factor given by (36). This form of a(t) is also used by Amirhashchi et al.[48] to study the evolution of dark energy models in a spatially homogeneous and isotropic FRW space-time with barotropic fluid and dark energy.

By using (36)into (28)-(30),we get the following expressions for scale factors:

$$A = p_1 \sinh(\alpha T) \exp\left[\frac{q_1}{k\alpha} \log\tanh\left(\frac{\alpha T}{2}\right)\right], \tag{37}$$

$$B = p_2 \sinh(\alpha T) \exp\left[\frac{q_2}{k\alpha} \log\tanh\left(\frac{\alpha T}{2}\right)\right], \tag{38}$$

$$C = p_3 \sinh(\alpha T) \exp\left[\frac{q_3}{k\alpha} \log\tanh\left(\frac{\alpha T}{2}\right)\right]. \tag{39}$$

Therefore, the metric of our solution can be written in the form

$$\begin{aligned} ds^2 = dT^2 - \sinh^2(\alpha T) \exp\left[2\frac{q_1}{k\alpha} \log\tanh\left(\frac{\alpha T}{2}\right)\right] dx^2 \\ - \sinh^2(\alpha T) \exp\left[2\frac{q_2}{k\alpha} \log\tanh\left(\frac{\alpha T}{2}\right)\right] dy^2 \\ - \sinh^2(\alpha T) \exp\left[2\frac{q_3}{k\alpha} \log\tanh\left(\frac{\alpha T}{2}\right)\right] dz^2 \end{aligned} \tag{40}$$

3 Some Physical and Kinematical Properties

The mean generalized Hubble parameter has the value given by

$$H = \alpha \coth(\alpha T) \tag{41}$$

while the spatial volume turns out to be

$$V = \sinh^3(\alpha T). \tag{42}$$

Scalar expansion θ has the value given by

$$\theta = 3H = 3\alpha \coth(\alpha T). \tag{43}$$

Deceleration parameter q is given as

$$q = -\tanh^2(\alpha T). \tag{44}$$

The deceleration parameter q is negative for all time T, which corresponds to model of an accelerating universe.
However, shear scalar σ is given below

$$\sigma^2 = \frac{1}{2k^2}\left(q_1^2 + q_2^2 + q_3^2\right) csch^2\left(\alpha T\right). \tag{45}$$

Anisotropic parameter A_m has the value

$$A_m = \frac{1}{3k^2\alpha^2}\left(q_1^2 + q_2^2 + q_3^2\right) sech^2\left(\alpha T\right) - 1. \tag{46}$$

From (6) and (37)- (39), the Ricci scalar R is given by

$$R = -2\left[3\alpha^2 - \frac{1}{k^2}\left(q_1q_2 + q_2q_3 + q_3q_1\right)csch^2(\alpha T)\right]. \tag{47}$$

Using the above background solution into (3) and (27),we get (with the help of mathematica)

$$f(R) = \frac{4}{\alpha}\arctan(0.25\alpha T) + \frac{1.16}{\alpha}\coth(0.25\alpha T) - \frac{0.083}{\alpha}\coth(0.25\alpha T)csch(0.25\alpha T)^2 + \frac{11}{64\alpha}csch(\alpha T)^2 - \frac{1}{128\alpha T}csch\left(\frac{\alpha T}{4}\right)^2 + \frac{0.125}{\alpha}csch(0.5\alpha T)^2 - \frac{0.0625}{\alpha}csch(0.5\alpha T)^4 + \frac{2}{\alpha}csch(\alpha T)^4 + \frac{35}{16\alpha}\log\tanh\left(\frac{\alpha T}{4}\right) + \frac{11}{128\alpha}sech\left(\frac{\alpha T}{2}\right)^4 + \frac{3}{2\alpha}sech\left(\frac{\alpha T}{T}\right) + \frac{1}{6\alpha}sech\left(\frac{\alpha T}{2}\right) - \frac{3}{\alpha}sech(\alpha T)^2 - \frac{1.167}{\alpha}\tanh(0.25\alpha T) - \frac{0.083}{\alpha}sech(0.25\alpha T)^2 * \tanh(0.25\alpha T). \tag{48}$$

From (7) and (10), we obtain the pressure and energy density as

$$p = 2(q_2 + q_3)6sech^2(\alpha T)coth(\alpha T) - \alpha^2 kcsch^2(\alpha T) - 2q_1\alpha csch^3(\alpha T) + 4k\alpha coth^2(\alpha T) - k\alpha^2 csch^2(\alpha T) + \frac{f(R)}{2}, \tag{49}$$

$$\rho = 3\alpha coth(\alpha T) - \left[3\alpha^2 + \frac{1}{k^2}(q_1^2 + q_2^2 + q_3^2)2csch^2(\alpha T)\right] - \frac{f(R)}{2}. \tag{50}$$

where f(R) is given in (48).

The analysis of the above result shows that the model (40) has an initial singularity at T=0. The spatial volume is zero at T=0 and expansion scalar is infinite, which shows that the universe starts evolving with zero volume at T=0. The average scale factor is zero at this epoch and hence the model has a point-type singularity at T=0. Hubble's parameter is constant for large time and consequently steady -state occurs in this model. The shear scalar becomes infinite at T=0 while it tends to zero when T$\rightarrow \infty$. As T increases, the average scale factor and spatial volume increase but the expansion scalar decreases. However, pressure and energy density tend to constant values as T$\rightarrow \infty$ accordingly our universe is dominated by dark energy for large time T. It is observed that anisotropic parameter A_m is constant which means that the universe expands with anisotropic expansion . We also find that $\frac{\sigma}{\theta}$ tends to zero as T $\rightarrow \infty$, which implies that the fluid behaves like as isotropic dark energy since q = -1 this model represents continuously expanding, shearing universe from the starts of the big-bang.

In this model behaviors of the Hubble parameter H, deceleration parameter q, Ricci scalar R, f(R), pressure p and energy density ρ to cosmic time T are plotted in Figures (1)- (6).

3.1 Model 2

We consider the power-law form of the average scale factor as

$$a = a_0 t^m \tag{51}$$

where m is a constant. The form was used by Singh and Chaubey [49], Singh and Kale [50] and Adhav [51].

Using equation (51) into (28)-(30), we obtain the following expression for scale factors:

$$A = p_1 a_0 t^m \exp\left[\frac{q_1 t^{1-m}}{k a_0 (1-m)}\right], \tag{52}$$

$$B = p_2 a_0 t^m \exp\left[\frac{q_2 t^{1-m}}{k a_0 (1-m)}\right], \tag{53}$$

$$A = p_3 a_0 t^m \exp\left[\frac{q_3 t^{1-m}}{k a_0 (1-m)}\right]. \tag{54}$$

Therefore, the metric of these solutions is

$$\begin{aligned} ds^2 = dt^2 - a_0^2 t^{2m} [\exp\left(\frac{2q_1 t^{1-m}}{k a_0 (1-m)}\right) dx^2 + \exp\left(\frac{2q_2 t^{1-m}}{k a_0 (1-m)}\right) dy^2 \\ + \exp\left(\frac{2q_1 t^{1-m}}{k a_0 (1-m)}\right) dz^2]. \end{aligned} \tag{55}$$

The physical parameters such as spatial volume V directional Hubble parameters H_i, Hubble parameter H, expansion scalar θ, deceleration parameter q shear scalar σ and anisotropic parameter A_m for the model (54) are given by

$$V = a_0^3 t_m^3, \tag{56}$$

$$H_i = \frac{m}{t} + \frac{q_i}{k a_0} \frac{1}{t^m}, i = 1, 2, 3, \tag{57}$$

$$H = \frac{m}{t}, \tag{58}$$

$$\theta = 3H = \frac{3m}{t}, \tag{59}$$

$$q = \frac{1}{m} - 1. \tag{60}$$

For m< 1, q> 0, therefore the metric (55) represents a decelerated expanding model with constant decelerating parameter. If m>1, q<0, and so the metric (55) indicates model of an accelerating expanding universe. Thus, the model is consistent with the present-day observations

$$\sigma^2 = \frac{1}{k^2 a_0^2 t^{2m}} \left(q_1^2 + q_2^2 + q_3^2\right), \tag{61}$$

$$A_m = \frac{1}{3k^2 a_0^2 m^2 t^{2(m-1)}} \left(q_1^2 + q_2^2 + q_3^2\right). \tag{62}$$

From equations (6),(52),(53) and (54), the expression for the Ricci scalar R becomes

$$R = -2\left[\frac{3m}{t^2}(2m-1) - \frac{1}{(ka_0)^2}\frac{1}{t^{2m}}\left(q_1q_2 + q_2q_3 + q_3q_1\right)\right]. \tag{63}$$

From equation (3), the expression for f(R)is therefore

$$f(R) = \frac{12k.a_0^2(1-2m)}{(2m+1)t^{2m+1}} + \frac{1}{k}\frac{1}{t^{4m}}\left(q_1q_2 + q_2q_3 + q_3q_1\right). \tag{64}$$

Pressure and energy density are obtained following sane fashion as model 1

$$p = \left[\frac{(2q_1^2 + 3q_1q_2 + 3q_1q_3 + q_2q_3)}{a_0^4 k t^{4m}}\right] + \frac{M_2}{t^{(2m+1)}} + \frac{M_3}{t^{(3m+1)}} - \frac{q_1 m}{k a_0^2}\frac{1}{t^{3m}} - \frac{M_1}{t^{2(m+1)}}, \tag{65}$$

$$\rho = \left[\frac{6ka_0^2(2m+1)}{(2m+1)t^{2m+1}}\right] - \frac{N_1}{t^{4m}} - \frac{3m(m-1)}{a_0^3 t^{2(m+1)}}. \tag{66}$$

where M_1, M_2, M_3 and N_1 are constants

We observe that the model has initial singularity at t=0 and shows the late time accelerated expansion of the universe . The kinematical and physical parameter H θ, σ^2, R, f(R), p and ρ become infinite at t$\to$ 0 and they vanish as t$\to \infty$ while the volume scale factor increases with time showing the late time accelerating universe. For finite time the model is anisotropic for all finite limit. We also find that $\frac{\sigma^2}{\theta^2}$ tends to zero if m$>$ 1. Therefore, the model isotropizes for large time if m$>$ 1, otherwise anisotropic for other finite values of m. The behaviors of physical and kinematical parameters are depicted in Figures (7)-(11).

4 Concluding Remarks

In this paper, we study a Bianchi type- I modified f(R) gravity theory in the presence of perfect fluid. The exact solutions to the corresponding field equations are obtained by using two appropriate forms of the average scale factor of the model that correspond to the negative deceleration parameter. These solutions correspond to accelerated expanding cosmological models having finite time singularity. All the physical and kinematical parameters are well behaved and decreasing functions of time and ultimately tend to zero. Anisotropy in the models are maintained throughout the passage of time . Thus, we hope that the models presented in this paper put amble light on the understanding of the evolution of the early universe. The cosmological models obtained in this paper within the framework of f(R) gravity theory are of considerable interest and will be useful in the study of present -day universe.

References

[1] Perlmutter,S., et al.: Astrophys. J.**517**,565(1999)

[2] Perlmutter,S.,: Phys. Scr. T. **117**,17(2005)

[3] Riess, A.G., et al.: Astron. J. **116**,1009(1998)

[4] Spergel, D.N., et al.: Astrophys. J.Suppl. Ser.**148**,175(2003)

[5] Tegmark, M., et al.: Phys.Rev.D **69**,10350(2004)

[6] Freese, K.M.: Lewis Phys.Lett. B **540**,1(2002)

[7] Freese, K.: Nucl.Phys. B **124**,50(2003)

[8] Wang, Y., et al.: Astrophys.J. **594**,25 (2003)

[9] Zhu, Z., Fujimoto, M.: Astrophys. J. **620**,12(2004)

[10] Dvali, G., et al.: Phys.Lett. B **485**,208 (2000)

[11] Dvali G., et al.: Phys. Rev. D **64**,084004(2001)

[12] Dvali, G., et al.: Phys. Rev. D **65**,024031(2002).

[13] Sahni, V., Starobinsky, A.: Int. J. Mod. Phys. D **9**,373 (2000)

[14] Sahni, V., Starobinsky, A.: Int. J.Phys.D **15**,2105(2006)

[15] Misner, C. W.: Ap. J. **151**, 431 (1968)

[16] Bertolami, O.: Nuovo Cimento B**93**, 36(1986)

[17] Bali, R., Upadhaya, R.D.: Astrophys Space Sci **283**, 97(2003)

[18] Bali, R., Anjali: Astrophys Space Sci **302**, 201(2006)

[19] Bali,R., Kumar U. P., Pradhan A.: Chin.Phys. Lett.**24**,2455(2007)

[20] Nojiri, S., Odintsov, S.D.: Phys. Rev.D **68**,123512(2003)

[21] Nojiri, S.,Odintsov, S.D.: Int.J.Geom.Meth.Mod.Phys.**4**,115(2007)

[22] Nojiri,S.,Odintsov, S.D.: J.Phys. Conf. Ser.**66**,012005 (2007)

[23] Nojiri, S., Odintsov, S.D.: Phys. Rev. D **78**,046006(2008)

[24] Sotiriou, T.P.: Class. Quantum Grav.**23**,5117 (2006)

[25] Santos, J. et al.: Phys.Lett.B **669**,14 (2008)

[26] Dev,A., et al.: Phys.Rev.D **78**,083515(2008)

[27] Lecian, O.M., Montani, G.: Class. Quantum Grav.**26**,045014(2009)

[28] Sotiriou, T.P., Gen.Relativ. Grav. **381**,407 (2006)

[29] Sotiriou, T.P., Phys. Lett. B **645**,389 (2007)

[30] Frolov, A.V.: Phys Rev. Lett.**101**,061103 (2008)

[31] Multamaki, T.,Vilja, I.: Phys. Rev. D **74**,064022 (2006)

[32] Multamaki, T.,Vilja, I.: Phys. Rev. D **76**,064021(2007)

[33] Carames,T.R.P., Bezerra, E.R.: Eur.Phys.J.C **64**,113(2009)

[34] Sharif, M.,Shamir, M.F.: Class Quantum Grav.**26**,235020(2009)

[35] Sharif,M.,Shamir, M.F.: Gen Relativ.Grav.**42**,1559(2010)

[36] Shamir, M.F., Int.J. Theor Phys. **50**,637(2010)

[37] Sharif, M.,Kausar, H.R.: arXiv:1102.4124v1(2011)

[38] Sharif, M.,Kausar, H.R.: Astrophys.Space Sci.**332**,463(2011)

[39] Sharif,M.,Kausar, H.R.: arXiv:1101,3342 (2011)

[40] Adhav, K.S.: Bulg.J.Phys.39,197(2012)

[41] Singh, V. Singh, C.P.: Astrophys.Space Sci **346**, 285(2013)

[42] Capozziello, S., Francaviglia, M.: Gen.Relativ.Grav.**46**, 357(2008)

[43] Saha, B., Mod.Phys.Lett.A **20**, 2127(2005)

[44] Johri, V.B.,Desikan, K.: Gen Relativ. Grav. **26**, 1217(1994)

[45] Uddin, K., et al.: Class.Quantum Grav. **24**,3951(2007)

[46] Pradhan,A., Jaiswal, R., Khare, R.K.: Astrophys. Space Sci.**343**,489(2013)

[47] Pradhan, A.,Jaiswal, R., Jotania, R.: Astrophy. Space Sci. **337**,401 (2012)

[48] Amirhashchi, H., Pradhan, A. Jaiswal, R.: Int. J.Theo. Phys.**52**,2735(2013)

[49] Singh, T., Chaubey, R.: Pramana J Phys. **71**, 447(2008)

[50] Singh, G.P., Kale, A.Y.: Astrophys. Space Sci.**331**, 27(2011)

[51] Adhav, K.S., et al.: Cent.Eur.J.Phys.**9**,919(2011)

[52] A. Pradhan, P. Pandey, and K.K. Rai, Czech. J. Phys. 56 (2006) 303.

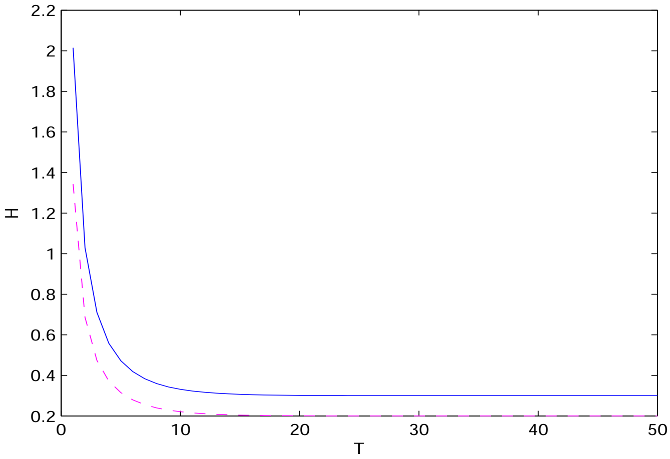

Fig. 1 The plot of Hubble parameter H verses cosmic time T, α_1 =0.3, α_2=0.2;

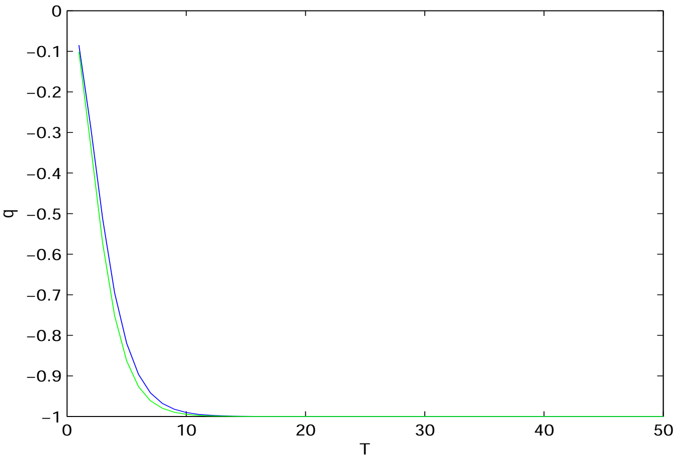

Fig. 2 The plot of deceleration parameter q verses cosmic time T, α_1 =0.3, α_2=0.2;

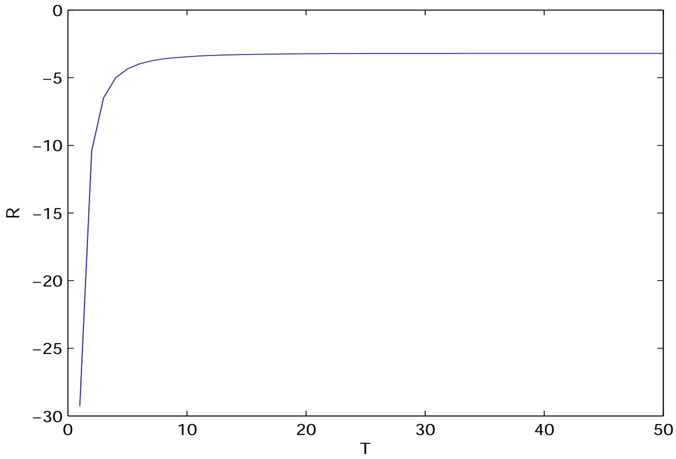

Fig. 3 The plot of scalar curvature R verses cosmic time T, α_1 =0.3, α_2=0.2;

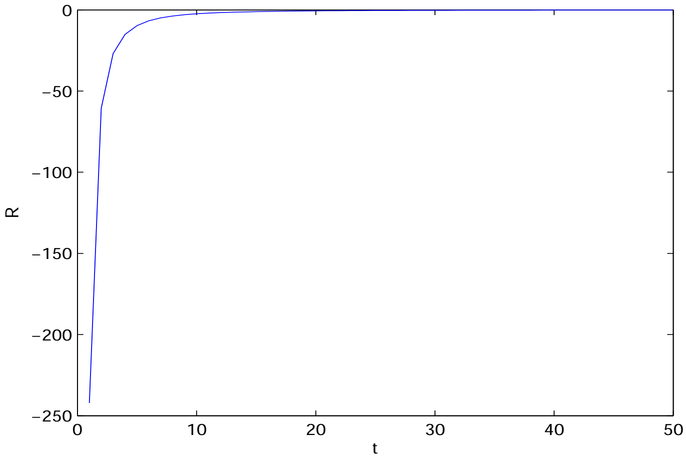

Fig. 4 The plot of f(R) verses cosmic time T, α_1 =0.3, α_2=0.2;

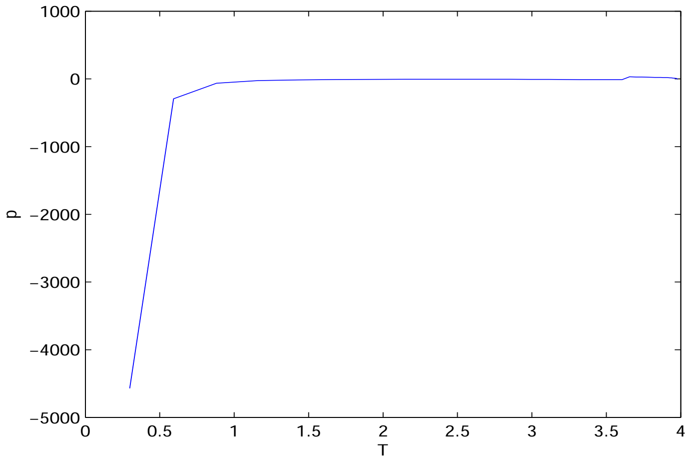

Fig. 5 The plot of pressure p verses cosmic time T, α_1 =0.3, α_2=0.2;

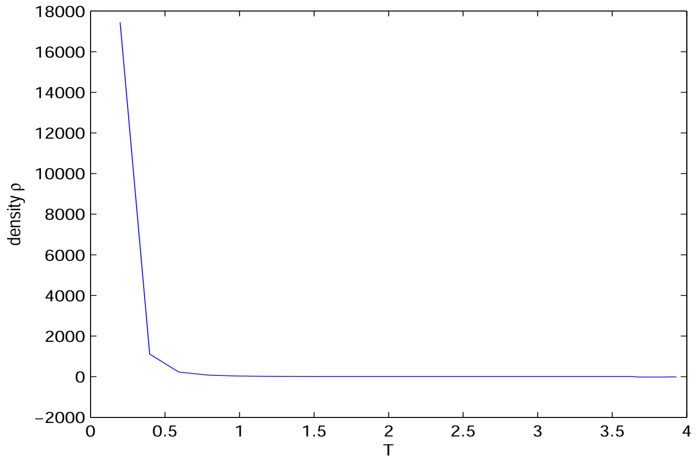

Fig. 6 The plot of density ρ verses cosmic time T,α_1 =0.3, α_2=0.2;

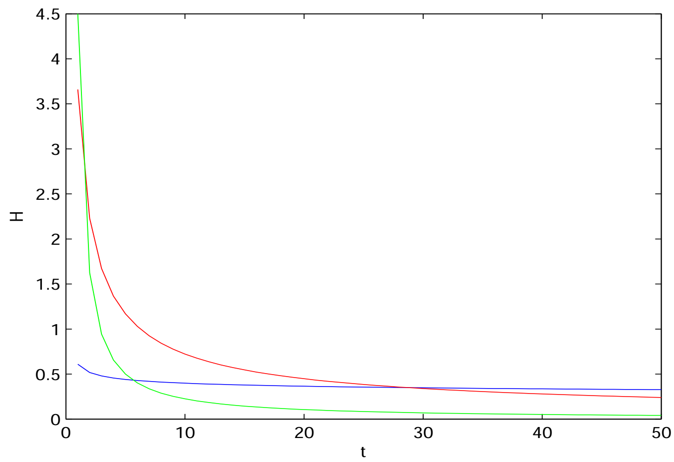

Fig. 7 The plot of Hubble parameter H verses cosmic time t, a=0.1, k=0.2, m_1=0.11, m_2=0.66, m_3=1;

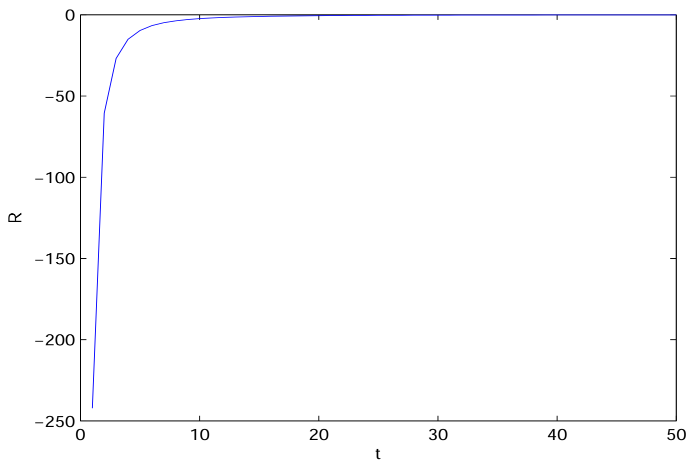

Fig. 8 The plot of scalar curvature R verses cosmic time t, a=0.1, k=0.2, m=1;

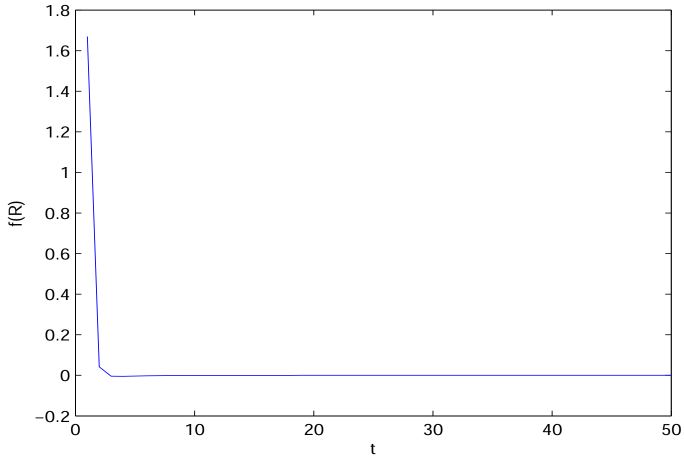

Fig. 9 The plot of f(R) verses cosmic time t, a=0.1, k=0.2, m=1;

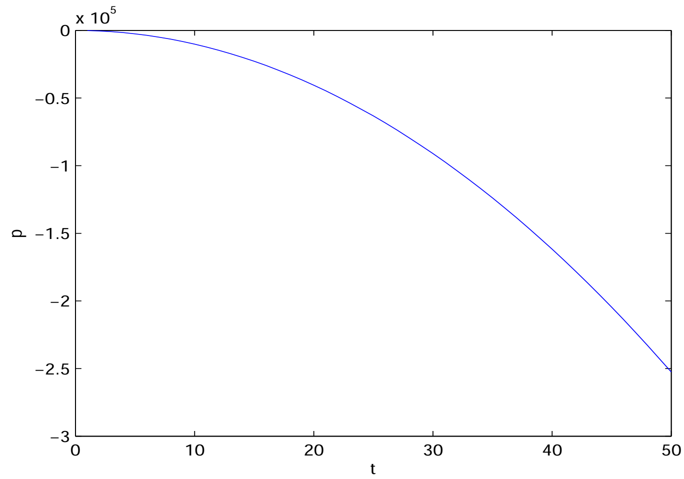

Fig. 10 The plot of pressure p verses cosmic time t,a=0.1, k=0.2, m=1;

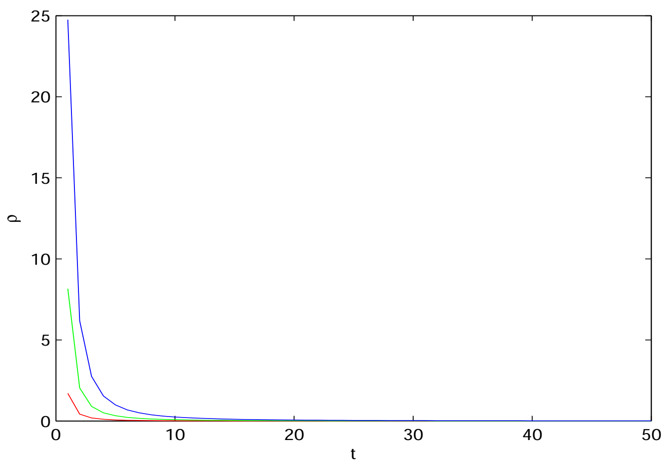

Fig. 11 The plot of density ρ verses cosmic time t,a=0.1, k=0.2, m=1;