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Neutrino Oscillations and Relativistic Effect

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Abstract: In light of the recent neutrino oscillation data confirming the tiny neutrino masses, we analyze quantum-mechanically the behavior during motion of the neutrino state, that can be represented as a quantum superposition of the Hamiltonian eigenstates, and investigate the revival of the flavor of the neutrino during its propagation to show a relativistic effect that appears.

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1 Introduction

The notion of particle mass, especially for neutrino, is an actual problem of particle physics where neutrinos have been a field of wide studies [1, 2, 3]. In the standard model of the particle physics (SM), the neutrinos were considered as massless fermions. However experimental and theoretical research, mostly in the second half of the 20th century, brought arguments in favor of the possibility that the neutrino has a non-zero (though tiny) mass. First, the neutrinos were found to be of several types – flavors – electron flavor e, (first detected by Cowan and Reines, 1956) [4], muon flavor μ (Lederman, Schwartz and Steinberger, 1962) [5], and tauon flavor τ (1975). (There is an ongoing research and a dispute on whether a fourth type, so-called sterile neutrino exists, first indications of it reported in 2007 [6].) The probability of detecting a particular neutrino flavor varies periodically during its flight, phenomenon called flavor oscillation. After being predicted first by B. Pontecorvo in 1957 [7], this oscillation has been observed in many experiments with different neutrino sources. Indeed, evidences of neutrino oscillations have been recently confirmed implying the non-zero neutrino masses [8, 9, 10, 11, 12, 13]. These oscillations seem to find a good explanation in that one and the same flavor state is a

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quantum superposition of different energy states. Those different energy states, actually, hint about non-zero (though tiny) different neutrino masses [8, 9, 10]. That makes the behavior of the neutrinos during their propagation, an important tool for investigating various issues beyond the SM [11, 12, 13].
It is well known that the special relativity implies that the lifetime of an unstable system according to a coordinate frame in which the system moves, is longer than that the lifetime of the same system according to a coordinate frame in which the system is at rest, by the relativistic factor [2] [14, 15, 16, 17]. However, the internal dynamics of a system doesn't depend on the velocity of its centre of mass with respect to the observer. These principles are going in the text below.
In what follows, motivated by the very recently confirmed tiny neutrino masses, we start with a brief review of the quantum mechanical treatment of a neutrino in the presumed rest frame, then we extend that treatment to the relativistic motion, and finally we show that a usual relativistic effects emerges. It is worth mentioning that neutrinos generated in a weak interaction processes are intially in a flavour eigenstate [18], and in this text we consider neutrinos which intially have a well-defined flavour.

2 The flavor state as a superposition of rest-mass states

Let's therefore denote by $\left|\upsilon^{f}_{\overrightarrow{v}}\right\rangle$ the state of a neutrino υ of a neutrino with defined, $f = e, \mu, \tau$, and in movement with a velocity $\overrightarrow{v}$, and by $\left|\upsilon^{f}_{\overrightarrow{0}}\right\rangle$ the state of a neutrino with the same flavour in the frame of coordinates in which it is a rest. A state $\left|\upsilon^{f}_{\overrightarrow{v}}\right\rangle$ can be expanded in a quantum superposition of the eigenstate $\left|\upsilon_{\overrightarrow{v},m}\right\rangle$ of the Hamiltonian,

$$\widehat{H}\left|\upsilon_{\overrightarrow{v},m}\right\rangle = E\left|\upsilon_{\overrightarrow{v},m}\right\rangle, \tag{1}$$

With the energy E and linear momentum $\overrightarrow{p}$ of a neutrino are determined by the rest mass m_0 and the velocity $\overrightarrow{v}$,

$$\begin{aligned}\overrightarrow{p} &= m\overrightarrow{\beta},\\ E &= \sqrt{\overrightarrow{p} + m_0^2}.\end{aligned} \tag{2}$$

Considering that the relativistic mass of a particle of rest-mass m_0 is $m = \gamma m_0$, one has $E = m_0\sqrt{\gamma^2\beta^2 + 1 =}\gamma m$, s.t. the expressions 2 may have the additional form,

$$\begin{aligned}\overrightarrow{p} &= \gamma m_0\overrightarrow{\beta} = m\overrightarrow{\beta},\\ E &= \gamma m_0 = m.\end{aligned} \tag{3}$$

[2] We work here with units defined by the convention $c = \hbar = 1$. Thus, the ratio $\beta = v/c$ is numerically equal v. The relativistic factor $\gamma = (1 - \beta^2)^{-1/2}$.

The eigenstates$\left|\upsilon_{\overrightarrow{v},m}\right\rangle$ are normalized to δ Dirac,

$$\langle \upsilon_{\overrightarrow{v},m} \mid \upsilon_{\overrightarrow{v}',m'} \rangle = \delta_{\overrightarrow{v},\overrightarrow{v}',m,m'}. \tag{4}$$

Therefore we can write,

$$\left|\upsilon^{f}_{\overrightarrow{v}}\right\rangle = \sum_{m} U^{*}_{f,\overrightarrow{v},m} \left|\upsilon_{\overrightarrow{v},m}\right\rangle , \tag{5}$$

$$\left|\upsilon_{\overrightarrow{v},m}\right\rangle = \sum_{f} U_{f,\overrightarrow{v},m} \left|\upsilon^{f}_{\overrightarrow{v}}\right\rangle . \tag{6}$$

The amplitudes $U^{*}_{f,\overrightarrow{v},m}$ are the elements of the Pontecorvo–Maki–Nakagawa–Sakata matrix[3].
Analogously, in the rest frame $\overrightarrow{v} = \overrightarrow{0}$, s.t. the state of a neutrino initially produced with a determined flavor f, as said above, can be represented as,

$$\left|\upsilon^{f}_{\overrightarrow{0}}\right\rangle = \sum_{m_0} U^{*}_{f,\overrightarrow{0},m_0} \left|\upsilon_{\overrightarrow{0},m_0}\right\rangle , \tag{7}$$

$$\left|\upsilon_{\overrightarrow{0},m}\right\rangle = \sum_{f} U_{f,\overrightarrow{0},m_0} \left|\upsilon^{f}_{\overrightarrow{0}}\right\rangle . \tag{8}$$

The significance of the amplitude $U^{*}_{f,\overrightarrow{0},m_0}$, is that its absolute square gives the probability to find the mass, at a measurement in the rest frame of a neutrino of initial flavor f.

3 The flavor persistence probability of a moving neutrino

Now we consider a reference frame in which the neutrino moves with a fixed velocity $\overrightarrow{v}$. The Lorentz transformation $\widehat{L}$ applied to a neutrino of mass m0 changes the mass, and since the state$\left|\upsilon_{\overrightarrow{0},m_0}\right\rangle$ is an energy eigenstate, in the moving frame it becomes a plane wave.

$$\left|\upsilon^{f}_{\overrightarrow{v},m}\right\rangle = \widehat{L} \left|\upsilon_{\overrightarrow{0},m_0}\right\rangle = e^{i\left(\overrightarrow{p}\,\overrightarrow{x}-Et\right)} \left|\upsilon_{\overrightarrow{0},m_0}\right\rangle , \tag{9}$$

with $\overrightarrow{p}$ and E given by either 2 or 3. Applying the Lorentz transformation on both sides of the upper line in 6, then using the rightmost equality in 7,

$$\widehat{L} \left|\upsilon^{f}_{\overrightarrow{0}}\right\rangle = \sum_{m_0} U^{*}_{f,\overrightarrow{0},m} e^{i\left(\overrightarrow{p}\,\overrightarrow{x}-Et\right)} \left|\upsilon_{\overrightarrow{0},m_0}\right\rangle . \tag{10}$$

[3] Usually, it isconsidered as 3×3 matrix, however, its unitarity is not yet sure as long as it is not sure wether the sterile neutrino -the 4th flavor flavor exist [18].

Finally, for obtaining the flavor oscillations we substitute here the states $\left|\upsilon_{\overrightarrow{0},m_0}\right\rangle$ with their expressions in the second line of 6,

$$\widehat{L}\left|\upsilon_{\overrightarrow{0}}^{f}\right\rangle=\sum_{m_0,f'}U_{f,\overrightarrow{0},m_0}^{*}U_{f',\overrightarrow{0},m}^{*}e^{i\left(\overrightarrow{p}\overrightarrow{x}-Et\right)}\left|\upsilon_{\overrightarrow{0}}^{f'}\right\rangle. \tag{11}$$

The amplitude of still finding the initial flavor f after a time interval t from the generation of the neutrino, and at a distance x from the source, is

$$A^{f}\left(t,x\right)=\left\langle\upsilon_{\overrightarrow{0}}^{f}\right|A^{f}\left|\upsilon_{\overrightarrow{0}}^{f'}\right\rangle. \tag{12}$$

From 10 and 9 one easily gets,

$$A^{f}\left(t,x\right)=\sum_{m_0}\left|U_{f,\overrightarrow{0},m_0}^{*}\right|^{2}e^{i\left(\overrightarrow{p}\overrightarrow{x}-Et\right)}. \tag{13}$$

Thus, the probability of persistence of the flavor f is

$$P^{f}\left(t,x\right)=\sum_{m_0}\left|U_{f,\overrightarrow{0},m_0}^{*}\right|^{4}+4\sum_{\substack{m_0,m_0'\\ m_0'\succ m_0}}\left|U_{f,\overrightarrow{0},m_0}^{*}\right|^{2}\left|U_{f,\overrightarrow{0},m_0'}^{*}\right|^{2}\cos\left[\left(\overrightarrow{p}-\overrightarrow{p}\right)\overrightarrow{x}-\left(E-E'\right)t\right]. \tag{14}$$

At this point it's worthy to mention that according to experimental results it is unlikely that the neutrinos with different masses in the superposition 10 possess the same velocity [19]. A more acceptable proposal turned to be that the particles have the same linear momentum [19, 20]. In this case, the x-dependent term in 14 disappears.
It is useful in continuation to replace $\overrightarrow{p}$ and E with their expressions in 3,

$$P^{f}\left(t\right)=\sum_{m_0}\left|U_{f,\overrightarrow{0},m_0}^{*}\right|^{4}+2\sum_{\substack{m_0,m_0'\\ m_0'\succ m_0}}\left|U_{f,\overrightarrow{0},m_0}^{*}\right|^{2}\left|U_{f,\overrightarrow{0},m_0'}^{*}\right|^{2}\cos\left[\left(m-m'\right)t\right]. \tag{15}$$

Thus, if the intensity of the neutrino of a given flavor f can be monitored in time, the change in phases depends on the differences between the relativistic masses.
Finally, if the matrix $U_{f,\overrightarrow{0},m_0}^{*}$ is unitary, see footnote 2, one can easily derive from the equality,

$$\sum_{m_0}\left|U_{f,\overrightarrow{0},m_0}^{*}\right|^{2}=1, \tag{16}$$

the simpler form for 15,

$$P^{f}\left(t\right)=1-4\sum_{\substack{m_0,m_0'\\ m_0'\succ m_0}}\left|U_{f,\overrightarrow{0},m_0}^{*}\right|^{2}\left|U_{f,\overrightarrow{0},m_0'}^{*}\right|^{2}\sin^{2}\left[\left(\frac{m-m'}{2}\right)t\right]. \tag{17}$$

It is interesting to compare this result with its counterpart in the rest frame $\overrightarrow{v}=\overrightarrow{0}$. There, $\beta=0$ and $\gamma=1$, and denoting the proper time by τ,

$$P^{f}_{\overrightarrow{0}}(\tau) = 1 - 4 \sum_{\substack{m_0, m'_0 \\ m'_0 \succ m_0}} \left| U^{*}_{f,\overrightarrow{0},m_0} \right|^2 \left| U^{*}_{f,\overrightarrow{0},m'_0} \right|^2 \sin^2 \left[\left(\frac{m - m'}{2} \right) \tau \right] . \tag{18}$$

Obviously the difference between the formulas 17 and 18 is the Lorentz transformations of the time when passing from the rest frame to a frame in which the neutrino moves.

4 Conclusion

The results 15 and 18 don't contain terms of the form $m_0^2/2E$, which appear frequently in the literature in the phases of the oscillations [21]. Given that the rest-masses are extremely small, our expressions with differences between relativistic masses may be preferable, in experiments in which the time elapsed between the generation and the measurement of the neutrinos is short, over expressions which involve differences between squares of rest-masses [21]. The way on which the expressions with $m_0^2/2E$ are obtained contain a couple of approximations, $p \gg m$, the velocities of the neutrinos with different rest-masses are considered all equal and equal further with the light velocity, etc. At least the assumption of equality of the velocities is not correct.

Also, we place a question mark over the statement in [21]: "In practice, our neutrino will be extremely relativistic, so we will be interested in evaluating the phase factor . . . with $t \approx L$", where t is the time in the lab frame, and $x = L$ is the lab-frame position where the particle is found. If the linear momentum is well-defined, the particle's position is undefined. It is well-known that the point where the particle is detected is just the result of the wave-packet collapse, and doesn't imply that the particle followed a trajectory $\overrightarrow{x} = \overrightarrow{\beta} t$ with $\beta \approx 1$. Moreover, if one wishes to admit a classical trajectory, the formula is $\overrightarrow{x} = \overrightarrow{x}_{initial} + \overrightarrow{\beta} t$. One has $x = L$ the position found in the measurement, $\beta \approx 1$, but initial $\overrightarrow{x}_{initial}$ is not known, s.t. t is not known.

The phenomenon of neutrino oscillations, as has been recently confirmed, still deserves deeper investigations. It is not an easy task, because the interaction of the neutrino with matter is so weak, that whole tanks of liquid are needed for capturing it [22]. Next, it is not yet decided whether the sterile neutrino exists, s.t. it is not sure that the U^* matrix is 3×3 and not 4×4, fact which introduces additional difficulty in interpreting results.

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