Geometry and Probablity: Statistical Geometrodynamics with Holography

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Abstract: The following paper makes an endeavor to derive from scratch, the Einstein gravity from purely novel and subtle set of statistical ansatz. The theory developed is that of statistical geometrodynamics in close conjunction with the standard statistical thermodynamics, thereby extending the Einstein theory to three laws involving the Holographic Principle in a totally different way. A Boltzmann-like formula is derived right after the second law of geometrodynamics. The author hopes that the following work will shed some light on the fundamental understanding of the underpinnings of gravity and information in a non-conventional way. The rest of the latter part of the paper consists of discussions and possible conclusions that could be drawn by the author at the time of writing the paper.

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The Holographic Principle worked out by tHooft and independently by Susskind [1] has played out a vital role in the understanding of gravity. The works of Bousso (see for example for a good survey [2] and also references therein for an exhaustive study of the Holographic Principle) has given a definitive direction to the study of the entropy bounds. Starting from the Bekenstein bound arising from the Geroch process to the work of Susskind through his own process of transforming any thermodynamic system into a blackhole and the application of this transformation to the Bekensteins Generalized Second Law (GSL) yielding the more rigorous and general spherical entropy bound, one finds a rather subtle and elegant formulation through the light-sheet formalism to the Bousso covariant entropy conjecture (also look up in [2] in the reference section under B for Bousso), now a theorem due to the proof provided by Flanagan et. al. [3]. Recently,

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Verlinde has made a revelation in gravitational physics [4] by demonstrating the emergence of Newtonian and Einsteinean gravity in steps of approximations from the laws of thermodynamics by the application of the Holographic Principle and bit dynamics. The work of Caticha [5](check also references in [5]) has taken definitive steps in the direction of construction of a statistical theory of geometrodynamics. The present work takes a completely different route to derive the Einstein gravity and further extends the theory. The geometrodynamic theory is considered from statistical postulates applied to hypothetical fundamental constituents of curved spacetime. The theory of geometrodynamics thus derived is considered independent of thermodynamics and yet a similar theory in its own right with a set of quantities analogous to entropy, thermodynamic probability, temperature, etc. There is thus no such entropy bound here. Everything is in terms of curvature and statistical geometrodynamics. The paper is a bold attempt to pave way in the positive direction of a fundamental understanding of the origins of space and time [6].

One always arrives at the Einstein field equations and his law of gravitation through the Principle of equivalence which asserts the equivalence of acceleration and gravitation. Then, there is the Einstein-Hilbert variational principle from which also one gets the Einstein field equations of gravitation which read

$$\boldsymbol{R}_{\mu\nu} - \frac{1}{2}\boldsymbol{g}_{\mu\nu}\boldsymbol{R} = \boldsymbol{T}_{\mu\nu}, \tag{1}$$

where, $\boldsymbol{R}_{\mu\nu}$ is the Ricci curvature tensor, $\boldsymbol{R}$ the corresponding scalar, $\boldsymbol{g}_{\mu\nu}$ is the fundamental or metric tensor and $\boldsymbol{T}_{\mu\nu}$ is the energy momentum tensor. The above equation embodies the fact that spacetime is curved by the presence of matter and mass is made to move by the curved spacetime according to the warp of the spacetime.

This involves tensor analytic manipulation of the Christoffel symbols of the second kind thereby defining the Ricci tensor. Yet another approach is that of the Bianchi identity of the second type which exhibit the principle of geometrodynamics that the boundary of a boundary of a boundary is zero. All these approaches involve deterministic and generically predictive propositions and interpretations. On the other hand, energy-momentum tensor represents matter and energy and these obey quantum statistical laws. There are quantum transitions involved in the matter represented by $\boldsymbol{T}_{\mu\nu}$. This quantum behavior should correspondingly be accounted for by the spacetime geometry as well. If matter fluctuates statistically then so should geometry. If matter obeys statistical laws then so should geometry at the quantum scales. The laws of physical statistics obeyed by $\boldsymbol{T}_{\mu\nu}$ lead to statistical thermodynamics. The laws of physical statistics obeyed by the geometric part of the Einstein law viz., the l.h.s. of eq (1) should lead to statistical geometrodynamics. Still, it is the aim of this paper to not argue this way by starting with the Einstein law and the eq (1). The aim is to rather start with a subtle and simple set of statistical postulates and ansatz and arrive at various results.

We begin by proposing the existence of geomets hypothetical quantum or fundamental objects of spacetime geometry which occupy different available geometrodynamic states in the statistical manner of speaking. The various geometrodynamic distributions are then arrived at by the gas of geomets. The resulting spacetime geometry is a direct consequence of the ensemble of the gas of geomets occupying the geometrodynamic states of different curvature probabilities. This is our first ansatz. So, now there are two hypothetical objects (i) geomets the fundamental constituents of curved spacetime and (ii) the geometrodynamic states. Since entropy for gravity as a thermodynamic system is non-concave, there are many possible geometrodynamic states and end-states. As such, the geometrodynamic probability and the geometrodynamic distribution function for a given ensemble of a gas of geomets determining the curvature in the bulk of spacetime become $N^2 = 4^2 = 16$ functions and are tensors of rank two. We denote them respectively by $\mathbf{\Gamma}_{\mu\nu}$ and $\boldsymbol{f}_{\mu\nu}$. Even though the number, $\boldsymbol{n}_s$, of the bits available for the description of the individual species of geomets is the same (is equivalent to the number of matter or radiation particles) the non-concave nature of entropy for gravity makes these tensors. Now from our simple hypothesis, we derive: (i) the Einstein law of geometrodynamics and follow it up with two additional laws based on the holographic principle in analogy with the laws of thermodynamics and (ii) we derive a formula connecting the Gauss-Bonnet mean curvature of the holographic surface bounding the bulk of the spacetime and the geometrodynamic probability $\mathbf{\Gamma}_{\mu\nu}$ in the bulk.

The quantity $\boldsymbol{\beta}$ appearing in statistics is inversely proportional to the absolute temperature $\boldsymbol{T}$ in statistical thermodynamics. In statistical geometrodynamics, we define the geometrodynamic probability $\mathbf{\Gamma}_{\mu\nu}$ by

$$ln\mathbf{\Gamma}_{\mu\nu} = \sum_s \boldsymbol{f}_{\mu\nu}\left(\boldsymbol{n}_s\right) \tag{2}$$

Now, we fix the following ansatz,

$$\sum_s \boldsymbol{n}_s = \boldsymbol{N} \text{ and } \sum_s \boldsymbol{n}_s \boldsymbol{\gamma}_{\mu\nu(s)} = \boldsymbol{T}_{\mu\nu} \tag{3}$$

Here, $\boldsymbol{\gamma}_{\mu\nu(s)}$ is the kinetic geometry of the species of the geomets. The kinetic geometry is defined as the geometry possessed by the geomet on account of its motion. This is a simple definition: moving bodies possess kinetic energy and moving elements of curved spacetime geometry the geomets possess kinetic geometry. This fixes up an exact yet abstruse analogy between thermodynamics and geometrodynamics and further strengthens ansatz (3) above. Pure statistical geometrodynamics should be geometrical in character. Any energy should be translated into geometry and vice-versa. In fact, the first condition of the ansatz eq (3) is simply the number conservation but the second condition of the ansatz (3) is the Principle of Equivalence of Gravitation and Inertia in disguise if one thinks carefully.

So, we fix up tensor multipliers as,

$$\boldsymbol{\alpha}_{\mu\nu} + \boldsymbol{\beta}\boldsymbol{\gamma}_{\mu\nu(s)} = \frac{\partial \boldsymbol{f}_{\mu\nu}}{\partial n_s}. \tag{4}$$

Then,

$$\delta ln\mathbf{\Gamma}_{\mu\nu} = \sum_s \frac{\partial f_{\mu\nu}}{\partial n_s}\delta n_s = \sum_s \left(\alpha_{\mu\nu} + \beta\gamma_{\mu\nu(s)}\right)\delta n_s$$
$$= \alpha_{\mu\nu}\sum_s \delta n_s + \beta\sum_s \gamma_{\mu\nu(s)}\delta n_s \tag{5}$$

Since n_s is fixed by ansatz (3),

$$\sum_s \delta n_s = 0. \tag{6}$$

And from the second part of (3),

$$\sum_s n_s\delta\gamma_{\mu\nu(s)} + \sum_s \gamma_{\mu\nu(s)}\delta n_s = \delta T_{\mu\nu}. \tag{7}$$

The first term in (7) represents the stretch in the spacetime; that is, the geometric work or in other words, the stressing curvature mathematically equal to $'' - \frac{1}{2}Rg_{\mu\nu}''$; that is

$$\sum_s n_s\delta\gamma_{\mu\nu(s)} = \sum_s n_s\frac{\partial\gamma_{\mu\nu(s)}}{\partial R}\delta R = -\frac{1}{2}g_{\mu\nu}\delta R, \tag{8}$$

so that

$$g_{\mu\nu} = -2\sum_s n_s\frac{\partial\gamma_{\mu\nu(s)}}{\partial R}.(\tag{9}$$

Also taking local derivatives w.r.t. time we have by Hamiltons Ricci flow

$$\frac{\partial g_{\mu\nu}}{\partial t} = -2\sum_s n_s\frac{\partial^2\gamma_{\mu\nu(s)}}{\partial R\partial t} = -2R_{\mu\nu}\left(g\right)(\tag{10}$$

so

$$\sum_s n_s\frac{\partial^2\gamma_{\mu\nu(s)}}{\partial R\partial t} = R_{\mu\nu}\left(g\right). \tag{11}$$

Now, the second term $''\sum_s \gamma_{\mu\nu(s)}\delta n_s''$ is the Ricci curvature which is the geometrodynamic warp accrued by the bulk of the spacetime, i.e., the curvature inside the holographic screen. Thus,

$$\delta R_{\mu\nu} = \sum_s \gamma_{\mu\nu(s)}\delta n_s. \tag{12}$$

From (7), (8) and (12), we have for the first law of geometrodynamics, the Einstein field equations (1) which we rewrite here as (1) The first law of geometrodynamics: $T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. Also,

$$\sum_s n_s\frac{\partial^2\gamma_{\mu\nu(s)}}{\partial R\partial t} = \sum_s \gamma_{\mu\nu(s)}\delta n_s$$

or

$$n_s\frac{\partial^2\gamma_{\mu\nu(s)}}{\partial R\partial t} = \gamma_{\mu\nu(s)}\delta n_s. \tag{13}$$

On the other hand,

$$\delta ln\Gamma_{\mu\nu} = \beta \sum_s \gamma_{\mu\nu(s)}\delta n_s = \beta\delta R_{\mu\nu}. \tag{14}$$

Therefore, $\beta\delta R_{\mu\nu}$ is a total differential and β, the intergrating factor of the Ricci curvature. The statistical theory leads naturally to the second law of geometrodynamics which we enunciate as follows: (2) $\delta R_{\mu\nu}$ has an integrating factor namely,

$$\delta K_{\mu\nu} = (1/N_{sat})\delta R_{\mu\nu}, \tag{15}$$

where, $K_{\mu\nu}$ is the Gauss-Bonnet mean curvature of the horizon or the holographic screen/surface. N_{sat} is the bit saturation. Bit saturation is the ratio of the number of bits required to describe the system, in the bulk of the spacetime including the bulk spacetime itself, to the number of bits available on the holographic screen. Hence, for quantum gravitational nature of the theory and for dimensional consistency, we insert the Planck length, λ_{Pl}, as

$$\beta = \frac{1}{\lambda_{Pl} N_{sat}}. \tag{16}$$

So,

$$\delta ln\Gamma_{\mu\nu} = \frac{1}{\lambda_{Pl} N_{sat}}\ \delta R_{\mu\nu}, \tag{17}$$

or

$$K_{\mu\nu} = \lambda_{Pl} ln\Gamma_{\mu\nu}. \tag{18}$$

According to Verlinde [4], $N_{sat} = -\frac{\phi}{2c^2}$ where ϕ represents the Newtonian gravitational potential. This N_{sat} is a positive number that vanishes at large distances. So, this coincides with the fact that the temperature also decreases with the expanding universe. Therefore, we extend the analogy with the third law of thermodynamics and propose that

(3) The Gauss-Bonnet mean curvature $K_{\mu\nu}$ of the evolving holographic surface tends to zero for pure gravity as the bit saturation N_{sat} tends to zero and becomes zero at zero bit saturation for the pure gravity situation.

Thus, we now have three laws of statistical geometrodynamics. The first law is the Einstein law of gravitation as given by the Einstein field equations. The second law connects the Ricci curvature (tensor) in the Einstein theory to the holographic screen/surface encompassing the Einstein bulk spacetime. And the third law is (more or less) purely about the holographic surface enclosing the Einstein bulk. Another statement for the third law of thermodynamics is that,

"in every irreversible thermodynamic process, the total entropy of the universe always increases". Similarly,

(3') In every irreversible geometrodynamic process, the Gauss-Bonnet mean curvature of the holographic screen bounding the bulk spacetime always increases.

Now, for empty spacetime, the number of bits available in the bulk will be constant and that on the holographic screen will also stay the same. So, we have the zeroeth law of geometrodynamics, viz.,

(0) The bit saturation for a system of empty spacetime bulk bounded by a holographic screen, is a constant.

Just as temperature is a concept that holds macroscopically and breaks down at the individual molecular/atomic level, the bit saturation is a concept that plays out a similar role. Now, temperature is defined as the average kinetic energy of all the particles in a thermodynamic system. At the particle level, there is the individual kinetic energy of the moving particles. For the whole bulk of spacetime and its enclosing Holographic screen, the bit saturation reduces to individual bits and thereby plays out an equivalent role. The definite physical boundaries of the object (matter) are blurred in the fine graining limit and vanish completely in the completely fine grained structure. Similarly, the spacetime manifold defined by the l.h.s. of (1) will also disappear at the quantum scales where the structure is sufficiently finely grained. Actually the ansatz (3) somehow seems to show a fundamental equivalence between quantum geometry and quantum matter and radiation by establishing an exact relationship between quanta of geometry and quanta of matter.

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