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Influence of Quintessential Matter on the Optical Properties of a Spherically Symmetric Black Hole

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Abstract: We study analytically the influence of a quintessential matter on the shadow of a black hole. Since we consider as starting point Kiselev's solution, this work is restricted to spherically symmetric and static situations where the shadow is circular. As particular examples we consider the Schwarzschild spacetime, Reissner-Nordstrom solution and, of course, the spherically symmetric black hole surrounded by quintessential matter. The perspectives of actually observing the influence of this kind of matter on the shadows of supermassive black holes are presented to find that very-long baseline interferometry (VLBI) in the near future may provide information about quintessence presence.

Keywords: Black Holes; Quintessential Matter; Kiselev's Solution; Schwarzschild Spacetime; Reissner-Nordstrom Solution; VLBI
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1 Introduction

The shadow of a black hole is to a two-dimensional dark zone as seen from an asymptotical observer. For a nonrotating black hole, the shadow is a perfect circle while for the rotating case, it has a deformed shape in the direction of the rotation due to the dragging effect [1, 2]. Today, two projects are under way to observe the shadow of Sgr A*, the supermassive black hole at the center of the Milky Way: the Event Horizon Telescope [3] and the BlackHoleCam [4]. This shadow is expected to have a diameter of 54 μarcsec and its observation would give important information about our galaxy. The projects use radio telescopes distributed over the Earth to obtain sub-millimeter Very Long Baseline Interferometry (VLBI) observations.

The optical properties and apparent shape of different rotating and non-rotating black

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holes have been investigated in the literature. One of the first studies, made by Hioki and Maeda [5], suggested that the spin parameter can be determined from the observation of the shadow of a Kerr black hole. Afterwards, several solutions were studied: black holes in string theory [6], non-commutative inspired solutions [7], braneworld rotating black holes [8, 9], black holes in extended Chern-Simons modified gravity [10], Kaluza-Klein solutions [11], the Einstein-Maxwell-Dilaton-Axion black hole [12], some regular black holes [13] and even rotating wormholes [14].

During the final decade of the XX century, astronomical observations of Supernovas type Ia suggested that the Universe has an accelerating expansion [15]. Cosmologists proposed different models in order to explain this behavior of the Universe and one of them is the existence of a dynamic scalar field (quintessence, phantoms or k-essence [16–18]). In this article we will work with the quintessence model, which incorporate a state parameter $\omega(t) = \frac{p}{\rho}$ (i.e. the ratio of pressure and energy density of the quintessence field) that may vary with time [19–21]. Recently, Kiselev [22] found a general solution of the Einstein field equations which recovers Schwarzschild and Reissner-Nrdstrom solutions as particular cases but also represents a static and spherically symmetric black hole surrounded with quintessence matter with a state parameter in the range $-1 < \omega < -\frac{1}{3}$. The value $\omega = -1$ can be identified with the description of dark matter as a cosmological constant Λ while the upper limit $\omega = -\frac{1}{3}$ incorporates a de-Sitter type horizon. Due to the interesting characteristic of this solution, in this paper we investigate the optical properties of this spacetime, calculating the influence of quintessence matter in the angular diameter of the shadow of a Kiselev's black hole.

2 Motion of Photons and Shadow of the Kiselev Black Hole

By solving Einstein's field equations, Kiselev [22] found a static, spherically symmetric solution surrounded by matter described by the line element

$$ds^2 = -\frac{\Delta}{r^2}dt^2 + \frac{r^2}{\Delta}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \tag{1}$$

with

$$\Delta = r^2 - 2Mr + Q^2 - \sigma r^{1-3\omega}. \tag{2}$$

M and Q are identified as the mass and the electric charge of the black hole, while σ is a normalization constant and ω is a state parameter describing the quintessence matter around the black hole, taking values in the range $-1 < \omega < -\frac{1}{3}$ and being related to the energy density through [22–24]

$$\rho = -\frac{\sigma}{2}\frac{3\omega}{r^{3(1+\omega)}}. \tag{3}$$

To determine the motion of the photons in the background given by this black hole, we consider the Hamilton-Jacobi equation,

$$\frac{\partial S}{\partial \lambda} = -\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^{\mu}} \frac{\partial S}{\partial x^{\nu}}, \tag{4}$$

where λ is an affine parameter along the geodesics, and S is the Jacobi action. Taking the rest mass of the particle moving in this spacetime as μ_0, the Hamilton-Jacobi equation can be separated and its solution can be written as

$$S = \frac{1}{2}\mu_0^2 \lambda - Et + L_z \phi + S_r(r) + S_\theta(\theta), \tag{5}$$

where S_r and S_θ are the radial and angular contributions, respectively. The constants of motion E and L_z are identified with the energy at infinity and the angular momentum in the φ direction, respectively. For photons we take $\mu_0 = 0$ and thus, equation (4) gives the null geodesics components in the r and θ directions as

$$r^2 \frac{dr}{d\lambda} = \pm\sqrt{V_{eff}(r)} \tag{6}$$

$$r^2 \frac{d\theta}{d\lambda} = \pm\sqrt{\Theta(\theta)} \tag{7}$$

where the effective potentials are

$$V_{eff}(r) = r^4 E^2 - \left(K + L_z^2\right)\Delta \tag{8}$$

and

$$\Theta(\theta) = K + L_z^2 - \frac{L_z^2}{\sin^2\theta} \tag{9}$$

with K the separation constant of the Hamilton-Jacobi equation. In order to obtain the shadow of the black hole, we study the radial motion equation,

$$r^2 \dot{r}^2 + V_{eff}(r) = 0, \tag{10}$$

where we use the constant *impact parameters* $\xi = \frac{L}{E}$ and $\eta = \frac{K}{E^2}$ to write

$$\frac{V_{eff}(r)}{E^2} = r^4 - \left(\eta + \xi^2\right)\Delta. \tag{11}$$

Since the boundary of the shadow of the black hole is determined by the circular photon orbits, we impose the conditions

$$\begin{cases} V_{eff}(r_0) & = 0 \\ \left.\frac{dV_{eff}}{dr}\right|_{r=r_0} & = 0 \end{cases} \tag{12}$$

leading to the equations

$$\begin{cases} r_0^4 - (\eta + \xi^2)\,\Delta(r_0) & = 0 \\ 4r_0^3 - (\eta + \xi^2)\left[2r_0 - 2M - \sigma(1 - 3\omega)\, r_0^{-3\omega}\right] & = 0 \end{cases}. \tag{13}$$

From these relations, we conclude that the parameters ξ and η satisfy the relation

$$\xi^2 + \eta = \frac{r_0^4}{r_0^2 - 2Mr_0 + Q^2 - \sigma r_0^{1-3\omega}}, \tag{14}$$

with r_0 the radius of the photon sphere given as the solution of the equation

$$r_0^2 - 3Mr_0 + 2Q^2 - \frac{3}{2}\sigma\left(1+\omega\right) r_0^{1-3\omega} = 0. \tag{15}$$

In order to obtain the shadow, we introduce a parameter space with the celestial coordinates defined as usual (a detailed derivation of these coordinates can be found in [25]),

$$\alpha = \lim_{r\to\infty}\left(-r^2 \sin\theta_0 \frac{d\varphi}{dr}\bigg|_{\theta\to\theta_0}\right) = -\xi\csc\theta_0, \tag{16}$$

$$\beta = \lim_{r\to\infty}\left(r^2 \frac{d\theta}{dr}\bigg|_{\theta\to\theta_0}\right) = \pm\sqrt{\eta - \xi^2\cot^2\theta_0}, \tag{17}$$

where θ_0 is the angle of inclination of the observer. These coordinates give the apparent position of the image in the plane that passes through the center of the black hole and that is orthogonal to the line joining the observer and the black hole [9]. In the (α, β) parameter space , the shadow of the black hole corresponds to the region not illuminated by the photon sources located at infinity and distributed uniformly in all directions and its boundary is given by a circle with radius a R determined by

$$\alpha^2 + \beta^2 = \xi^2 + \eta = \frac{r_0^4}{r_0^2 - 2Mr_0 + Q^2 - \sigma r_0^{1-3\omega}} = R^2. \tag{18}$$

Now we will consider some specific cases of Kiselev's solution to analyze the behavior of the shadow.

3 Schwarzschild and Reissner-Nordström Solutions

Reissner-Nordström black hole is recovered from metric (1) by taking $\sigma = 0$. From (15) one obtains the value

$$r_0^{RN} = \frac{3M + \sqrt{9M^2 - 8Q^2}}{2} \tag{19}$$

and hence, the radius of the shadow of this black hole is

$$R^{RN} = \frac{1}{2\sqrt{2}} \frac{\left(3M + \sqrt{9M^2 - 8Q^2}\right)^2}{\sqrt{15M^2 + 5M\sqrt{9M^2 - 8Q^2} - 2Q^2}}. \tag{20}$$

Taking $Q = 0$ we obtain Schwarzschild black hole, for which the radius of the circular orbit becomes the well known radius of the photon sphere, $r_0^{Sch} = 3M$ and the shadow's radius is simply [5]

$$R^{Sch} = 3\sqrt{3}M. \tag{21}$$

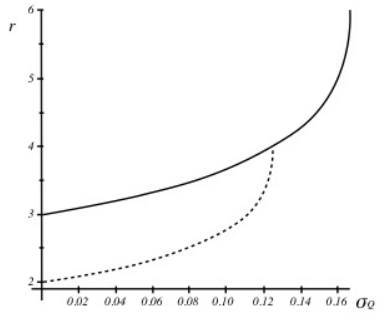

Fig. 1 Horizon radius (dotted curve) and radius of the photon sphere (continuous curve) for a non-rotating black hole surrounded by quintessence as function of the parameter σ_Q. We take units with $M = 1$.

4 The Spherically Symmetric Black Hole Surrounded by Quintessence

The special case of a spherically symmetric black hole surrounded by quintessence is obtained by setting $Q = 0$ and $\sigma = \sigma_Q$ in metric (1). We will consider the particular value $\omega = -\frac{2}{3}$ of the parameter describing quintessence, as done by several authors ([22–24]). Thus the metric describing spacetime has the function

$$\Delta = r^2 - 2Mr - \sigma_Q r^3, \tag{22}$$

where the parameter σ_Q has units of length^{-1}. From here, the radius of the event horizon is [23, 24]

$$r_H = \frac{1 - \sqrt{1 - 8M\sigma_Q}}{2\sigma_Q}. \tag{23}$$

from which one obtains a condition on the parameter σ_Q for its existence,

$$\sigma_Q \leq \frac{1}{8M}. \tag{24}$$

On the other hand, equation (15) gives this time the radius of the photon sphere as

$$r_0^Q = \frac{1}{\sigma_Q}\left(1 - \sqrt{1 - 6M\sigma_Q}\right) \tag{25}$$

and its existence needs the restriction

$$\sigma_Q \leq \frac{1}{6M}. \tag{26}$$

From Figure 1, it can be seen that the existence of the photon sphere in the outer region is restricted to values of $\sigma_Q \leq \frac{1}{8M}$. Since our interest is the shadow of the black hole, we will restrict our analysis to these values.

Using this result, the radius of the shadow of the black hole can be written as a function of the quintessence parameter σ_Q as

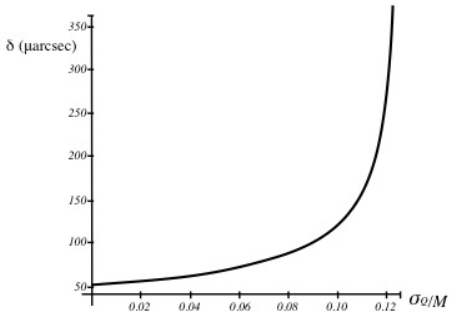

Fig. 2 Angular diameter δ of the circular shadow cast by Sgr A* as seen from Earth and modeling it as a non-rotating black hole surrounded by quintessence matter as function of the parameter σ_Q.

$$R_Q = \frac{1}{\sigma_Q}\sqrt{\frac{\left(1-\sqrt{1-6M\sigma_Q}\right)^3}{\sqrt{1-6M\sigma_Q}+4M\sigma_Q-1}}. \tag{27}$$

As a particular application consider our galactic center source, Sgr A*, which, according to references [26] and [27], has an estimated mass of $M \simeq 4.3 \times 10^6 M_\odot$ and is located at a distance of $D \simeq 8.3$ kpc. Using Synge's [28] treatment, one can obtain easily the angular radius of the shadow α as a function of the parameter σ_Q, which is depicted in Figure 2. Note that in vacuum, $\sigma_Q = 0$, we recover the well known result for a Schwarzschild's black hole candidate of $\alpha_{Sch} \simeq 27$ μarcsec or equivalently an angular diameter of the shadow of $\delta_{Sch} = 2\alpha_{Sch} \simeq 54$ μarcsec. This value is expected to be resolvable with Very Long Baseline Interferometry (VLBI) soon ([3, 4]). However, the existence of the quintessence matter around the black hole and increasing the value of the parameter σ_Q from 0 to its maximum value $\left(\frac{1}{8M}\right)$, produces a grow in the radius of the shadow.

Finally, we want to emphasize that in this work the gravitational field was not supposed to be weak and we have found that the presence of quintessence matter around the compact object leads to a shadow that is different from the predicted for Schwarzschild's black hole. A further work would be to generalize the analysis to the case of axially symmetric and stationary situations to include rotating black holes surrounded by an adequate mass-energy distribution. It is expected that the shadow will be no longer circular, and the presence of quintessence will have effects not only on the size of the shadow but also on its shape.

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