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Are Black Holes Compatible with a Varying G?

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Abstract: We investigate models where the gravitational constant G varies with time. Applying this to Black Holes, we conclude that given this circumstance, they cannot exist.

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1 Introduction

Cosmologies with varying G have been considered, e.g. in Brans-Dicke theory, Dirac cosmology and Hoyle-Narlikar theory and Sidharth's own 1997 model of fluctuational cosmology which correctly predicted an accelerating universe with dark energy and a small cosmological constant when the ruling Standard Big Bang model asserted the exact opposite. As is well known, this last was observed to be the case, the very next year by Perlmutter and others [1, 2, 3, 4]. Further, Sidharth has in a series of papers, shown that G can reproduce all results of General Relativity, from the bending of light, precession of the perihelion of Mercury right up to the shortening of the time periods of binary pulsars and so on [5, 6, 7]. In the above work, we have

$$\dot{G}/G \approx \left(1 - \frac{t}{T}\right) \tag{1}$$

(Throughout we consider CGS units). The result of observations from different fields like Paleontology, Geology and Astronomy, give [8]:

$$|\dot{G}|G| \leq 10^{-11}$$

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2 Effect on the Schwarzchild Radius

2.1 First approach

We know that the Schwarzchild radius of a black hole is given by

$$R_s = \frac{2GM_b}{c^2} \tag{2}$$

Now, as a first approximation we differentiate equation (2) with respect to time keeping M_b constant we obtain

$$\dot{R}_s = \frac{2\dot{G}M_b}{c^2} \tag{3}$$

From these two relations we have

$$\frac{\dot{R}_s}{R_s} = \frac{\dot{G}}{G}$$

Again, it has been found that [8]

$$\frac{\dot{G}}{G} < or \approx -10^{-11}$$

Let us denote, $\alpha = 10^{-11}$ and therefore, we have

$$\frac{\dot{R}_s}{R_s} = \frac{\dot{G}}{G} = -\alpha \tag{4}$$

which shows that R_s is exponentially decreasing with time. The physical interpretation for this decrement can be attributed to the *Hawking radiation*. Now, as a second approximation we take the mass M_b of the black hole as a variable such that

$$\dot{R}_s = \frac{2\dot{G}M_b}{c^2} + \frac{2G\dot{M}_b}{c^2} \tag{5}$$

Again, from Beckenstein's relation for a black hole we have

$$\mathrm{d}M_b = T_a \mathrm{d}(\frac{k_B c}{4\hbar G} A)$$

where, $A = 4\pi R_s^2$ is the area of the black hole. Therefore, we obtain

$$\frac{\mathrm{d}M_b}{\mathrm{d}t} = \beta[\frac{2R_s\dot{R}_s}{G} - \frac{R_s^2\dot{G}}{G^2}] \tag{6}$$

where, $\beta = \frac{1}{\hbar}T_a k_B c\pi$, and we have taken T_a the temperature of the black hole to be slowly varying with time ($\approx$ constant). Now, using the result obtained in the first approximation, namely $\frac{\dot{R}_s}{R_s} = \frac{\dot{G}}{G} = -\alpha$ we would get

$$\dot{M} = -\beta\frac{\alpha R_s^2}{G} \tag{7}$$

Now, using relations (2), (3) and (6) we obtain from equation (4)

$$\dot{R}_s = -\alpha R_s - 2\beta\alpha R_S^2 \tag{8}$$

Here, if we put $\frac{\mathrm{d}R_s}{\mathrm{d}t} = 0$ then we find the stationary point to be

$$R_s = -\frac{1}{2\beta}$$

Now, integrating this we have

$$\int_0^t \mathrm{d}t = -\frac{1}{\alpha}\int_{R_0}^{R} \frac{\mathrm{d}R_s}{R_S(2\beta R_s + 1)} \tag{9}$$

where, the moment when black hole is born with Schwarzchild radius R_0 is measured as $t = 0$ and after $t = 0$ this radius starts to decrease; $t > 0$ is the time when the initial Schwarzchild radius has decreased to R such that $R_0 \gg R$. Therefore, we obtain

$$t = \frac{1}{\alpha}[\ln|\frac{2\beta R + 1}{R}| - \ln|\frac{2\beta R_0 + 1}{R_0}|] \tag{10}$$

This can also be written as

$$t = \frac{1}{\alpha}[\ln(1 + \frac{1}{2\beta R}) - \ln(1 + \frac{1}{2\beta R_0})]$$

Since, $\beta \gg 1$ we will have $|\frac{1}{2\beta R}| \ll 1$ and thus we can write this as

$$t = \frac{1}{\alpha}[\sum_{n=1}^{\infty}\frac{1}{n}(\frac{1}{2\beta R})^n - \sum_{n=1}^{\infty}\frac{1}{n}(\frac{1}{2\beta R_0})^n]$$

Neglecting terms for $n \geq 2$ we obtain

$$t = \frac{1}{\alpha}[\frac{1}{2\beta R} - \frac{1}{2\beta R_0}] \tag{11}$$

Now, since $R_0 \gg R$ we obtain

$$t \approx \frac{1}{\alpha}\frac{1}{2\beta R} \tag{12}$$

If we consider a black hole with $R_0 > 10^{10} cms$ or, $R_0 \approx 10^{10} cms$ and $R \approx 10^{-5} cms$ just for the sake of fixing a number, then we find that

$$t \approx \frac{10^{-6}}{T_a}\ s$$

where, $\beta \approx 10^{22} T_a$, T_a being the temperature of the black hole. This result shows that even if a large black hole exists it will become microscopic in a few micro seconds from the moment of it's birth. Owing to this reason, a black hole is almost undetectable.

2.2 Second Approach

Now, in equation (2) if we consider M_b varying with time then we have

$$\dot{R}_s = \frac{2\dot{G}M_b}{c^2} + \frac{2G\dot{M}_b}{c^2} \tag{13}$$

Dividing both sides by $\frac{2GM_b}{c^2}$ we get

$$\frac{\dot{R}_s}{R_s} = \frac{\dot{G}}{G} + \frac{\dot{M}}{M}$$

$$\dot{R}_s = -\alpha R_s + \frac{\dot{M}}{M} R_s \tag{14}$$

Using equation (6) we have

$$\dot{R}_s = -\alpha R_s + \frac{\beta R_s^2}{GM}(2\dot{R}_s + \alpha R_s)$$

from whence we obtain

$$\dot{R}_s = -\alpha R_s + \frac{2\beta R_s}{c^2}(2\dot{R}_s + \alpha R_s) \tag{15}$$

This can also be written as

$$\dot{R}_s = -\alpha R_s \frac{1 - \frac{2\beta R_s}{c^2}}{1 - \frac{4\beta R_s}{c^2}}$$

Integrating this we have

$$\int \frac{1 - \frac{4\beta R_s}{c^2}}{R_s(1 - \frac{2\beta R_s}{c^2})} \mathrm{d}R_s = -\alpha \int \mathrm{d}t \tag{16}$$

$$\Rightarrow \quad \ln[\frac{2\beta R^2}{c^2} - R] = -\alpha t \tag{17}$$

where we have written R instead of R_s. Now, this relation is different from (12) since the approach taken is slightly different. The significance of equation (12) and (17) will be manifest in the next section where we show that black holes or the so-called point of singularity cannot be attained, as shown recently by Mersini-Houghton [10].
Now, let us consider equation (12)

$$t \approx \frac{1}{\alpha}\frac{1}{2\beta R}$$

Here, if we consider that $R = 0$ then we have

$$t = \infty \tag{18}$$

This implies that the life time (t) is infinite, which means that the point $R = 0$ or more explicitly the point of singularity is not reached by the collapsing star within a finite period of time.

Again, from equation (17) we can derive the quadratic equation

$$\frac{2\beta R^2}{c^2} - R = \exp(-\alpha t) \tag{19}$$

Solving this equation we have

$$R = \frac{1 \pm \sqrt{1 + \frac{8\beta}{c^2}\exp(-\alpha t)}}{\frac{4\beta}{c^2}} \tag{20}$$

Now, from this equation we find that when $R = 0$

$$\exp(-\alpha t) = 0$$

which gives $t = \infty$, the same as equation (18) and thus the life time becomes infinite. Evidently, we find that whatever approach we take the point of singularity is never reached within a finite time. This is consistent with the result obtained by Mersini-Houghton [10].

3 Conclusions

We have shown that given a time varying G, black holes cannot exist with a realistic lifetime. This conclusion has received support. The author (Sidharth) had shown over a decade ago that black holes with mass $< 10^5$ times the sun's mass would not have any realistic lifetime [4, 9]. More recently Hawking himself suggested that black holes would not have any event horizon while Mersini-Houghton [10] has shown that black holes cannot form in the first place, as the Hawking radiation would prevent any collapse [11, 12]. Finally, it may be mentioned that Greiner and Hess have concluded, using pseudo-complex space, that Black Holes do not exist [13]. We on the other hand have deduced that a time varying G is incompatible with Black Holes.

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