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Quantum Transitions of Minimum Energy for Hawking Quanta in Highly Excited Black Holes: Problems for Loop Quantum Gravity?

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Abstract: By analysing some recent results by Yoon, which arise from loop quantum gravity and from the assumption of the locality of photon emission in a black hole, we argue that they are not consistent with our recent semi-classical results for highly excited black holes. Maybe that the results by Yoon can be correct for non-highly excited black holes, but, in any case, our analysis renders further problematical the match between loop quantum gravity and semi-classical theory.

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It has been suggested [1] that the Hawking radiation spectrum [2] could be discrete if one quantized the area spectrum in a way that the allowed area is the integer multiples of a single unit area. Indeed, the Hawking radiation spectrum looks to be continuous in loop quantum gravity if the area spectrum is quantized in such a way that there are not only a single unit area [3, 4].

Recently, by assuming the locality of photon emission in a black hole, Yoon argued that the Hawking radiation spectrum is discrete in the framework of loop quantum gravity even in the case that the allowed area is not simply the integer multiples of a single unit area [5]. Yoon's result arises from the selection rule for quantum black holes [5].

On the other hand, by analysing Hawking radiation as tunnelling, Parikh and Wilczek

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showed that the radiation spectrum cannot be strictly thermal [6, 7]. In fact, the energy conservation implies that the black hole contracts during the process of radiation[6, 7]. Thus, the horizon recedes from its original radius to a new, smaller radius [6, 7]. The consequence is that black holes cannot strictly emit thermally [6, 7]. This is consistent with unitarity [6] and has profound implications for the black hole information puzzle because arguments that information is lost during black hole's evaporation rely in part on the assumption of strict thermal behavior of the spectrum [6-8].

Working with $G = c = k_B = \hbar = \frac{1}{4\pi\epsilon_0} = 1$ (Planck units), the probability of emission is [2, 6, 7]

$$\Gamma \sim \exp(-\frac{\omega}{T_H}), \tag{1}$$

where $T_H \equiv \frac{1}{8\pi M}$ is the Hawking temperature and ω the energy-frequency of the emitted radiation.

Parikh and Wilczek released a remarkable correction, due to an exact calculation of the action for a tunnelling spherically symmetric particle, which yields [6, 7]

$$\Gamma \sim \exp[-\frac{\omega}{T_H}(1-\frac{\omega}{2M})]. \tag{2}$$

This important result, which takes into account the conservation of energy, enables a correction, the additional term $\frac{\omega}{2M}$ [6, 7]. We recently finalized the tunneling framework by Parikh and Wilczek in [20], by showing that the probability of emission (2) is correctly associated to the two distributions [20]

$$< n >_{boson} = \frac{1}{\exp\left[-4\pi n\left(M-\omega\right)\omega\right]-1}, \quad < n >_{fermion} = \frac{1}{\exp\left[-4\pi n\left(M-\omega\right)\omega\right]+1}, \tag{3}$$

for bosons and fermions respectively, which are *non* strictly thermal.

In various frameworks of physics and astrophysics the deviation from the thermal spectrum of an emitting body is taken into account by introducing an *effective temperature* which represents the temperature of a black body that would emit the same total amount of radiation [9]. The effective temperature can be introduced for black holes too [9]. It depends on the energy-frequency of the emitted radiation and is defined as [9]

$$T_E(\omega) \equiv \frac{2M}{2M-\omega}T_H = \frac{1}{4\pi(2M-\omega)}. \tag{4}$$

Then, eq. (2) can be rewritten in Boltzmann-like form [9]

$$\Gamma \sim \exp[-\beta_E(\omega)\omega] = \exp(-\frac{\omega}{T_E(\omega)}), \tag{5}$$

where $\beta_E(\omega) \equiv \frac{1}{T_E(\omega)}$ and $\exp[-\beta_E(\omega)\omega]$ is the *effective Boltzmann factor* appropriate for an object with inverse effective temperature $T_E(\omega)$ [9]. The ratio $\frac{T_E(\omega)}{T_H} = \frac{2M}{2M-\omega}$ represents the deviation of the radiation spectrum of a black hole from the strictly thermal feature [9]. If M is the initial mass of the black hole *before* the emission, and $M-\omega$ is the final mass of the hole *after* the emission [9], eqs. (2) and (4) enable the introduction of the *effective mass* and of the *effective horizon* [9]

$$M_E \equiv M - \frac{\omega}{2}, \; r_E \equiv 2M_E \tag{6}$$

of the black hole *during* the emission of the particle, i.e. *during* the contraction's phase of the black hole [9]. The *effective quantities* T_E, M_E and r_E are average quantities. M_E is the average of the initial and final masses, r_E is the average of the initial and final horizons and T_E is the inverse of the average value of the inverses of the initial and final Hawking temperatures (*before* the emission $T_{H\ \text{initial}} = \frac{1}{8\pi M}$, *after* the emission $T_{H\ \text{final}} = \frac{1}{8\pi(M-\omega)}$) [9]. Notice that the analysed process is *discrete* instead of *continuous* [9]. In fact, the black hole's state before the emission of the particle and the black hole's state after the emission of the particle are different countable black hole's physical states separated by an *effective state* which is characterized by the effective quantities [9]. Hence, the emission of the particle can be interpreted like a *quantum transition* of frequency ω between the two discrete states [9]. The tunnelling visualization is that whenever a tunnelling event works, two separated classical turning points are joined by a trajectory in imaginary or complex time [6, 9].

In [9] we used the concepts of effective quantities and the discrete character of Hawking radiation to argue a natural correspondence between Hawking radiation and black hole's quasi-normal modes. A problem concerning previous attempts to associate quasi-normal modes to Hawking radiation was that ideas on the continuous character of Hawking radiation did not agree with attempts to interpret the frequency of the quasi-normal modes [10]. In fact, the discrete character of the energy spectrum of black hole's quasi-normal modes should be incompatible with the spectrum of Hawking radiation whose energies are of the same order but continuous [10]. Actually, the issue that Hawking radiation is not strictly thermal and, as we have shown, it has discrete instead of continuous character, removes the above difficulty [9]. In other words, the discrete character of Hawking radiation permits to interpret black hole's quasi-normal frequencies in terms of energies of physical Hawking quanta too [9]. In fact, quasi-normal modes are damped oscillations representing the reaction of a black hole to small, discrete perturbations [9, 11-13]. A discrete perturbation can be the capture of a particle which causes an increase in the horizon area [11-13]. Hence, if the emission of a particle which causes a decrease in the horizon area is a discrete rather than continuous process, it is quite natural to assume that it is also a perturbation which generates a reaction in terms of countable quasi-normal modes [9]. This natural correspondence between Hawking radiation and black hole's quasi-normal modes permits to consider quasi-normal modes in terms of quantum levels not only for absorbed energies like in [11, 12, 13], but also for emitted energies like in [9]. This issue endorses the idea that, in an underlying unitary quantum gravity theory, black holes can be considered highly excited states [9, 11, 12, 13] and looks consistent with Yoon's approach in loop quantum gravity [5].

The intriguing idea that black hole's quasi-normal modes carry important information about black hole's area quantization is due to the remarkable works by Hod [11, 12]. Hod's original proposal found various objections over the years [9, 13] which have been answered in a good way by Maggiore [13], who refined Hod's conjecture. In [9] we further improve the Hod-Maggiore conjecture by taking into account the non-strict thermal character and, in turn, discrete rather than continuous character of Hawking radiation spectrum.

In particular, we found the solution for the absolute value of the quasi-normal frequencies in the case of large n (highly excited black hole) [9]

$$(\omega_0)_n = M - \sqrt{M^2 - \frac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2(n+\frac{1}{2})^2}}, \tag{7}$$

where n is the *quantum "overtone"number*, see [9] for details. Eq. (7) is based on Eq. (7) in [9], which is

$$\begin{array}{c} \omega_n = \ln 3 \times T_E(|\omega_n|) + 2\pi i(n+\frac{1}{2}) \times T_E(|\omega_n|) + \mathcal{O}(n^{-\frac{1}{2}}) = \\ \\ = \frac{\ln 3}{4\pi(2M-|\omega_n|)} + \frac{2\pi i}{4\pi(2M-|\omega_n|)}(n+\frac{1}{2}) + \mathcal{O}(n^{-\frac{1}{2}}). \end{array} \tag{8}$$

We intuitively derived eq. (8) in [9]. A rigorous analytical derivation of it can be found in the Appendix of [21].

We also found that, for large n, an emission involving the levels n and $n-1$ of a Schwarzschild black hole having an original mass M gives a variation of energy [9]

$$\begin{array}{c} \Delta E = (\omega_0)_n - (\omega_0)_{n-1} = f(M,n) \equiv \\ \\ \equiv \sqrt{M^2 - \frac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2(n-\frac{1}{2})^2}} - \sqrt{M^2 - \frac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2(n+\frac{1}{2})^2}}. \end{array} \tag{9}$$

The result of eq. (9) also hold for a Kerr black hole in the case $M^2 \gg J$, where J is the angular momentum of the black hole [22].The analysis in [5] shows that the Hawking radiation spectrum is truncated below a certain frequency and hence there is a minimum energy of an emitted particle [5]

$$E_{min} \approx \alpha T_H = \frac{\alpha}{8\pi M}, \tag{10}$$

where $\alpha = 1.49$ in the case of isolated horizon framework [14, 15], $\alpha = 2.46$ in the case of the Tanaka-Tamaki scenario [16] and $\alpha = 4.44$ in the case of the Kong-Yoon scenario [17-19].

Hence, by considering an excited black hole, we argue that they should exist values of n that we label as n_* for which

$$f(M, n_*) = E_{min}. \tag{11}$$

In other words, we search the minimum energy of an emitted particle for a fixed Hawking temperature which corresponds to two neighboring levels. In fact, if the two levels are not neighboring the emitted energy will be higher. Thus, n_* will be the values of n for which Hawking quanta having minimum energy can be emitted in emissions involving two neighboring levels (n_* and $n_* - 1$).

A black hole excited at a level $n_* - 1$ has a mass [9]

$$M_{n_*-1} \equiv M - (\omega_0)_{n-1} \tag{12}$$

which, by using eq. (7) becomes

$$M_{n_*-1} = \sqrt{M^2 - \frac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2(n_* - \frac{1}{2})^2}} \tag{13}$$

Considering eqs. (9), (10) and (11) one gets

$$\begin{aligned} &\sqrt{M^2 - \tfrac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2(n_* - \tfrac{1}{2})^2}} \approx \\ &\approx \sqrt{M^2 - \tfrac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2(n_* + \tfrac{1}{2})^2}} + \tfrac{\alpha}{8\pi\sqrt{M^2 - \frac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2(n_* - \frac{1}{2})^2}}}. \end{aligned} \tag{14}$$

For $n \gg 1$ eq. (14) is well approximated by

$$\begin{aligned} &\sqrt{M^2 - \tfrac{1}{2}n_* + \tfrac{1}{4}} \approx \\ &\approx \sqrt{M^2 - \tfrac{1}{2}n_* - \tfrac{1}{4}} + \tfrac{\alpha}{8\pi\sqrt{M^2 - \frac{1}{2}n_* + \frac{1}{4}}}. \end{aligned} \tag{15}$$

Putting

$$\begin{aligned} &M^2 - \tfrac{1}{2}n_* + \tfrac{1}{4} \equiv x \\ &\tfrac{\alpha}{8\pi} \equiv \beta \end{aligned} \tag{16}$$

eq. (15) becomes

$$\sqrt{x} \approx \sqrt{x - \frac{1}{2}} + \frac{\beta}{\sqrt{x}} \tag{17}$$

which can be simplified as

$$x - \beta \approx \sqrt{x - \frac{1}{2}}\sqrt{x}. \tag{18}$$

By squaring eq. (18) one easily solve for x:

$$x \approx \frac{2\beta^2}{4\beta - 1}. \tag{19}$$

Hence, by using eq. (16) one gets immediately

$$n_* \approx 2M^2 + \frac{1}{2} - \frac{\alpha^2}{8\pi\,(\alpha - 2\pi)}. \tag{20}$$

Thus, we find a sole value of n_* for which the Hawking radiation spectrum is truncated below a certain frequency of minimum energy for a transition between two neighboring levels n_* and $n_* - 1$ of an highly excited black hole. By using eqs. (20) and (13) the black hole's mass at the level $n_* - 1$ results

$$M_{n_*-1} = \sqrt{M^2 - \tfrac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2 (n_* - \tfrac{1}{2})^2}} \approx$$
$$\sqrt{M^2 - \tfrac{1}{2} n_* + \tfrac{1}{4}} \approx \sqrt{\tfrac{\alpha^2}{16\pi(\alpha - 2\pi)}} \tag{21}$$

We note that the mass (21) and its correspondent Hawking temperature

$$(T_H)_{n_*-1} \equiv \frac{1}{8\pi M_{n_*-1}} \approx \frac{\sqrt{\pi\,(\alpha - 2\pi)}}{2\pi\alpha} \tag{22}$$

are imaginary for $\alpha < 2\pi$. Then, they are imaginary also for $\alpha = 1.49$ (isolated horizon framework [14, 15]), $\alpha = 2.46$ (Tanaka-Tamaki scenario [16]) and $\alpha = 4.44$ (Kong-Yoon scenario [17-19]). This implies that that transitions between two neighboring levels look forbidden for highly excited black holes if one assumes the correctness of the analysis in [5]. Thus, in highly excited black holes, the minimum energy found by Yoon [5] should always correspond to transitions between two levels which are not neighboring.

Let us analyse this case in detail. A black hole excited at a level m has a mass [9]

$$M_m \equiv M - (\omega_0)_m \tag{23}$$

which, by using eq. (7) becomes

$$M_m = \sqrt{M^2 - \frac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2 m^2}}. \tag{24}$$

Considering two levels which are not neighboring, i.e. m and n with $n - m \geq 2$, eq. (7) implies that one needs the condition

$$\sqrt{M^2 - \tfrac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2 m^2}} \approx$$
$$\approx \sqrt{M^2 - \tfrac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2 n^2}} + \frac{\alpha}{8\pi\sqrt{M^2 - \frac{1}{4\pi}\sqrt{(\ln 3)^2 + 4\pi^2 m^2}}}. \tag{25}$$

For $m, n \gg 1$ eq. (25) is well approximated by

$$\sqrt{M^2 - \tfrac{1}{2} m} \approx$$
$$\approx \sqrt{M^2 - \tfrac{1}{2} n} + \frac{\alpha}{8\pi\sqrt{M^2 - \frac{1}{2} m}}. \tag{26}$$

Putting

$$M^2 - \tfrac{1}{2}m \equiv x$$

$$M^2 - \tfrac{1}{2}n \equiv y \tag{27}$$

$$\tfrac{\alpha}{8\pi} \equiv \beta, \quad x \geq 0, \quad y \geq 0$$

eq. (26) becomes

$$\sqrt{x} \approx \sqrt{y} + \frac{\beta}{\sqrt{x}} \tag{28}$$

which can be simplified as

$$x - \beta \approx \sqrt{y}\sqrt{x}. \tag{29}$$

By squaring eq. (29) and by dividing for x one gets

$$y \approx x + \frac{\beta^2}{x} - 2\beta. \tag{30}$$

By using eqs. (27) the condition $n - m \geq 2$ implies also $x - y \gtrsim 1$, which, in turn, gives

$$2\beta - \frac{\beta^2}{x} \gtrsim 1. \tag{31}$$

Eq. (31) is easily solved for x:

$$x \lesssim \frac{\beta^2}{2\beta - 1} = \frac{\alpha^2}{16\pi\left(\alpha - 4\pi\right)}, \tag{32}$$

which is not consistent with the third of eqs. (27) because the quantity $\frac{\alpha^2}{16\pi(\alpha-4\pi)}$ is always negative for $\alpha = 1.49$ (isolated horizon framework [14, 15]), $\alpha = 2.46$ (Tanaka-Tamaki scenario [16]) and $\alpha = 4.44$ (Kong-Yoon scenario [17-19]). In other words, the mass (24) results imaginary in this case too.

Hence, in this work we have shown that the results by Yoon [5], which arise from loop quantum gravity, are not consistent with our semi-classical results for highly excited black holes. Maybe the results in [5] can be correct for non-highly excited black holes, but, in any case, our analysis renders further problematical the match between loop quantum gravity and semi-classical theory.

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