Interacting and Non-interacting Two-Fluid Atmosphere for Dark Energy in FRW Universe

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Abstract: The evolution of the equation of state (EoS) parameter for dark energy (DE) within the scope of a spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) model filled with barotropic fluid and dark energy is examined. To get the deterministic solution we choose the scale factor $a(t) = -\frac{1}{t} + t^2$, $t > 1$ which yields a time dependent deceleration parameter (DP) $q = -2\left(\frac{t^3-1}{2t^3+1}\right)^2$, representing a model which generates an accelerating phase at the present epoch. We consider the two cases of an interacting and non-interacting two-fluid (barotropic and dark energy) scenario and obtained exact solution. It is observed that in both interacting and non-interacting cases, EoS parameter for DE is decreasing function of time and always varying in quintessence region for all open, closed and flat models. The cosmic jerk parameter in our derived models is also found to be in good agreement with the recent data of astrophysical observations under suitable conditions. The physical aspects of the model and stability of the corresponding solutions are also discussed.

Keywords: FRW Universe; Dark Energy; Variable Deceleration Parameter
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1. Introduction

It is now well-accepted in astrophysics that the observable universe is in a phase of rapid expansion whose rate of expansion is increasing, so called ‘accelerated expansion. This

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phenomenon is commonly termed as ‘dark energy (DE) in the literature, and suggests that a cosmic dark fluid possessing negative pressure and positive energy density. Although the phenomenon of dark energy in cosmic history is very recent $z \approx 0.7$, it has opened new areas in cosmology research. There is good evidence that a mysterious form of dark energy accounts for about two-third of matter and energy in the Universe. The direct evidence comes from distance measurements of type Ia Supernovae (SNe Ia) as standard candles which indicate the expansion of the universe is speeding up, not slowing down [1]−[4]. In addition, measurements of microwave background radiation [5] and the galaxy power spectrum [6], Sloan Digital Sky Survey [7, 8], WMAP [9] and Chandra X-ray observatory [10] also indicate the existence of dark energy. These observations have reopened the quest for the cosmological constant which was introduced by Einstein [11] in 1917, but later abandoned in 1931 and infamously cited as his greatest blunder [12]. The simplest candidate for the dark energy is the energy density of the quantum vacuum (or cosmological constant) for which $p = -\rho$. However, the inability of particle theories to compute the energy of the quantum vacuum - contributions from well understood physics amount to 10^{55} times critical density − casts a dark shadows on the cosmological constant [13]. For recent review, the readers are advised to see the references of Padmanabhan [14], Jassal et al. [15], Copeland et al. [16], Perivolaropoulos [17] and Mia et al. [18].

The idea of a late time accelerating universe is usually associated with unknown physical processes necessitating either new fields in high energy physics or modifications of gravity on very large scales. In following the former route, the predominant concept nowadays is the existence of a new component with sufficiently negative pressure, named dark energy [14, 19, 20], which is fully characterized by its equation of state (EoS), $\omega(t) = p_D(t)/\rho_D(t)$, where $p_D(t)$ and $\rho_D(t)$ are, respectively, the dark component pressure and energy density. The value $\omega = -1$ characterizes the vacuum energy (Λ), which is conceptually the simplest model (ΛCDM). Although these scenarios constitute a kind of benchmark model in the analysis of observational data, the large discrepancy between the theoretically predicted and “observed” values of Λ has incited the study of dynamical dark energy (DDE) models. In this case, Λ is treated as a dynamical quantity, whereas its constant EoS, $\omega = -1$, is preserved. This includes, among others, models based on renormalization group running Λ [21]−[24] and vacuum decay [25]−[28]. Other DE models, in which the potential energy density associated with a dynamical scalar field (ϕ) dominates the dynamics of the low-redshift Universe, have also been extensively discussed in the current literature (see, e.g., Ratra & Peebles [29]; Wetterich [30]; Caldwell et al. [31]). The EoS parameter in this class of models is necessarily a function of time and may take values > -1 [quintessence] or < -1 [phantom] (Caldwell [32]; Faraoni [33]; Alcaniz [34]). In this regard, it is still worth mentioning that some recent observational analysis [35], implying that a transition from $\omega > -1$ to $\omega < -1$ might have happened at low redshifts, have also insisted the development of models with a Λ boundary crossing (Nojiri & Odintsov [36]; Feng et al. [37]; Stefancic [38, 39]). The Quintom dark energy is a proposal that explains that the recent observations that mildly favor the EoS energy

ω crossing -1 near the past. An interacting two-fluid scenario for quintom dark energy was investigate by Xin [40].

The cosmological evolution of a two-field dilation model of dark energy was investigated by Liang et al. [41]. The viscous dark tachyon cosmology in interacting and non-interacting cases in non-flat FRW Universe was studied by Setare et al. [42]. Recently, several authors [43]−[54] have studied dark energy models in different context. In this paper we study the evolution of the dark energy parameter within the framework of a FRW cosmological model filled with two fluids (barotropic and dark energy) by considering the scale factor $a(t) = -\frac{1}{t} + t^2$ $(t > 1)$ and obtained more interesting results. The cosmological implications of this two-fluid scenario will be discussed in detail in this paper for both non-interacting and interacting cases. The out line of the paper is as follows: In Sect. 2, the metric and the basic equations are described. Sections 3 and 5 deal with non-interacting and interacting two-fluid models respectively and their physical significances. The cosmic jerk parameter is discussed in Sect. 4. Physical acceptability and the stability of corresponding solutions are analyzed in Sect. 6. Finally, conclusions are summarized in the last Sect. 7.

2. The Metric and Field Equations

We consider the spherically symmetric Friedmann-Robertson-Walker (FRW) metric as

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]. \tag{1}$$

Here $a(t)$ is the scale factor and the curvature constants k are $-1, 0, +1$ for open, flat and closed models of the universe respectively, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, where r, θ and ϕ are comoving coordinates.

The Einstein's field equations (with $8\pi G = 1$ and $c = 1$) read as

$$R_i^j - \frac{1}{2}R\delta_i^j = -T_i^j, \tag{2}$$

where the symbols have their usual meaning and T_i^j is the two fluid energy-momentum tensor consisting of dark field and barotropic fluid.

In a co-moving coordinate system, Einstein's field equations (2) for the line element (1) lead to

$$p_{tot} = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right), \tag{3}$$

and

$$\rho_{tot} = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right), \tag{4}$$

where $p_{tot} = p_m + p_D$ and $\rho_{tot} = \rho_m + \rho_D$. Here p_m and ρ_m are pressure and energy density of barotropic fluid and p_D & ρ_D are pressure and energy density of dark fluid respectively.

The Bianchi identity $G^j_{i;j} = 0$ leads to $T^j_{i;j} = 0$ which yields

$$\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}\left(\rho_{tot} + p_{tot}\right) = 0. \tag{5}$$

The EoS of the barotropic fluid and dark field are given by

$$\omega_m = \frac{p_m}{\rho_m}, \tag{6}$$

and

$$\omega_D = \frac{p_D}{\rho_D}, \tag{7}$$

respectively.

3. Non-interacting Two-Fluid Model

First, we consider that two-fluid do not interact with each other. Therefore, the general form of conservation equation (5) leads us to write the conservation equation for the dark and barotropic fluid separately as,

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\left(\rho_m + p_m\right) = 0, \tag{8}$$

and

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}\left(\rho_D + p_D\right) = 0. \tag{9}$$

Integration of (5) leads to

$$\rho_m = \rho_0 a^{-3(1+\omega_m)}, \tag{10}$$

where ρ_0 is an integrating constant. By using Eq. (10) in Eqs. (3) and (4), we first obtain the ρ_D and p_D in term of scale factor $a(t)$

$$\rho_D = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0 a^{-3(1+\omega_m)}. \tag{11}$$

and

$$p_D = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0\omega_m a^{-3(1+\omega_m)}. \tag{12}$$

The understanding of the global evolution of the observationally conformable universe, mathematically encoded in the dynamics of its scale factor a, is of utmost importance in explaining practically all cosmological phenomena. One of the most intriguing aspects of this evolution is the recently established late-time transition from a decelerated to an

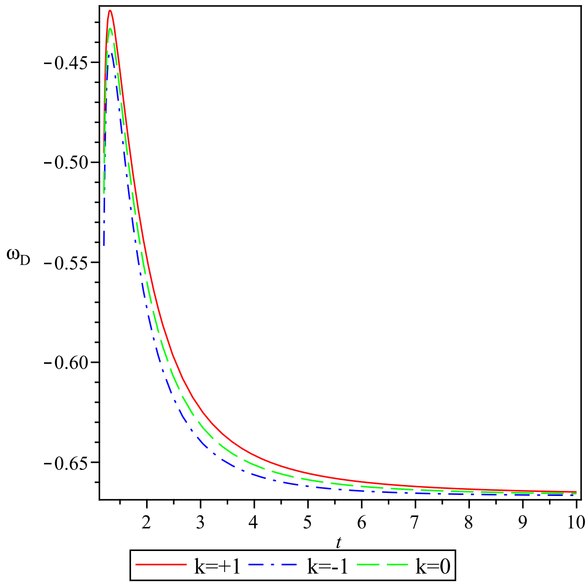

Fig. 1 The plot of EoS parameter ω_D vs. t for $\rho_0 = 1$ and $\omega_m = 0.5$ in non-interacting two-fluid model

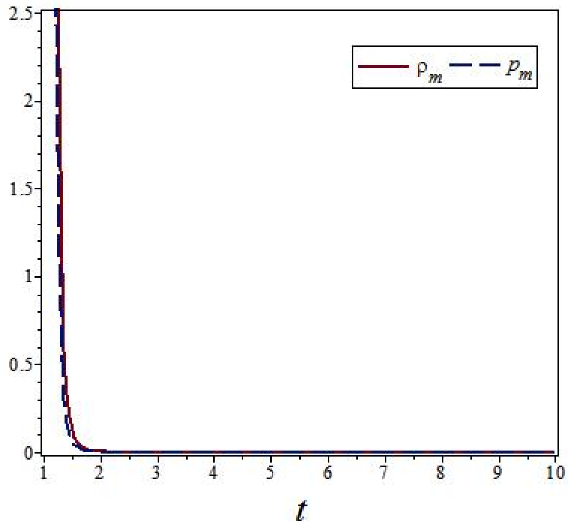

Fig. 2 The plot of ρ_m and p_m vs. t for $\rho_0 = 1$ and $\omega_m = 0.5$ in non-interacting two-fluid model

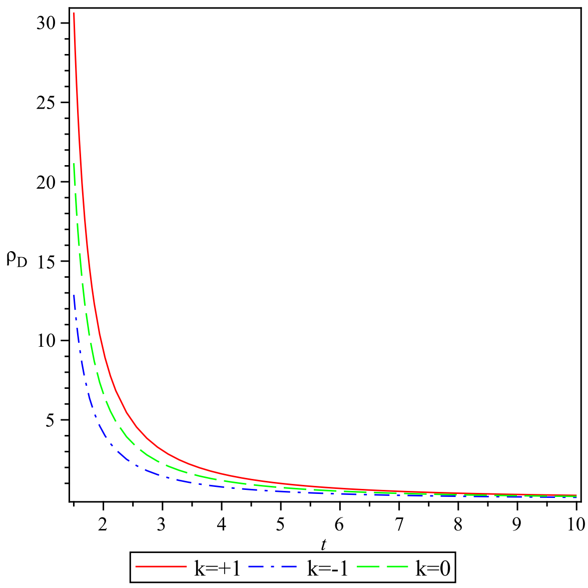

Fig. 3 The plot of ρ_D vs. t for $\rho_0 = 1$ and $\omega_m = 0.5$ in non-interacting two-fluid model

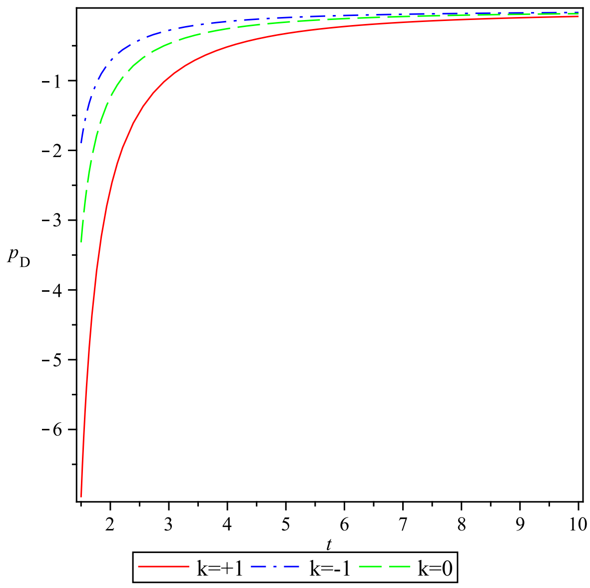

Fig. 4 The plot of p_D vs. t for $\rho_0 = 1$ and $\omega_m = 0.5$ in non-interacting two-fluid model

accelerating regime of the expansion of the Universe. Now we take following *ansatz* for the scale factor, where increase in term of time evolution

$$a(t) = -\frac{1}{t} + t^2, \quad t > 1. \tag{13}$$

Recently, the *ansatz* (13) is used by Pradhan [55] in studying accelerating dark energy models with anisotropic fluid in Bianchi type-VI_0 space-time. The relation (13) is also used by Pradhan et al. [56] to study Bianchi type-I cosmological models in scalar-tensor theory of gravitation. The motivation to choose such scale factor is behind the fact that the universe is accelerated expansion at present and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. So, in general, the DP is not a constant but time variable. By the above choice of scale factor yields a time dependent DP.

By using this scale factor in Eqs. (11) and (12), the ρ_D and p_D are obtained as

$$\rho_D = \frac{3}{(t^3-1)^2}\left[\frac{(1+2t^3)^2}{t^2} + kt^2\right] - \rho_0(-\frac{1}{t} + t^2)^{-3(1+\omega_m)}, \tag{14}$$

and

$$p_D = -\frac{1}{(t^3-1)^2}\left[\frac{4t^3(2t^3-1)+5}{t^2} + kt^2\right] - \rho_0\omega_m(-\frac{1}{t} + t^2)^{-3(1+\omega_m)}, \tag{15}$$

respectively. By using Eqs. (14) and (15) in Eq. (7), we find the equation of state of dark field in term of time as

$$\omega_D = -\left\{\frac{\frac{1}{(t^3-1)^2}\left[\frac{4t^3(2t^3-1)+5}{t^2} + kt^2\right] + \rho_0\omega_m(-\frac{1}{t} + t^2)^{-3(1+\omega_m)}}{\frac{3}{(t^3-1)^2}\left[\frac{(1+2t^3)^2}{t^2} + kt^2\right] - \rho_0(-\frac{1}{t} + t^2)^{-3(1+\omega_m)}}\right\}. \tag{16}$$

The behavior of EoS for dark energy in term of cosmic time t is shown in Fig. 1. It is observed that for open, closed and flat universes, the EoS parameter of DE ω_D is a decreasing function of time and always varying in quintessence region. The rapidity of its decrease at the early stage depends on the type the universe, while later on it tends to the same constant value independent to the types of universe.

The expressions for the matter-energy density Ω_m and dark-energy density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{t^2(t^3-1)^2}{3(1+2t^3)^2}\rho_0(-\frac{1}{t} + t^2)^{-3(1+\omega_m)}, \tag{17}$$

and

$$\Omega_D = \frac{\rho_D}{3H^2} = 1 + \frac{kt^4}{(1+2t^3)^2} - \frac{t^2(t^3-1)^2}{3(1+2t^3)^2}\rho_0(-\frac{1}{t} + t^2)^{-3(1+\omega_m)} \tag{18}$$

respectively. Adding Eqs. (17) and (18), we obtain density parameter

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{kt^4}{(1+2t^3)^2}. \tag{19}$$

The behavior of ρ_m and p_m in terms of cosmic time t are shown in Fig. 2. Both are positive decreasing function of time and converges to zero for sufficiently large times. Figure 3 depicts the variation of energy density of dark fluid ρ_D versus t. Here, we observe that ρ_D decreases as time increases in all the three open, closed and flat universes. Figure 4 depicts pressure p_D for dark fluid versus t. It is observed that p_D is always negative in all open, closed and flat universes and finally tends to zero as expected. It is worth to mention that since at $t = 1$, the scale factor a, total energy density ρ, and pressure p tend to infinity, one may consider this point as the analogous of the Big Bang in this model.

From the right hand side of Eq. (19), it is clear that in flat universe ($k = 0$), $\Omega = 1$ and in open universe ($k = -1$), $\Omega < 1$ and in closed universe ($k = +1$), $\Omega > 1$. But at late time we see for all flat, open and closed universes $\Omega \to 1$. This result is compatible with the observational results. Since our model predicts a flat universe for large times and the present-day universe is very close to flat, the derived model is also compatible with the observational results. The variation of density parameter with cosmic time has been shown in Fig. 5.

We define the deceleration parameter q as usual, i.e.

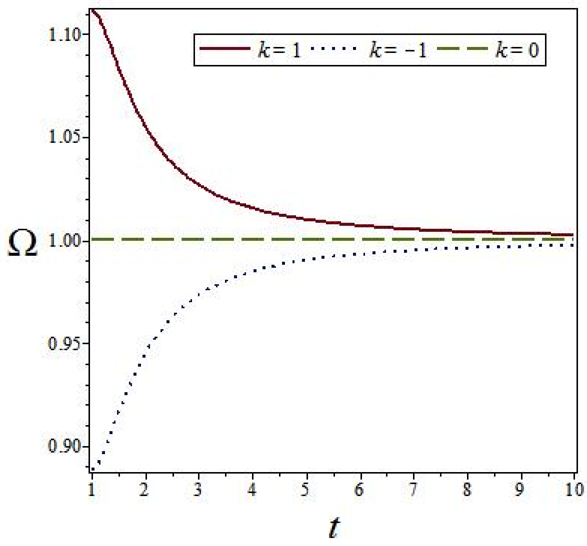

Fig. 5 The plot of density parameter Ω vs. t in non-interacting two-fluid model

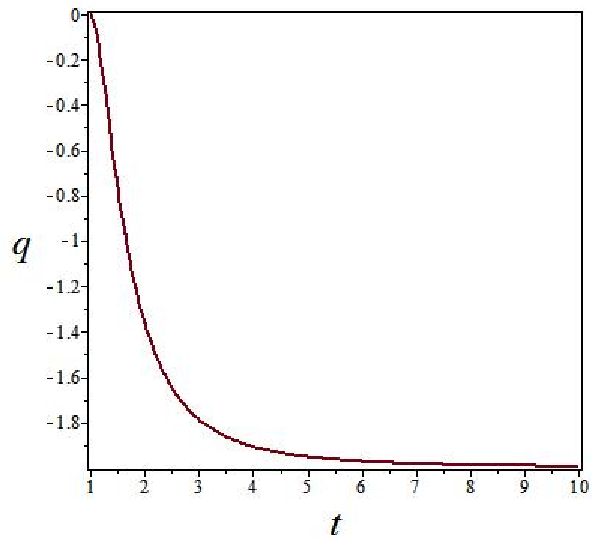

Fig. 6 The plot of deceleration parameter q vs. t

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}. \tag{20}$$

Using Eqs. (3) and (4), we may rewrite Eq. (20) as

$$q = \frac{1}{6H^2}\left[\rho_m(1 + 3\omega_m) + \rho_D(1 + 3\omega_D)\right]. \tag{21}$$

On the other hand, using Eq. (13) into Eq. (20), we find

$$q = -2\left(\frac{t^3 - 1}{2t^3 + 1}\right)^2. \tag{22}$$

Figure 6 is the plot of deceleration parameter q versus time t. From this figure we observe that q is always negative showing that the universe is accelerating at present epoch.

4. Cosmic Jerk Parameter

A convenient method to describe models close to Λ CDM is based on the cosmic jerk parameter j, a dimensionless third derivative of the scale factor with respect to the cosmic time. A deceleration-to-acceleration transition occurs for models with a positive value of j_0 and negative q_0. Flat Λ CDM models have a constant jerk $j = 1$. The jerk parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a}. \tag{23}$$

and in terms of the scale factor to cosmic time

$$j(t) = \frac{(a^2H^2)''}{2H^2}. \tag{24}$$

where the 'dots' and 'primes' denote derivatives with respect to cosmic time and scale factor, respectively. The jerk parameter appears in the fourth term of a Taylor expansion of the scale factor around a_0

$$\frac{a(t)}{a_0} = 1 + H_0(t - t_0) - \frac{1}{2}q_0H_0^2(t - t_0)^2 + \frac{1}{6}j_0H_0^3(t - t_0)^3 + O\left[(t - t_0)^4\right], \tag{25}$$

where the subscript 0 shows the present value. One can rewrite Eq. (23) as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H}. \tag{26}$$

Eqs. (22) and (26) reduce to

$$j(t) = 6\frac{(t^3 - 1)^2}{(2t^3 + 1)^3}. \tag{27}$$

This value overlaps with the value $j \simeq 2.16$ obtained from the combination of three kinematical data sets: the gold sample of type Ia supernovae (Riess et al. [57]), the SNIa data from the SNLS project (Astier et al. [58]), and the X-ray galaxy cluster distance measurements (Rapetti et al. [59]) at $t \simeq 0.5$ but as we have already mentioned our special choose of the scale factor restricts time t to be greater than 1. However, for $t > 1$, j is always less than 1 which is in agreement with the results given in Ref [60] in which it has been approved that for $\omega > -1$, the jerk parameter should be less than 1.

5. Interacting Two-Fluid Model

In this section, we consider the interaction between dark and barotropic fluids. For this purpose we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\left(\rho_m + p_m\right) = Q, \tag{28}$$

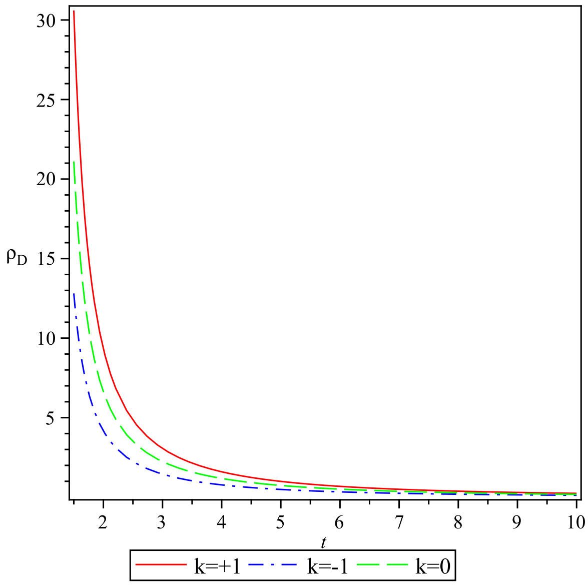

Fig. 7 The plot of ρ_D a Vs. t for $\rho_0 = 1$, $\omega_m = 0.5$ and $\sigma = 0.3$ in interacting two-fluid model

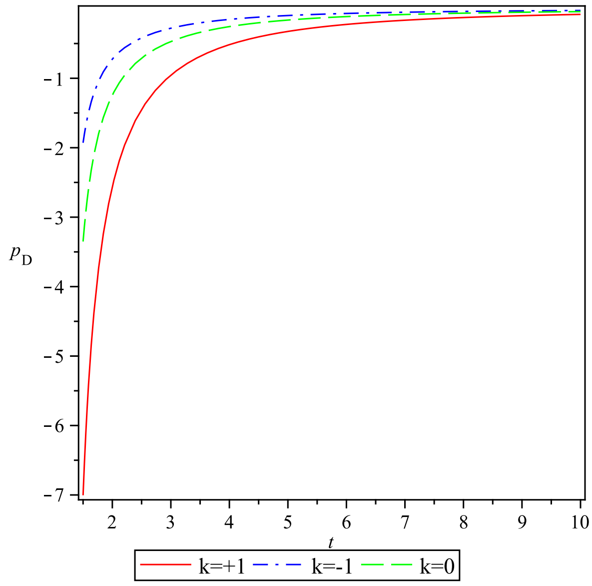

Fig. 8 The plot of p_D a Vs. t for $\rho_0 = 1$, $\omega_m = 0.5$ and $\sigma = 0.3$ in interacting two-fluid model

and

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}(\rho_D + p_D) = -Q. \tag{29}$$

The quantity Q expresses the interaction between the dark components. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q > 0$. $Q > 0$ ensures that the second law of thermodynamics stands fulfilled [61]. Here we emphasize that the continuity Eqs. (28) and (29) imply that the interaction term (Q) should be proportional to a quantity with units of inverse of time i.e $Q \propto \frac{1}{t}$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Following Amendola et al. [62] and Gou et al. [63], we consider

$$Q = 3H\sigma\rho_m, \tag{30}$$

where σ is a coupling constant. Using Eq. (30) in Eq. (28) and after integrating the resulting equation, we obtain

$$\rho_m = \rho_0 a^{-3(1+\omega_m-\sigma)}. \tag{31}$$

By using Eq. (31) in Eqs. (3) and (4), we again obtain the ρ_D and p_D in term of scale factor $a(t)$.

$$\rho_D = 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0 a^{-3(1+\omega_m-\sigma)}, \tag{32}$$

and

$$p_D = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \rho_0\omega_m a^{-3(1+\omega_m-\sigma)}, \tag{33}$$

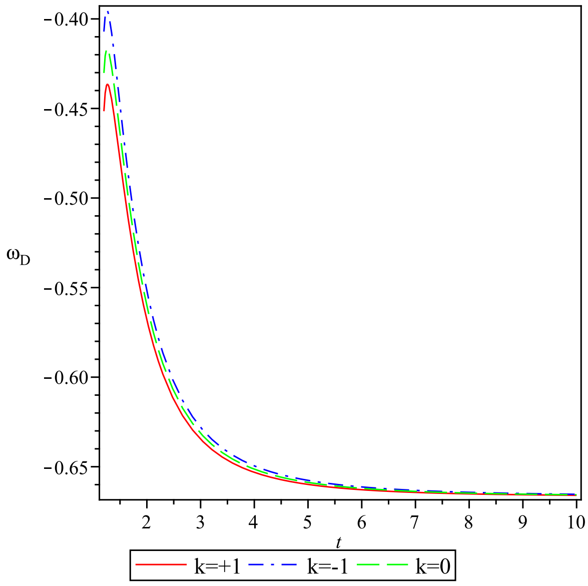

Fig. 9 The plot of EoS parameter ω_D vs. t for $\rho_0 = 1$, $\omega_m = 0.5$ and $\sigma = 0.3$ in interacting two-fluid model

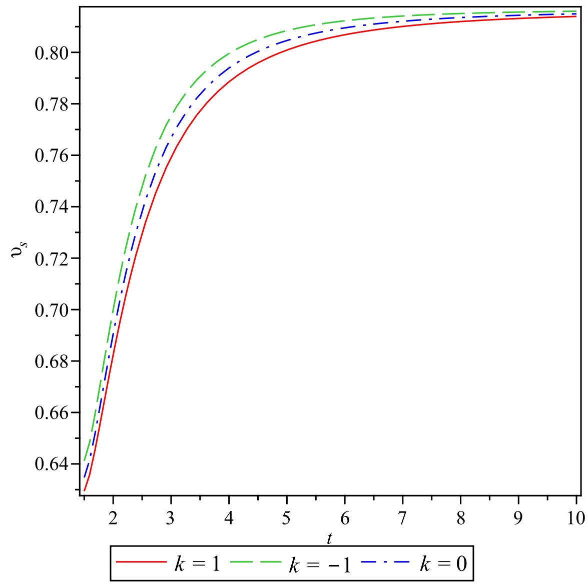

Fig. 10 The plot of sound speed υ_s vs. t for non-interacting two-fluid scenario

respectively. Putting the value of $a(t)$ from Eq. (13) in Eqs. (32) and (33), we obtain

$$\rho_D = \frac{3}{(t^3-1)^2}\left[\frac{(1+2t^3)^2}{t^2} + kt^2\right] - \rho_0(-\frac{1}{t} + t^2)^{-3(1+\omega_m-\sigma)}, \tag{34}$$

and

$$p_D = -\frac{1}{(t^3-1)^2}\left[\frac{4t^3(2t^3-1)+5}{t^2} + kt^2\right] - \rho_0\omega_m(-\frac{1}{t} + t^2)^{-3(1+\omega_m-\sigma)}, \tag{35}$$

respectively. Using Eqs. (34) and (35) in Eq. (7), we can find the EoS parameter of dark field as

$$\omega_D = -\left[\frac{\frac{1}{(t^3-1)^2}\left[\frac{4t^3(2t^3-1)+5}{t^2} + kt^2\right] + \rho_0\omega_m(-\frac{1}{t} + t^2)^{-3(1+\omega_m-\sigma)}}{\frac{3}{(t^3-1)^2}\left[\frac{(1+2t^3)^2}{t^2} + kt^2\right] - \rho_0(-\frac{1}{t} + t^2)^{-3(1+\omega_m-\sigma)}}\right]. \tag{36}$$

Figure 7 depicts the variation of energy density of dark fluid ρ_D versus t. Here, we observe that ρ_D decreases as time increases in all the three open, closed and flat universes. Figure 8 depicts pressure p_D for dark fluid versus t. It is observed that p_D is always negative in all open, closed and flat universes and finally tends to zero as expected.

The behavior of EoS in term of cosmic time t is shown in Fig. 9. It is observed that like the minimal coupling case, the EoS parameter is a decreasing function of time and always varying in quintessence region for all closed, open and flat universes, the rapidity of its decrease at the early stage depends on the type of universe. At the later stage of evolution it tends to the same constant value independent to the types of the universe.

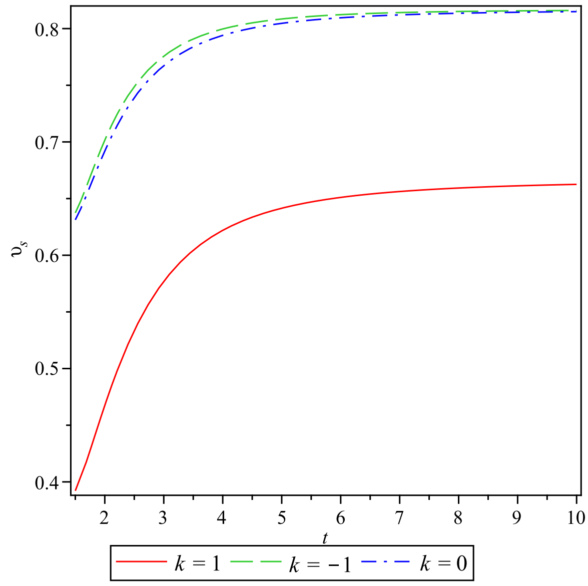

Fig. 11 The plot of sound speed υ_s vs. t for interacting two-fluid scenario

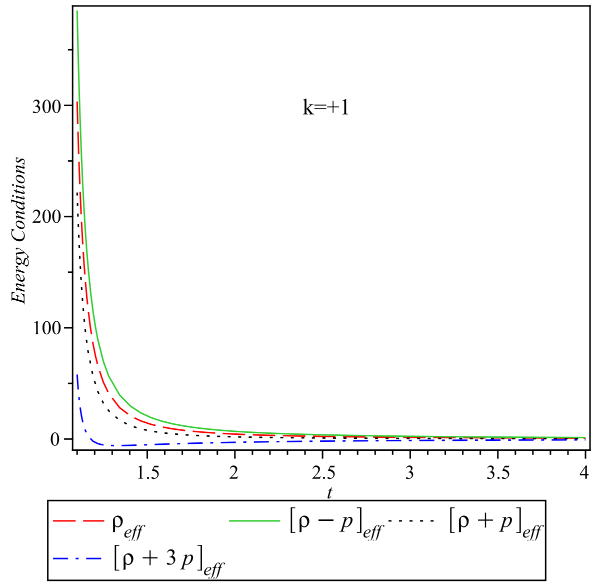

Fig. 12 The plot of energy conditions vs. t in non-interacting two-fluid model

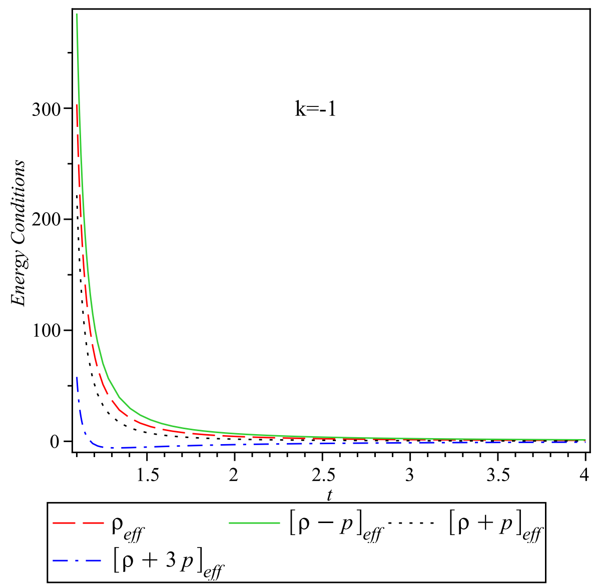

Fig. 13 The plot of energy conditions vs. t in non-interacting two-fluid model

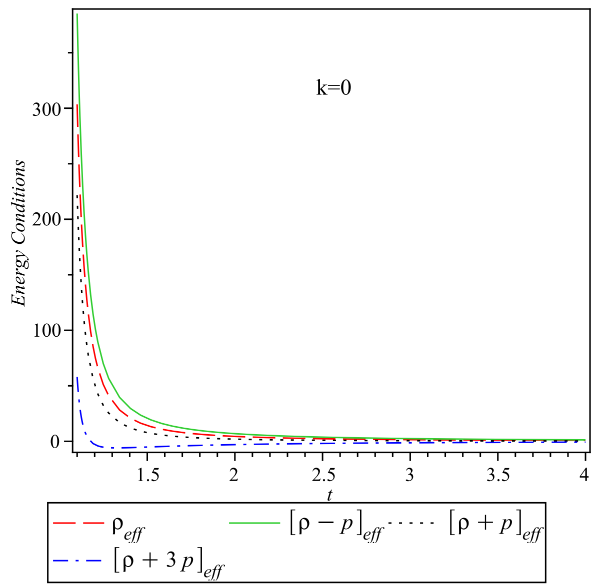

Fig. 14 The plot of energy conditions vs. t in non-interacting two-fluid model

The expressions for the matter-energy density Ω_m and dark-energy density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{t^2(t^3-1)^2}{3(1+2t^3)^2}\rho_0(-\frac{1}{t}+t^2)^{-3(1+\omega_m-\sigma)}, \tag{37}$$

and

$$\Omega_D = \frac{\rho_D}{3H^2} = 1 + \frac{kt^4}{(1+2t^3)^2} - \frac{t^2(t^3-1)^2}{3(1+2t^3)^2}\rho_0(-\frac{1}{t}+t^2)^{-3(1+\omega_m-\sigma)} \tag{38}$$

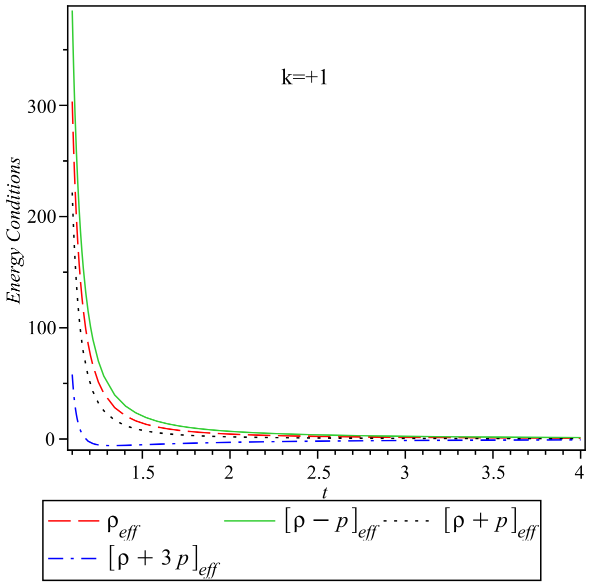

Fig. 15 The plot of energy conditions vs. t in interacting two-fluid model

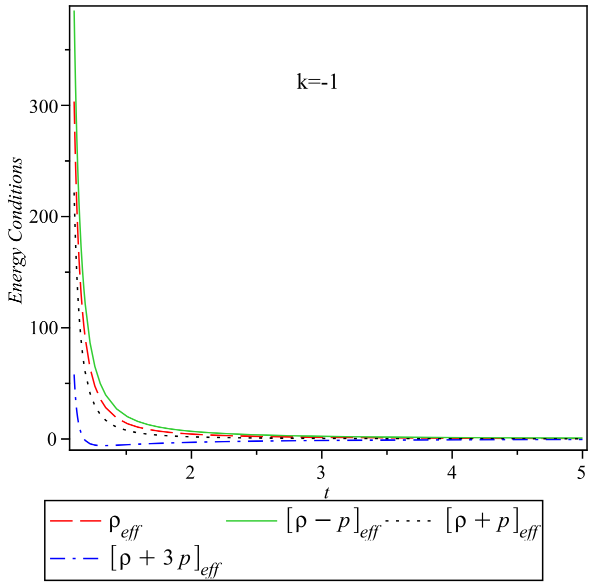

Fig. 16 The plot of energy conditions vs. t in interacting two-fluid model

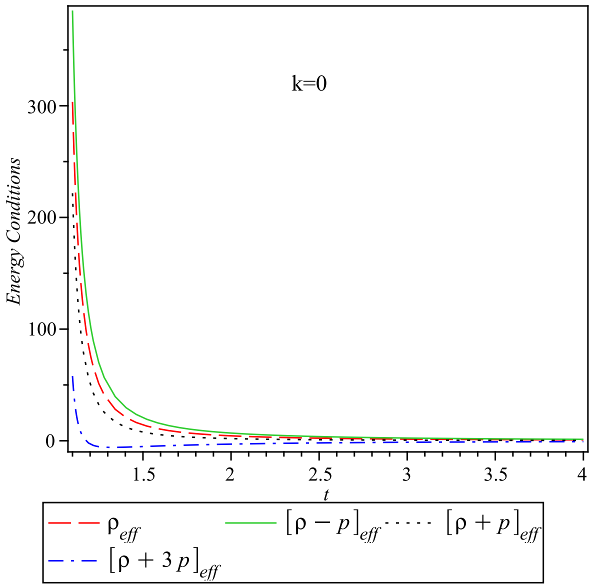

Fig. 17 The plot of energy conditions vs. t in interacting two-fluid model

respectively. From Eqs. (37) and (38), we obtain

$$\Omega = \Omega_m + \Omega_D = 1 + \frac{kt^4}{(1 + 2t^3)^2}, \tag{39}$$

which is the same as Eq. (19). Therefore, we observe that in interacting case the density parameter has the same properties as in non-interacting case. The expressions for deceleration parameter and jerk parameter are also same as in the case of non-interacting case.

Studying the interaction between the dark energy and ordinary matter will open a possibility of detecting the dark energy. It should be pointed out that evidence was recently provided by the Abell Cluster A586 in support of the interaction between dark energy and dark matter [64, 65].

6. Physical acceptability and stability of solutions

For the stability of corresponding solutions in both non-interacting and interacting models, we should check that our models are physically acceptable. For this, firstly it is required that the velocity of sound should be less than velocity of light i.e. within the range $0 \leq \upsilon_s = \left(\frac{dp}{d\rho}\right) \leq 1$.

In our non-interacting and interacting models, we obtained the sound speeds as

$$\upsilon_s = \frac{dp}{d\rho} = \frac{\frac{6(\frac{4t^3(2t^3-1)+5}{t^2}+kt^2)t^2}{(t^3-1)^3} - \frac{\frac{12t^2(2t^3-1)+24t^5}{t^2}-\frac{2(4t^3(2t^3-1)+5)}{t^3}+2kt}{(t^3-1)^2} + \frac{2.25(\frac{1}{t^2}+2t)}{(-\frac{1}{t}+t^2)^{5.5}}}{-\frac{18(\frac{(1+2t^3)^2}{t^2}+kt^2)t^2}{(t^3-1)^3} + \frac{3(12+24t^3-\frac{2(1+2t^3)^2+2kt}{t^3})}{(t^3-1)^2} + \frac{4.5(\frac{1}{t^2}+2t)}{(-\frac{1}{t}+t^2)^{5.5}}} \tag{40}$$

and

$$\upsilon_s = \frac{\frac{6(\frac{4t^3(2t^3-1)+5}{t^2}+kt^2)t^2}{(t^3-1)^3} - \frac{\frac{12t^2(2t^3-1)+24t^5}{t^2}-\frac{2(4t^3(2t^3-1)+5)}{t^3}+2kt}{(t^3-1)^2} + \frac{0.72(\frac{1}{t^2}+2t)}{(-\frac{1}{t}+t^2)^{4.6}}}{-\frac{18(\frac{(1+2t^3)^2}{t^2}+kt^2)t^2}{(t^3-1)^3} + \frac{3(12+24t^3-\frac{2(1+2t^3)^2+2kt}{t^3})}{(t^3-1)^2} + \frac{3.6(\frac{1}{t^2}+2t)}{(-\frac{1}{t}+t^2)^{4.6}}} \tag{41}$$

respectively. In both cases we observe that $\upsilon_s < 1$. From Figs. 10 & 11, we observe that in both non-interacting and interacting cases $\upsilon_s < 1$.

Secondly, the weak energy conditions (WEC) and dominant energy conditions (DEC) are given by

(i) $\rho_{eff} \geq 0$, (ii) $\rho_{eff} - p_{eff} \geq 0$ and (iii) $\rho_{eff} + p_{eff} \geq 0$.

The strong energy conditions (SEC) are given by $\rho_{eff} + 3p_{eff} \geq 0$.

Figures $12 - 14$ depict the variation of L.H.S. of WEC, DEC and SEC versus t for closed, open and flat models of the universe for non-interacting two-fluid scenario. Figures $15 - 17$ depict the variation of L.H.S. of WEC, DEC and SEC versus t for closed, open and flat models of the universe for interacting two-fluid scenario.

From the Figs. $12 - 17$, we observe that

- The WEC and DEC for the closed universe in both non-interacting and interacting cases are satisfied.
- In both open and flat models, the WEC and DEC are satisfied throughout the entire evolution of the universe.

- The SEC for both non-interacting and interacting cases is violated in early stages of the evolution of the universe whereas it satisfies at present epoch for all three open, closed and flat models.

Therefore, on the basis of above discussions and analysis, our corresponding solutions are physically acceptable.

A rigorous analysis on the stability of the corresponding solutions can be done by invoking a perturbative approach. Perturbations of the fields of a gravitational system against the background evolutionary solution should be checked to ensure the stability of the exact or approximated background solution [66]. Now we will study the stability of the background solution with respect to perturbations of the metric. Perturbations will be considered for all three expansion factors a_i via

$$a_i \rightarrow a_{Bi} + \delta a_i = a_{Bi}(1 + \delta b_i) \tag{42}$$

We will focus on the variables δb_i instead of δa_i from now on for convenience. Therefore, the perturbations of the volume scale factor $V_B = \Pi_{i=1}^3 a_i$, directional Hubble factors $\theta_i = \frac{\dot{a}_i}{a_i}$ and the mean Hubble factor $\theta = \sum_{i=3}^3 \frac{\theta_i}{3} = \frac{\dot{V}}{3V}$ can be shown to be

$$V \rightarrow V_B + V_B \sum_i \delta b_i, \quad \theta_i \rightarrow \theta_{Bi} + \sum_i \delta\dot{b}_i, \quad \theta \rightarrow \theta_B + \frac{1}{3}\sum_i \delta b_i \tag{43}$$

One can show that the metric perturbations δb_i, to the linear order in δb_i, obey the following equations

$$\sum_i \delta\ddot{b}_i + 2\sum \theta_{Bi}\delta\dot{b}_i = 0, \tag{44}$$

$$\delta\ddot{b}_i + \frac{\dot{V}_B}{V_B}\delta\dot{b}_i + \sum_j \delta\dot{b}_j\theta_{Bi} = 0, \tag{45}$$

$$\sum \delta\dot{b}_i = 0. \tag{46}$$

From above three equations, we can easily find

$$\delta\ddot{b}_i + \frac{\dot{V}_B}{V_B}\delta\dot{b}_i = 0, \tag{47}$$

where V_B is the background volume scale factor. In our case, V_B is given by

$$V_B = t^6. \tag{48}$$

Using above equation in equation (47) and after integration we get

$$\delta b_i = c_i t^{-5}, \tag{49}$$

where c_i is an integration constant. Therefore, the "actual" fluctuations for each expansion factor $\delta a_i = a_{Bi}\delta b_i$ is given by

$$\delta a_i \rightarrow c_i t^{-3}, \tag{50}$$

where $a_{Bi} \rightarrow t^2$. From above equation it is obvious that δa_i approaches zero as $t \rightarrow \infty$. Consequently, the background solution is stable against the perturbation of the graviton field.

7. Concluding Remarks

In this paper we have investigated the role of two fluid either minimally or directly coupled in the evolution of the dark energy parameter. It is found that in both non-interacting and interacting cases EoS parameter is a decreasing function of time and for flat, open and closed universes always varying in quintessence region. The rapidity of its decrease at the early stage depends on the type of universe. But at the later stage of evolution it tends to the same constant value independent to the types of the universe.

To get an accelerating model of the universe at present epoch, we have proposed a special scale factor which is given by Eq. (13). This special choice of scale factor yields a time dependent deceleration parameter (see Eq. (22)). The cosmic jerk parameter in our derived models is also found to be in good agreement with the recent data of astrophysical observations namely the gold sample of type Ia supernovae [57], the SNIa data from the SNLS project [58] and the X-ray galaxy cluster distance measurements [59].

Our proposed solutions are physically stable and acceptable. Thus, the solutions obtained in this paper may be useful for better understanding of the characteristic of DE in the evolution of universe within the framework of FRW.

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